~~The Hamiltonian: $\boxed{H = \int \frac{d^3k}{(2\pi)^3\, 2\varepsilon_k}\, \varepsilon_k \sum_{\lambda=\pm} \hat{a}^+_{\vec{k},\lambda}\, \hat{a}_{\vec{k},\lambda}}$~~

~~⇒ no longitudinal photon contribution to Hamiltonian and, therefore, time-evolution.~~

~~⇒ proper way is to require that all physical states have $\partial_\mu A^{\mu\,(+)} |\psi\rangle = 0$ ← positive frequencies while quantizing by adding a term like $\lambda(\partial_\mu A^\mu)^2$ to the Hamiltonian. (a constraint)~~

~~Time evolution: $\boxed{-i\frac{\partial}{\partial t} A_\mu = [H, A_\mu]}$ as usual.~~

Quark Model and Group Theory

Quarks.

Many meson & baryon resonances were discovered in the 1960's. People wanted to organize the data: they started noticing some pattern.

Take p & n (known from 1932): both are nucleons with spin 1/2. They have almost identical masses: $M_p \simeq 938$ MeV, $M_n = 940$ MeV. Proton has charge $+e$, neutron has charge 0.

++

Other similar groups of particles were found:

- pions are π^+, π^-, π^0 ~ spin-0 mesons.

$m_{\pi^\pm} = 140\ MeV$, $m_{\pi^0} = 136\ MeV$

rho-mesons: ρ^+, ρ^0, ρ^- ~ spin-1 mesons, $m_\rho = 770 MeV$

Δ-baryons: $\Delta^{++}, \Delta^+, \Delta^0, \Delta^-$ ~ spin-$3/2$ baryons
(aka excited nucleons, N^*)

$m_\Delta \simeq 1232 MeV$

~ to organize these introduce a new quantum number called isospin. Works like the angular momentum operator in the "isospin space". If $\vec{I}$ is the isospin operator =>

=> can construct $\vec{I}^{\,2}$ & I_z:

$\vec{I}^{\,2}$ has eigenvalues $I(I+1)$, $I = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$

I_z — " — $-I, -I+1, \ldots, +I$ =>

=> multiplicity $= 2I+1$.

=> can classify hadrons as states $|I, I_3\rangle$:

$$\text{proton}: \quad p = |\overset{I}{\tfrac{1}{2}}, \overset{I_3}{+\tfrac{1}{2}}\rangle$$

$$\text{neutron}: \quad n = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$$

$\Rightarrow N = \begin{pmatrix} p \\ n \end{pmatrix}$ iso-doublet

(neglect mass difference) $2I+1 = 2 \Rightarrow I = 1/2$.

Pions: $2I+1=3 \Rightarrow I=1 \Rightarrow$

$$\left.\begin{array}{l} \;I \;\; I_3 \\ \pi^+ = |1,1\rangle \\ \pi^0 = |1,0\rangle \\ \pi^- = |1,-1\rangle \end{array}\right\} \text{iso-triplet!}$$

ρ-mesons:

$$\left.\begin{array}{l} \rho^+ = |1,1\rangle \\ \rho^0 = |1,0\rangle \\ \rho^- = |1,-1\rangle \end{array}\right\} \begin{array}{l}\text{iso-triplet!} \\ \text{(now spin-1)}\end{array}$$

Δ-baryon: $2I+1=4 \Rightarrow I=3/2 \Rightarrow$

$$\left.\begin{array}{l} \phantom{\Delta^{++} = |}\;I \;\; I_3 \\ \Delta^{++} = |\frac{3}{2},\frac{3}{2}\rangle \\ \Delta^+ = |\frac{3}{2},\frac{1}{2}\rangle \\ \Delta^0 = |\frac{3}{2},-\frac{1}{2}\rangle \\ \Delta^- = |\frac{3}{2},-\frac{3}{2}\rangle \end{array}\right\} \text{4-multiplet}$$

$\Rightarrow$ different <u>representations</u> of <u>group</u> SU(2) isospin.

(we'll discuss groups shortly)

Consider 2 processes: (i) $p + p \to d + \pi^+$
(ii) $p + n \to d + \pi^0$

(i) LHS: $|\tfrac{1}{2}, \tfrac{1}{2}\rangle |\tfrac{1}{2}, \tfrac{1}{2}\rangle = |1, 1\rangle$

↖ total isospin state

RHS: deuteron has isospin $= 0 \Rightarrow \pi^+ = |1,1\rangle$

$\Rightarrow$ amplitude $\propto \langle 1,1 | 1,1\rangle \sim 1$.

(ii) LHS: $|\tfrac{1}{2}, \tfrac{1}{2}\rangle |\tfrac{1}{2}, -\tfrac{1}{2}\rangle = \frac{1}{\sqrt{2}}\left(|1,0\rangle + |0,0\rangle\right)$

as
$$\left.\begin{aligned} |1,0\rangle &= \frac{1}{\sqrt{2}}\left(|p\rangle|n\rangle + |n\rangle|p\rangle\right) \\ |0,0\rangle &= \frac{1}{\sqrt{2}}\left(|p\rangle|n\rangle - |n\rangle|p\rangle\right) \end{aligned}\right\} \Rightarrow |p\rangle|n\rangle = \frac{1}{\sqrt{2}}\left(|1,0\rangle + |0,0\rangle\right)$$

RHS: $\pi^0 = |1,0\rangle \Rightarrow$

amplitude $\propto \frac{1}{\sqrt{2}}\left(\langle 1,0| + \langle 0,0|\right)|1,0\rangle \sim \frac{1}{\sqrt{2}}$

$$\Rightarrow \frac{\sigma_{pp \to d\pi^+}}{\sigma_{pn \to d\pi^0}} \propto \frac{|M_{pp \to d\pi^+}|^2}{|M_{pn \to d\pi^0}|^2} = \frac{1}{1/2} = 2.$$

~ agrees with experiment!

~ all given by Clebsch-Gordan coefficients.