Homework Set No. 1, Physics 834

Deadline – Wednesday, October 3, 2007

Jackson Problems 1.3, 1.4, 1.5 (each worth 10 points) along with the following one:

1. (10 pts) Delta-functions: similar to what we did in class, show that $\delta(x) = \lim_{\epsilon \to 0} \delta_\epsilon(x)$ by demonstrating that

(i) $\lim_{\epsilon \to 0} \delta_\epsilon(x) = 0$ for $x \neq 0$ and $\lim_{\epsilon \to 0} \delta_\epsilon(x = 0) = \infty$

(ii) $\lim_{\epsilon \to 0} \left[\int_{-\infty}^{\infty} dx\, f(x)\, \delta_\epsilon(x) \right] = f(0)$

for the following functions:

$$\delta_\epsilon(x) = \left\{ \frac{1}{\pi} \frac{\epsilon}{x^2 + \epsilon^2}, \quad \frac{2}{\pi} \frac{\sin^2(x/2\epsilon)}{x^2/\epsilon}, \quad \frac{e^{-|x|/\epsilon}}{2\,\epsilon} \right\} \tag{1}$$

You can use $f(x) = 1$ in (ii) for simplicity. Invent at least one more representation of the delta-function.