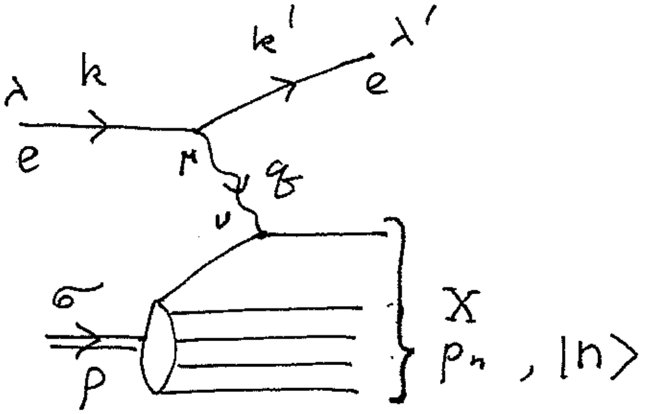
Parton Model and Deep Inelastic Scattering

DIS



$e(k) + \text{proton}(p) \rightarrow$

$\rightarrow e(k') + X$

Rest frame of the proton: $p = \overset{t\;\;x\;\;y\;\;z}{(m_p, 0, 0, 0)}$

$k = (\varepsilon, 0, 0, k) \simeq (\varepsilon, 0, 0, \varepsilon)$ (energy ~ many GeV, neglect m_e)

$k' = (\varepsilon', \varepsilon' \sin\Theta, 0, \varepsilon' \cos\Theta)$

Define:

$\rightarrow Q^2 \equiv -q^2 = -(k-k')^2 \simeq 2k\cdot k' = 2\varepsilon\varepsilon'(1-\cos\theta) =$

$= 4\varepsilon\varepsilon' \sin^2\frac{\theta}{2}$

$\nu \equiv \frac{p\cdot q}{m_p} = \varepsilon - \varepsilon'$ ← only in p's rest frame

← energy lost by e^-.

$\rightarrow x \equiv \frac{Q^2}{2p\cdot q} = \frac{Q^2}{2m_p\nu}$ Bjorken x variable

$\hat{s} \equiv (p+q)^2 \Rightarrow x = \frac{Q^2}{Q^2+\hat{s}}$

$x = \frac{Q^2}{2p\cdot q} = \frac{-q^2}{(p+q)^2-p^2-q^2} = \frac{Q^2}{\hat{s}-m_p^2+Q^2} \simeq \frac{Q^2}{\hat{s}+Q^2}$ if $Q^2 \gg m_p^2$

Q^2 and x are important / independent !

$\left(\nu = \frac{Q^2}{2xm} \right)$

Interaction amplitude:

$$T_{\sigma,\lambda,\lambda'}(n) = +ie\,\bar{u}_{\lambda'}(k')\,\gamma_\mu\,u_\lambda(k)\;\frac{-ig^{\mu\nu}}{q^2}\cdot$$

$$\cdot\, ie\,\langle n|\,j_\nu(0)\,|p,\sigma\rangle$$

where $j_\nu(x) = \sum_f e_f\,\bar{q}_f(x)\,\gamma_\nu\,q_f(x)$

with $e_f = +2/3,\ -1/3,\ \ldots$ (quark flavors) electric charges

j_ν is EM current

Let's calculate the cross-section $\left(\overset{|\vec{c}-\vec{0}|}{\overset{\shortparallel}{|\vec{v}_1-\vec{v}_2|}}=1\right)$:

$$d\sigma = \underset{\substack{\uparrow\\ \text{spin averaging}}}{\frac{1}{4}}\sum_{\sigma,\lambda,\lambda'}\sum_n |T_{\sigma,\lambda,\lambda'}(n)|^2\,(2\pi)^4\delta^4(q+p-p_n)\cdot\frac{d^3k'}{\underbrace{2m\;2\mathcal{E}}_{\text{incoming particles}}\;2\mathcal{E}'(2\pi)^3} = \frac{e^4}{Q^4}\cdot$$

$$\cdot\,\overbrace{\frac{1}{2}\sum_{\lambda,\lambda'}\left[\bar{u}_{\lambda'}(k')\,\gamma_\mu\,u_\lambda(k)\right]^*\left[\bar{u}_{\lambda'}(k')\,\gamma_\nu\,u_\lambda(k)\right]}^{l_{\mu\nu}}\cdot$$

$$\underbrace{\cdot\,\frac{1}{2}\sum_{\sigma,n}\langle n|\,j^\mu(0)\,|p,\sigma\rangle^*\,\langle n|\,j^\nu(0)\,|p,\sigma\rangle\cdot}_{\shortparallel\;4\pi m_p W^{\mu\nu}}$$

$$\cdot\,\underline{(2\pi)^4\,\delta(q+p-p_n)}\;\frac{d^3k'}{2m\,2\mathcal{E}\;\;2\mathcal{E}'}\;\frac{1}{(2\pi)^3}$$

Therefore

$$\boxed{\frac{d\sigma}{d^3k'} = \frac{\alpha_{EM}^2}{Q^4 \quad \varepsilon\cdot\varepsilon'} \quad l_{\mu\nu} \; W^{\mu\nu}}$$ (rest frame of proton)

$$l_{\mu\nu} = \frac{1}{2}\sum_{\lambda,\lambda'} \left[\bar{u}_{\lambda'\alpha}(k')\,(\gamma_\mu)_{\alpha\beta}\, u_{\lambda\beta}(k)\right]^* \bar{u}_{\lambda'\alpha'}(k')\,\cdot$$

see next page

$$\cdot(\gamma_\nu)_{\alpha'\beta'}\, u_{\lambda\beta'}(k) = \frac{1}{2}\sum_{\lambda,\lambda'} \bar{u}_\lambda(k)\,\gamma_\mu\, u_{\lambda'}(k')\,\cdot$$

$$\cdot\bar{u}_{\lambda'}(k')\,\gamma_\nu\, u_\lambda(k) = \frac{1}{2}\,\mathrm{Tr}\left[\gamma_\mu\;\gamma\cdot k'\;\gamma_\nu\;\gamma\cdot k\right]$$

as $\sum_{\lambda'} u_{\lambda'}(k')\,\bar{u}_{\lambda'}(k') = \gamma\cdot k' + m_e \simeq \gamma\cdot k'$

Using $\mathrm{Tr}\left[\gamma_\alpha\gamma_\beta\gamma_\mu\gamma_\nu\right] = 4\left[g_{\alpha\beta}\,g_{\mu\nu} + g_{\alpha\nu}\,g_{\beta\mu} - g_{\alpha\mu}\,g_{\beta\nu}\right]$

we get

$$\boxed{l_{\mu\nu} = 2\left(k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu}\, k\cdot k'\right)}$$

$$W_{\mu\nu} = \frac{1}{4\pi m}\,\frac{1}{2}\sum_{\sigma,n} \langle n|\,j^\mu(0)\,|p,\sigma\rangle^* \langle n|\,j^\nu(0)\,|p,\sigma\rangle\,\cdot$$

$$\cdot(2\pi)^4\,\delta^4(q+p-p_n) = \frac{1}{4\pi m}\,\frac{1}{2}\sum_{\sigma,n}\int d^4x\; e^{iq\cdot x}\,\cdot$$

$$\cdot\langle p,\sigma|\,\underset{\underset{\displaystyle e^{ip\cdot x}\, j^\mu(0)\, e^{-ip\cdot x}}{\parallel}}{j^\mu(x)}\,|n\rangle\,\langle n|\,j^\nu(0)\,|p,\sigma\rangle$$

(Heisenberg picture)

$$\left[\bar{u}_{\lambda'}(k')\gamma^\mu u_\lambda(k)\right]^* \underset{\uparrow}{=} \left[u^+_{\lambda'}\gamma^0\gamma^\mu u_\lambda\right]^+ =$$

as this is a scalar

$$= u^+_\lambda \underbrace{(\gamma^0)^2}_{\mathbb{1}} \gamma^{+\mu}\gamma^{\overset{+}{0}} u_{\lambda'} = \bar{u}_\lambda(k)\gamma^0\gamma^{+\mu}\gamma^{+0}u_{\lambda'}$$

now, $\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \Rightarrow \gamma^{+0} = \gamma^0 \Rightarrow \gamma^0\gamma^{+\mu}\gamma^{+0} = \gamma^0\gamma^{+\mu}\gamma^0$

Let's find $\gamma^0\gamma^{+\mu}\gamma^0$:

$\mu = 0 \Rightarrow \gamma^0\gamma^{+0}\gamma^0 = \gamma^0$

$\mu = i \Rightarrow \gamma^0\gamma^{+i}\gamma^0 = -\gamma^0\gamma^i\gamma^0 = \gamma^i(\gamma^0) =$

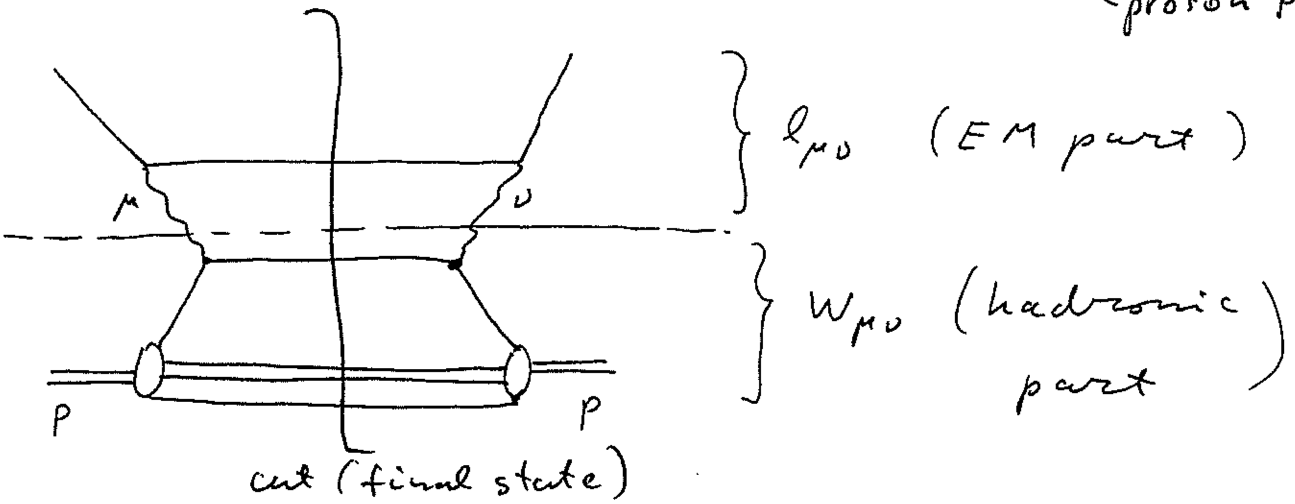
$= \gamma^i$

$$\Rightarrow \boxed{\gamma^0\gamma^{+\mu}\gamma^0 = \gamma^\mu}$$

$\Rightarrow$ get $\bar{u}_\lambda(k)\gamma^\mu u_{\lambda'}(k')$ as desired.

$$\boxed{W_{\mu\nu} = \frac{1}{4\pi m_p}\int d^4x\, e^{iq\cdot x}\langle p|\, j_\mu(x)\, j_\nu(0)\,|p\rangle}$$

(averaged over σ) ← proton polarization



$$W_{\mu\nu}(p,q) = A(x,Q^2)\, p_\mu p_\nu + B(x,Q^2)\, q_\mu q_\nu + C(x,Q^2)\, g_{\mu\nu} +$$

$$+ D(x,Q^2)\,(p_\mu q_\nu + p_\nu q_\mu) + E(x,Q^2)\,(p_\mu q_\nu - p_\nu q_\mu) +$$

$$+ F(x,Q^2)\; \varepsilon_{\mu\nu\rho\sigma}\, p^\rho q^\sigma$$

$F = 0$ in $\gamma^* p$, $\gamma^* A$ (F comes from γ_5's, appears in ν DIS).

(1) $q_\mu W^{\mu\nu} = 0$ (current conservation)

$q_\mu \to \partial_\mu \Rightarrow \partial_\mu j^\mu(x) = 0$

$$A\, p_\nu\,(p\cdot q) + B\, q_\nu \cdot q^2 + C\, q_\nu + D\,(p\cdot q\, q_\nu + q^2 p_\nu)$$

$$+ E\,(p\cdot q\;\; q_\nu - q^2 p_\nu) = 0$$

(2) $$q_\nu W^{\mu\nu} = 0 = A\, p_\mu\,(p\cdot q) + B\, q^2 q_\mu \overset{+Cq_\mu}{\vee} + D(p\cdot q\;\, q_\mu +$$

$$+ q^2 p_\mu) + E\,(p_\mu q^2 - p\cdot q\; q_\mu) = 0$$