Last time

DIS

$$\boxed{\begin{aligned}\frac{d\sigma}{d^3k'} &= \frac{4\alpha_{EM}^2}{Q^4}\left[2W_1 \sin^2\frac{\theta}{2} + W_2 \cos^2\frac{\theta}{2}\right] \\ &= \frac{4\alpha_{EM}^2}{Q^4}\left[\frac{2}{m_p}F_1(x,Q^2)\sin^2\frac{\theta}{2} + \frac{2m_p x}{Q^2}F_2(x,Q^2)\cos^2\frac{\theta}{2}\right]\end{aligned}}$$

W_1, W_2 = structure functions

F_1, F_2 = — " — , $F_1 = m_p W_1$

$F_2 = \nu W_2$

The Parton Model (cont'd)

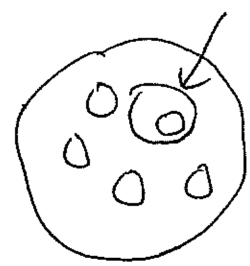
Infinite momentum Frame:

$$\begin{array}{cccccc} & t & x & y & z & \\ p^\mu \simeq & (p + \frac{m^2}{2p}, & 0, & 0, & p) & \end{array}$$

$$\begin{array}{ccccc} & t & x, y & z & \\ q^\mu = & (q^0, & q_\perp, & 0) & \end{array}$$

$p \gg Q$, p is very large

$\Delta x_\perp \sim \frac{1}{Q}$



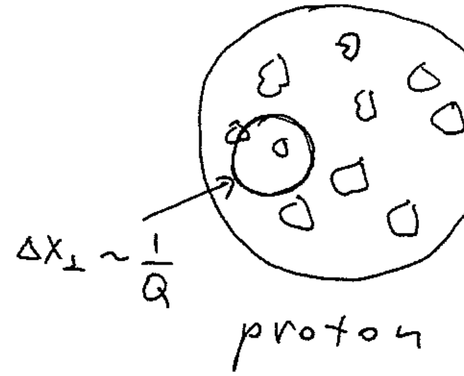
$q^0 = \frac{Q^2}{2xp} \ll q_\perp \Rightarrow \boxed{Q^2 \simeq -q_\perp^2} \Rightarrow$

$\frac{1}{Q} = \Delta x_\perp \sim$ distance probed by γ^* in DIS

$Q^2 = \not{q}^2 \Rightarrow$ photon acts like a <u>microscope</u> in <u>transverse</u> plane:

$\Delta x_\perp \cdot q_\perp \sim 1 \qquad (\hbar = 1)$

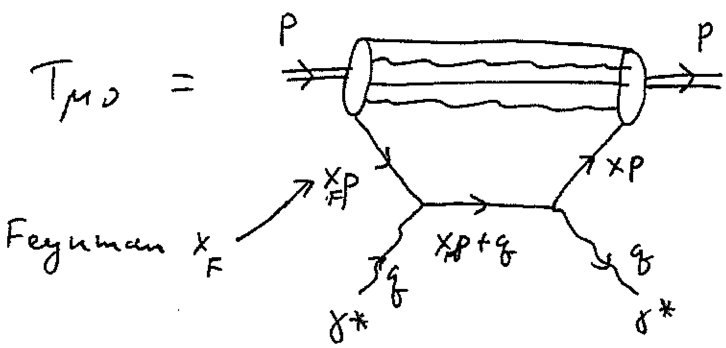
$\Delta x_\perp \sim \frac{1}{q_\perp} \sim \frac{1}{Q}$



large Q ~ resolve just 1 quark

Define $$T_{\mu\nu} = \frac{1}{4\pi m_p} \int d^4x \, e^{iq\cdot x} \, \frac{1}{2}\sum_\sigma \langle p,\sigma | T j_\mu(x) j_\nu(0) | p,\sigma\rangle$$

$W_{\mu\nu} = 2\,\mathrm{Im}\,(i\,T_{\mu\nu})$ (optical theorem)

$T_{\mu\nu} =$ 

"Forward Amplitude"

Def Feynman-x: the fraction of proton's longitudinal momentum carried by struck quark

typical interaction time in proton's rest frame is $\frac{1}{\Lambda_{QCD}} \Rightarrow$ boost to get $\frac{p}{m}\frac{1}{\Lambda} \equiv \tau_\Lambda$

int. time of DIS is $\tau_{DIS} \approx \frac{1}{q^0}$, where

$q^0 \approx \frac{Q^2}{2x_B p}$ $= \frac{m\nu}{p}$ ← as $m\nu = \frac{Q^2}{2x}$ is struck quark's velocity: $\tau_{DIS} \approx \frac{2x_B p}{Q^2}$.

time-ordered product: (denoted T)

$$\boxed{T\, j_\mu(x)\, j_\nu(y) \equiv \theta(x^0-y^0)\, j_\mu(x)\, j_\nu(y) + \theta(y^0-x^0)\, j_\nu(y)\, j_\mu(x)}$$

note: currents do not commute with each other in general $\Rightarrow$ not a trivial object.

$$2\,\mathrm{Im}\left(i\,T_{\mu\nu}\right) = 2\,\mathrm{Im}\Big[i \cdot \frac{1}{4\pi m_p}\int d^4x\, e^{iq\cdot x}\langle p|\,\theta(x^0)\, \underbrace{j_\mu(x)}_{=e^{ip\cdot x} j_\mu(0) e^{-ip\cdot x}}\, \underbrace{\ }_{\sum_n |n\rangle\langle n|} j_\nu(0) +$$

$$+\,\theta(-x^0)\, j_\nu(0)\, j_\mu(x)\,|p\rangle\Big] = 2\cdot\frac{1}{4\pi m_p}\sum_n \mathrm{Re}\Big\{\int d^4x\, e^{iq\cdot x + ip\cdot x - ip_n\cdot x}$$

$$\cdot\,\theta(x^0)\,\langle p|j_\mu(0)|n\rangle\langle n|j_\nu(0)|p\rangle + \int d^4x\, e^{iq\cdot x + ip_n\cdot x - ip\cdot x}\,\theta(-x^0)$$

$$\cdot\langle p|j_\nu(0)|n\rangle\langle n|j_\mu(0)|p\rangle\Big\} = \sum_n 2\,\frac{1}{4\pi m_p}\Big\{(2\pi)^3\,\delta(\vec{q}+\vec{p}-\vec{p}_n)\cdot\mathrm{Re}$$

$$\left(\frac{-1}{i(q^0+p^0-p_n^0+i\varepsilon)}\langle p|j_\mu(0)|n\rangle\langle n|j_\nu(0)|p\rangle\right) + \underbrace{(2\pi)^3\,\delta(\vec{q}+\vec{p}_n-\vec{p})\cdot}_{\text{not physical} \Rightarrow \text{drop}}$$

$$\cdot\,\mathrm{Re}\left(\frac{1}{i(q^0+p_n^0-p^0-i\varepsilon)}\langle p|j_\nu(0)|n\rangle\langle n|j_\mu(0)|p\rangle\right)\Big\}$$

$\propto \delta(q^0+p_n^0-p^0)$; $\overset{!}{=} 0$; not physical $\Rightarrow$ drop (after including $\delta(q^0+p_n^0-p^0)$)

$$= 2\,\frac{1}{4\pi m_p}\sum_n (2\pi)^3\,\delta(\vec{q}+\vec{p}-\vec{p}_n)\,(-)\left(\mathrm{Im}\,\frac{1}{q^0+p^0-p_n^0+i\varepsilon}\right)\cdot\langle p|j_\mu(0)|n\rangle$$

$$\langle n|j_\nu(0)|p\rangle = \Big|\text{as } \mathrm{Im}\,\frac{1}{x+i\varepsilon} = -\pi\,\delta(x) = \frac{1}{4\pi m_p}\sum_n (2\pi)^4\,\delta^4(q+p-p_n)$$

$$\cdot\langle p|j_\mu(0)|n\rangle\langle n|j_\nu(0)|p\rangle = W_{\mu\nu} \quad \text{as desired.}$$

(can convolute with $\varepsilon_\mu^*\varepsilon_\nu$ to get this to work out, use various polarizations to pick up components) if x is small ($\lesssim 1$) and Q is large

$\Rightarrow$ $\tau_{DIS} \ll \tau_\Lambda$ interaction is "instantaneous"

as $\frac{2xp}{Q^2} \ll \frac{p}{m}\frac{1}{\Lambda}$ $\Rightarrow$ as $2xm\Lambda < 2m\Lambda \ll Q^2$

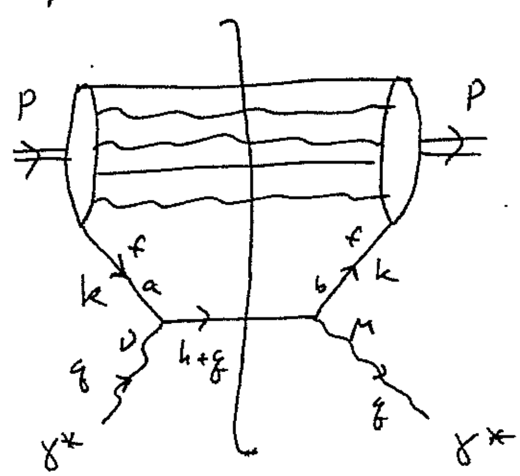
Define light cone variables:

for vector V^μ one has $V^+ = \frac{1}{\sqrt{2}}(V^0 + V^3)$

$\underline{V} = (V^1, V^2)$ $\qquad V^- = \frac{1}{\sqrt{2}}(V^0 - V^3)$

(2d transverse vector)

$V_1 \cdot V_2 = V_{1\mu} V_2{}^\mu = V_{1+}V_{2-} + V_{1-}V_{2+} - \underline{V}_1 \cdot \underline{V}_2$

$W_{\mu\nu} =$ 

$\Big\}$ $A^f_{ab}(p,k)$ — Dirac indices

e.g.

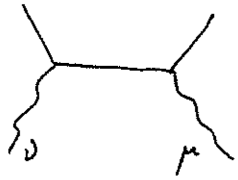
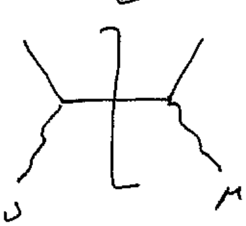
$p_+ = \frac{p}{\sqrt{2}}$

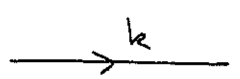
$p_- = \frac{m^2}{2\sqrt{2}p}$

$p_+ \gg p_-$

as $W_{\mu\nu} = 2\,\mathrm{Im}(i\,T_{\mu\nu})$

cut (Im part of a propagator)

$T_{\mu\nu} =$  $\qquad W_{\mu\nu} =$ 



$\frac{i}{k^2 - m^2 + i\varepsilon}$ $\Rightarrow$  $2\pi\,\delta^{(+)}(k^2 - m^2)$

as $2\,\mathrm{Im}\frac{-1}{k^2 - m^2 + i\varepsilon} = +2\pi\,\delta^{(+)}(k^2 - m^2)$

We write

$$W_{\mu\nu} = \frac{1}{\cancel{4\pi}\,2\,m} \sum_f e_f^2 \int d^4k \; A^f_{ab}(p,k) \left[\gamma_\mu \gamma_0 (k+q) \gamma_\nu\right]_{ba}$$

$$\cdot \cancel{(2\pi)}\, \delta\left((k+q)^2\right)$$ where A^f_{ab} is the rest of the diagram (see p. 90).

Start calculating assuming that

$$Q^2 \gg k^2, \; \underline{k}\cdot\underline{q} \quad , \quad k_+ \gg k_- \quad (\text{IMF})$$

$$\therefore \quad (k+q)^2 = k^2 + 2k_+ q_- + 2k_- q_+ - \underline{k}\cdot\underline{q} - Q^2$$

$q_3 = 0 \Rightarrow q_+ = q_- \Rightarrow$ as $k_+ \gg k_- \Rightarrow$ drop $2k_- q_+$

dropping $k^2, \underline{k}\cdot\underline{q} \ll Q^2$ get

$$(k+q)^2 \simeq 2k_+ q_- - Q^2$$

$$\Rightarrow \delta\left((k+q)^2\right) \simeq \delta\left(2k_+ q_- - Q^2\right) = \delta\left(\frac{k_+}{p_+} 2p_+ q_- - Q^2\right)$$

as $p\cdot q \simeq p_+ q_- \Rightarrow$ and $x_{Bj} = \dfrac{Q^2}{2p\cdot q}$

$$\Rightarrow \delta\left((k+q)^2\right) \simeq \frac{x_{Bj}}{Q^2}\, \delta\left(x_{Bj} - \frac{k_+}{p_+}\right)$$

$$\Rightarrow \boxed{x_{Bj} = \frac{k_+}{p_+}}$$ Feynman x = Bjorken x

physical meaning: light cone momentum fraction of the struck quark!