To differentiate one needs to remember that

$$\frac{\partial t_{ret}}{\partial t} = -\frac{\partial F/\partial t}{\partial F/\partial t_{ret}} = \frac{1}{1-\hat{n}\cdot\vec{\beta}}.$$

$$\frac{\partial R}{\partial t} = \frac{\partial t_{ret}}{\partial t}\frac{\partial R}{\partial t_{ret}} = \frac{1}{1-\hat{n}\cdot\vec{\beta}}\frac{(\vec{x}-\vec{r})\cdot(-\vec{V})}{R} =$$

$$= -\frac{\vec{R}\cdot\vec{V}}{R}\frac{1}{1-\hat{n}\cdot\vec{\beta}} = -\frac{\hat{n}\cdot\vec{V}}{1-\hat{n}\cdot\vec{\beta}} = -c\frac{\hat{n}\cdot\vec{\beta}}{1-\hat{n}\cdot\vec{\beta}}$$

as $t_{ret} = t - \frac{R}{c} \Rightarrow \vec{\nabla} t_{ret} = -\frac{1}{c}\vec{\nabla} R =$

$$= -\frac{1}{c}\frac{\vec{R}}{R} - \frac{1}{c}\frac{\partial R}{\partial t_{ret}}\vec{\nabla} t_{ret} \Rightarrow \vec{\nabla} t_{ret} = -\frac{\vec{R}}{cR(1-\vec{\beta}\cdot\hat{n})}$$

$$\Rightarrow \vec{\nabla} t_{ret} = \frac{-\hat{n}}{c(1-\hat{n}\cdot\vec{\beta})} = -\frac{1}{c}\vec{\nabla} R.$$

$\Rightarrow$ after straightforward (but tedious) calculations get

$$\boxed{\vec{E} = e\left[\frac{\hat{n}-\vec{\beta}}{\gamma^2(1-\vec{\beta}\cdot\hat{n})^3R^2}\right]_{ret} + \frac{e}{c}\left[\frac{\hat{n}\times[(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{(1-\vec{\beta}\cdot\hat{n})^3 R}\right]_{ret} \qquad \vec{B} = [\hat{n}\times\vec{E}]_{ret}}$$

first term ~ just due to velocity, "velocity field"

the 2nd term is due to acceleration $\Rightarrow$

$\Rightarrow$ "acceleration field"

$\Rightarrow$ if a particle moves with constant velocity $\Rightarrow$ 2nd term is absent and the 1st term can be obtained by a simple boost from the charge's rest frame, where one only has Coulomb $\vec{E}$ - field.

Power Radiated by an Accelerated Charge.

Imagine a non-relativistic motion, $|\vec{\beta}| \ll 1$, but with non-negligible acceleration: $|\dot{\vec{\beta}}| \sim$ large.

Radiation is given by the $\dot{\vec{\beta}}$ - term:

$$\vec{E}_{rad} = \frac{e}{c} \left[\frac{\hat{n} \times [(\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}}]}{(1 - \vec{\beta} \cdot \hat{n})^3 R} \right]_{ret} \approx \Big| \text{ as } |\vec{\beta}| \ll 1$$

$$\approx \frac{e}{c} \left[\frac{\hat{n} \times (\hat{n} \times \dot{\vec{\beta}})}{R} \right]_{ret}$$

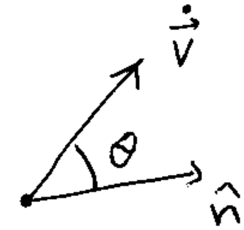
The Poynting vector is given by $(\vec{B} = \hat{n} \times \vec{E})$

$$\vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{B} = \frac{c}{4\pi} \vec{E} \times (\hat{n} \times \vec{E}) = \frac{c}{4\pi} \Big[\hat{n} \, |\vec{E}|^2 -$$

$$- \vec{E} \underbrace{(\hat{n} \cdot \vec{E})}_{=0} \Big] \Rightarrow \quad \vec{S} = \frac{c}{4\pi} \hat{n} \, |\vec{E}|^2 \quad \Rightarrow$$

for $\vec{E}_{rad}$.

$$\frac{dP}{d\Omega} = \frac{c}{4\pi} |\vec{E}|^2 R^2 = \frac{e^2}{4\pi c} \left|\left[\hat{n} \times (\hat{n} \times \dot{\vec{\beta}}) \right]\right|^2$$



$$\Rightarrow |\hat{n} \times (\hat{n} \times \dot{\vec{\beta}})|^2 = |\dot{\vec{\beta}}|^2 \sin^2\theta$$

$$\Rightarrow \frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3} |\dot{\vec{v}}|^2 \sin^2\theta$$

<u>Example:</u> imagine a charge oscillating along x-axis: $\vec{x}(t) = d \cdot \sin(\omega t)\, \hat{x} \Rightarrow$

$$\Rightarrow \vec{v} = + d\,\omega \cos(\omega t)\, \hat{x}\ , \quad \dot{\vec{v}} = - d\omega^2 \sin(\omega t)\, \hat{x}$$

$\Rightarrow$ time averaged $\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{e^2}{4\pi c^3} \frac{1}{2} d^2 \omega^4 \sin^2\theta$

dipole moment $|\vec{p}| = e d$, $k = \frac{\omega}{c} \Rightarrow$

$\Rightarrow \left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi} k^4 |\vec{p}|^2 \sin^2\theta$ ~ just dipole radiation, as expected!

$$P = \int d\Omega \frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3} |\dot{\vec{v}}|^2 \cdot 2\pi \cdot \frac{4}{3} = \frac{2}{3} \frac{e^2}{c^3} |\dot{\vec{v}}|^2$$

$$P = \frac{2}{3} \frac{e^2}{c^3} |\dot{\vec{v}}|^2$$

Larmor formula (non-relativistic limit)