Neutrino Masses and Oscillations

$\Rightarrow$ neutrinos have a mass (SNO, Super-K '03)

$\Rightarrow$ lepton number is not conserved (can have $\nu_e \to \nu_\mu$, etc.)
assume that $\Rightarrow$ may have ν's mixing

$\Rightarrow$ Similar to quarks have

mass eigenstates $\neq$ weak eigenstates
(ν_1, ν_2, ν_3) $(\nu_e, \nu_\mu, \nu_\tau)$

$\Rightarrow$ for simplicity, consider 2 generations:

$$\begin{pmatrix} \nu_\mu \\ \nu_e \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Pontecorvo '58, '68
Maki et al '62
Gribov & Pontecorvo '69

θ = mixing angle

Consider an EW process which produces either ν_e or ν_μ. Specifically, let's say ν_μ is produced:

$$|\nu_\mu(t=0)\rangle = \cos\theta\,|\nu_1(t=0)\rangle + \sin\theta\,|\nu_2(t=0)\rangle$$

Now, $|\nu_1(t)\rangle = e^{-iE_1 t}|\nu_1(0)\rangle$, $|\nu_2(t)\rangle = e^{-iE_2 t}|\nu_2(0)\rangle$

where $E_1 = \sqrt{p^2+m_1^2}$, $E_2 = \sqrt{p^2+m_2^2}$

$\Rightarrow$ assume a relativistic beam of neutrinos, all having the same momentum $\vec{p}$ $\Rightarrow$ plane wave factor $e^{i\vec{p}\cdot\vec{x}}$ is implied implicitly in all wave functions

Similarly, electron neutrino state is

$$|\nu_e(0)\rangle = -\sin\theta\,|\nu_1(0)\rangle + \cos\theta\,|\nu_2(0)\rangle.$$

After time t we have (for the muon neutrino state):

$$|\nu_\mu(t)\rangle = \cos\theta\,|\nu_1(t)\rangle + \sin\theta\,|\nu_2(t)\rangle =$$

$$= \cos\theta\, e^{-iE_1t}\,|\nu_1(0)\rangle + \sin\theta\, e^{-iE_2t}\,|\nu_2(0)\rangle.$$

Probability that the state $|\nu_\mu(t)\rangle$ is still a muon neutrino is

$$P(\nu_\mu\to\nu_\mu) = |\langle\nu_\mu(0)|\,\nu_\mu(t)\rangle|^2 = \left|\cos^2\theta\, e^{-iE_1t} + \sin^2\theta\, e^{-iE_2t}\right|^2$$

$$= \cos^4\theta + \sin^4\theta + \cos^2\theta\sin^2\theta\; 2\cos[(E_1-E_2)t]$$

$$= 1 + 2\sin^2\theta\cos^2\theta\left\{\cos[(E_1-E_2)t] - 1\right\}$$

$$= 1 - \sin^2 2\theta\, \sin^2\left(\frac{E_1-E_2}{2}t\right).$$

For small neutrino masses write $E_i = \sqrt{p^2+m_i^2} \simeq p + \frac{m_i^2}{2p}$

$$\Rightarrow E_1 - E_2 = \frac{m_1^2}{2E_1} - \frac{m_2^2}{2E_2} \simeq \frac{m_1^2-m_2^2}{2E}$$ as $E_1 \simeq E_2$ up to higher-order corrections

$$\boxed{P(\nu_\mu\to\nu_\mu) = 1 - \sin^2(2\theta)\,\sin^2\left(\frac{1.27\,\Delta m^2 L}{E}\right)}$$

with $\Delta m^2 \equiv m_2^2 - m_1^2 \left(\frac{eV^2}{c^4}\right)$, $L = tc$ (meters), E is in MeV.

as $P(\nu_\mu \to \nu_\mu) + P(\nu_\mu \to \nu_e) = 1 \Rightarrow$

$$\boxed{P(\nu_\mu \to \nu_e) = \sin^2(2\theta)\sin^2\left(\frac{1.27\,\Delta m^2 L}{E}\right)}$$

$\Rightarrow \nu_\mu$ can turn into ν_e & vice versa

$\Rightarrow$ neutrino oscillations!

Solar neutrino problem: # ν_e's from the Sun was ~3 times smaller than expected from solar models (w/o oscillations).

(Ray Davies '68 ~ experiment, John Bahcall '80 ~ Solar theory)

⇝ SNO experiment in 2003 measured ν_e and ν_μ from the sun: total # of neutrinos was in agreement with solar models $\Rightarrow$ oscillations!

⇝ see also Super-Kamiokande, KamLAND, Daya Bay.

$\Rightarrow$ for 3 neutrino flavors write:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

unitary

Common parameterization:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}\, e^{i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \cdot$$

$$\cdot \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \delta = \text{CP violating phase}$$

$$\theta_{12} \simeq 33^\circ, \quad \theta_{23} = 40^\circ, \quad \theta_{13} \simeq 8.7^\circ$$

$$\Delta m_{12}^2 = 8 \times 10^{-5}\, eV^2 \qquad \Delta m_{32}^2 \simeq 2.4 \times 10^{-3}\, eV^2$$

~ note large mixing angles

$\Rightarrow$ mass hierarchy has not been established for neutrinos (i.e., is it $m_1 < m_2 < m_3$ or $m_3 > m_2 > m_1$, or something else).