Pions: $2I+1=3 \Rightarrow I=1 \Rightarrow$

$$\left.\begin{aligned} & \quad\;\; I \;\; I_3 \\ \pi^+ &= |1, 1\rangle \\ \pi^0 &= |1, 0\rangle \\ \pi^- &= |1, -1\rangle \end{aligned}\right\} \text{iso-triplet!}$$

$$\rho\text{-mesons:} \quad \left.\begin{aligned} \rho^+ &= |1,1\rangle \\ \rho^0 &= |1,0\rangle \\ \rho^- &= |1,-1\rangle \end{aligned}\right\} \begin{array}{l}\text{iso-triplet!} \\ \text{(now spin-1)}\end{array}$$

$\Delta$-baryon: $2I+1=4 \Rightarrow I=3/2 \Rightarrow$

$$\left.\begin{aligned} & \quad\;\; I \;\;\; I_3 \\ \Delta^{++} &= \left|\tfrac{3}{2}, \tfrac{3}{2}\right\rangle \\ \Delta^+ &= \left|\tfrac{3}{2}, \tfrac{1}{2}\right\rangle \\ \Delta^0 &= \left|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle \\ \Delta^- &= \left|\tfrac{3}{2}, -\tfrac{3}{2}\right\rangle \end{aligned}\right\} \text{4-multiplet}$$

$\Rightarrow$ different <u>representations</u> of <u>group</u> SU(2) isospin.

(we'll discuss groups shortly)

example:

Consider 2 processes: (i) $p + p \to d + \pi^+$

(ii) $p + n \to d + \pi^0$

(i) LHS: $|\frac{1}{2}, \frac{1}{2}\rangle |\frac{1}{2}, \frac{1}{2}\rangle = \underset{\uparrow \text{ total isospin state}}{|1, 1\rangle}$

RHS: deuteron has isospin $= 0 \Rightarrow \pi^+ = |1, 1\rangle$

$\Rightarrow$ amplitude $\propto \langle 1, 1 | 1, 1\rangle \sim 1$.

(ii) LHS: $|\frac{1}{2}, \frac{1}{2}\rangle |\frac{1}{2}, -\frac{1}{2}\rangle = \frac{1}{\sqrt{2}}\left(|1, 0\rangle + |0, 0\rangle\right)$

as
$$\left.\begin{aligned} |1, 0\rangle &= \frac{1}{\sqrt{2}}\left(|p\rangle|n\rangle + |n\rangle|p\rangle\right) \\ |0, 0\rangle &= \frac{1}{\sqrt{2}}\left(|p\rangle|n\rangle - |n\rangle|p\rangle\right) \end{aligned}\right\} \Rightarrow |p\rangle|n\rangle = \frac{1}{\sqrt{2}}\left(|1,0\rangle + |0,0\rangle\right)$$

RHS: $\pi^0 = |1, 0\rangle \Rightarrow$

amplitude $\propto \frac{1}{\sqrt{2}}\left(\langle 1, 0| + \langle 0, 0|\right)|1, 0\rangle \sim \frac{1}{\sqrt{2}}$

$$\Rightarrow \frac{\sigma_{pp \to d\pi^+}}{\sigma_{pn \to d\pi^0}} \propto \frac{|M_{pp \to d\pi^+}|^2}{|M_{pn \to d\pi^0}|^2} = \frac{1}{1/2} = 2.$$

~ agrees with experiment!

~ all given by Clebsch-Gordan coefficients.

$$\left( |J, M\rangle = \sum_{m_1, m_2} \underbrace{\langle j_1, m_1; j_2, m_2 | J, M\rangle}_{\text{Clelsch-Gordan coefficient}} |j_1, m_1\rangle \otimes |j_2, m_2\rangle \right)$$

$M = m_1 + m_2$

Note that for pions & $\rho$'s: $Q = I_3$

where $Q$ is the electric charge. (in units of $|e|$)

For $\Delta$'s: $Q = I_3 + \frac{1}{2}$, same for protons!

Def. Baryon number ~ a quantum number counting the # of baryons: $B = +1$ for baryons, $B = -1$ for anti-baryons.

$$\Rightarrow \boxed{Q = I_3 + \frac{B}{2}}$$

However, there are also kaons:

(can see by looking at decay)

$K^+, K^0$ form an isospin-doublet $\Rightarrow I = 1/2 \Rightarrow$

$\Rightarrow K^+ = |\frac{1}{2}, \frac{1}{2}\rangle$, $K^0 = |\frac{1}{2}, -\frac{1}{2}\rangle$ $\Rightarrow$ $B = 0$ (mesons, spin-0)

$\Rightarrow$ would expect $Q = I_3$, but really only

$Q = I_3 + \frac{1}{2}$ works.

$K^-, \bar{K}^0$ are also spin-0 mesons, also form a doublet
($K^-$ is the anti-particle for $K^+$, $\bar{K}^0$ is for $K^0$)

$\bar{K}^0 = |\frac{1}{2}, \frac{1}{2}\rangle$, $K^- = |\frac{1}{2}, -\frac{1}{2}\rangle$ $\Rightarrow$ $Q = I_3 - \frac{1}{2}$ now...

Def. A new quantum number of strangeness:

for mesons $\boxed{Q = I_3 + \frac{S}{2}}$ $\Rightarrow$ $K^+, K^0$ have $S = +1$
$\bar{K}^0, K^-$ have $S = -1$.

Elementary particles can be characterized by their quantum number using $J^{PC}$ notation.

J ~ the spin of a particle

P ~ <u>parity</u>: $\mathbb{P}: \vec{x} \to -\vec{x}$

$$P|\psi\rangle = K|\psi\rangle \Rightarrow P^2|\psi\rangle = |\psi\rangle = K^2|\psi\rangle \Rightarrow K = \pm 1$$

all particles have definite parity $= \pm 1$.

<u>mesons</u>:

pions: $P = -1$.

$\rho$'s: $P = -1$

<u>baryons</u>: Δ-baryon, proton $P = +1$

C ~ <u>charge conjugation</u>: C transforms particles into anti-particles (particles with the same mass & spin and opposite quantum numbers, e.g. holes in Dirac sea)

if $|\psi\rangle$ is an eigenstate of C, then

$$C|\psi\rangle = \eta|\psi\rangle \Rightarrow C^2|\psi\rangle = \eta^2|\psi\rangle = |\psi\rangle$$

$$\Rightarrow \eta^2 = 1 \Rightarrow \eta = \pm 1.$$

$C|\gamma\rangle = -|\gamma\rangle$ $\quad$ $|\gamma\rangle$ is a photon (change charge from + → − ⇒ get a − sign)

$\Rightarrow$ as $\pi^0 \to \gamma\gamma \Rightarrow C|\pi^0\rangle = |\pi^0\rangle \Rightarrow C = +1$ for $\pi^0$.

~ not all particles are eigenstates of C. (e.g. $\pi^\pm$ are <u>not</u>!)

(need to be its own anti-particle to be an eigenstate of C)

=> proton: $J = \frac{1}{2}$, $P = +1$ => $\left(\frac{1}{2}\right)^{+}$

$\Delta$ : $J = \frac{3}{2}$, $P = +1$ => $\frac{3}{2}^{+}$

pion ($\pi^0$): $J = 0$, $P = -1$, $C = +1$ => $0^{-+}$

$\pi^{\pm}$ : $J = 0$, $P = -1$ => $0^{-}$

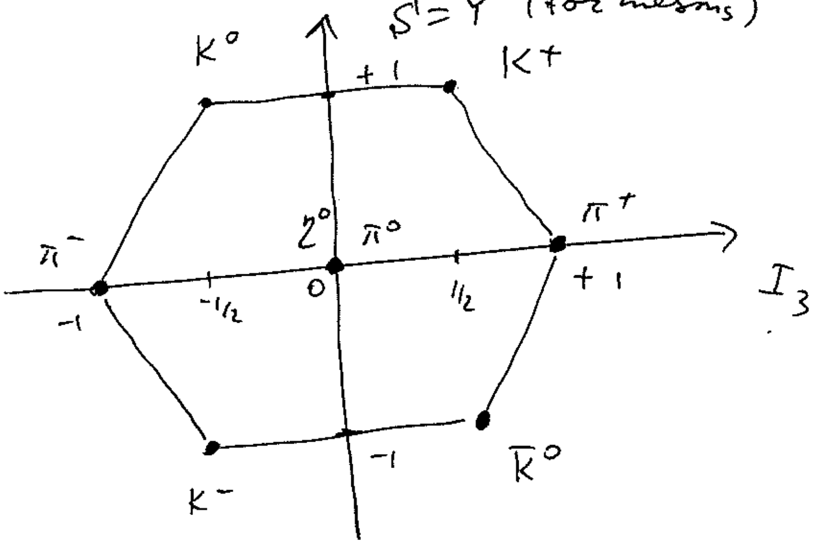
$\rho^0$: $J = 1$, $P = -1$ => $1^{--}$ ; $\rho^{\pm}$ : $J = 1$, $P = -1$ => $1^{-}$.
$C = -1$

---

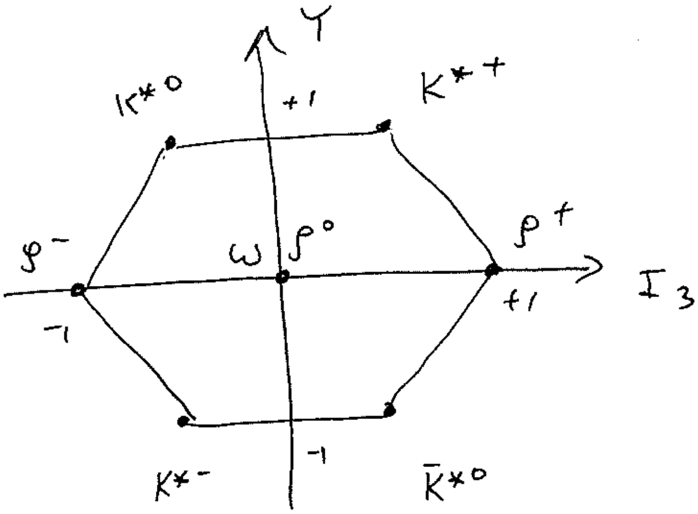
general formula: $\boxed{Q = I_3 + \frac{B}{2} + \frac{S}{2}}$ Gell-Mann Nishijima ('53, '55)

Def. $Y = B + S$ => $\boxed{Q = I_3 + \frac{Y}{2}}$ , $Y$ ~ hypercharge.

Gell-Mann & Ne'eman (1961): take, say, all $0^-$ mesons: $\pi$'s, $K$'s, $\eta$:

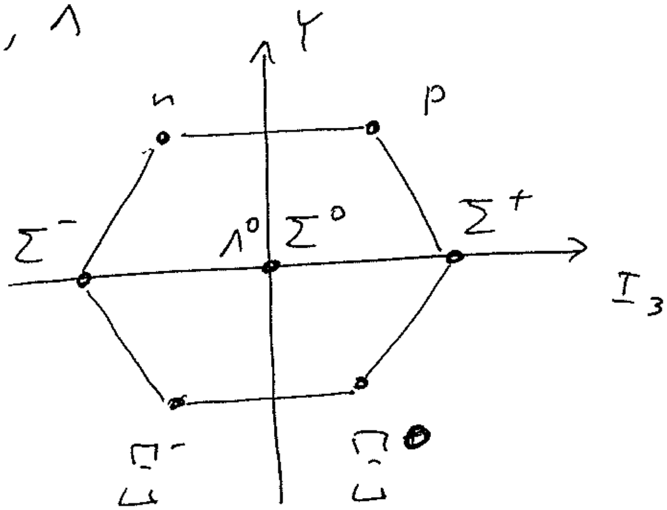


spin-1 mesons ($1^-$):



The "Eightfold Way".

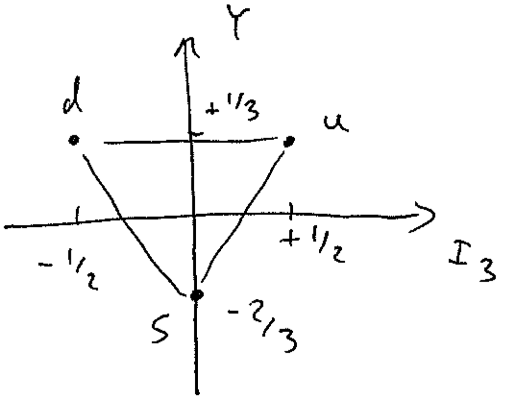
Baryons: $\frac{1}{2}^+$ : p, n, $\Sigma$, $\Xi$, $\Lambda$



⇒ there must be some sub-structure for all these symmetries.

⇒ Gell-Mann & Ne'eman suggested that there exist

3 quarks: u, d, s (up, down, strange)
spin-1/2 particles.



$B = +\frac{1}{3}$ for each quark

$I_3 = +\frac{1}{2}$ for u, $-\frac{1}{2}$ for d, 0 for s

$S = 0$ for u, d, $S = -1$ for strange

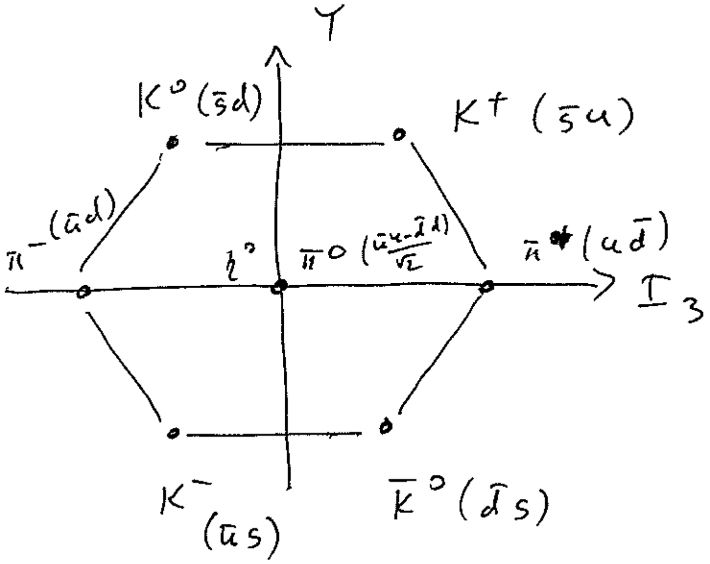
$Y = B + S \Rightarrow Y = +\frac{1}{3}$, u, d, $-\frac{2}{3}$, for s

⇒ if $\pi^+ = u\bar{d}$ , $\pi^- = \bar{u}d$ , $\pi^0 = \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)$

$K^+ \sim \bar{s}u$, $K^0 = \bar{s}d$, $\bar{K}^0 = \bar{d}s$ , $K^- = \bar{u}s$

$\eta^0 = \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s)$

$\pi^+ = u\bar{d} \Rightarrow Y = B + S = \frac{1}{3} - \frac{1}{3} = 0$ ; $I_3 = +\frac{1}{2} - (-\frac{1}{2}) = 1$.



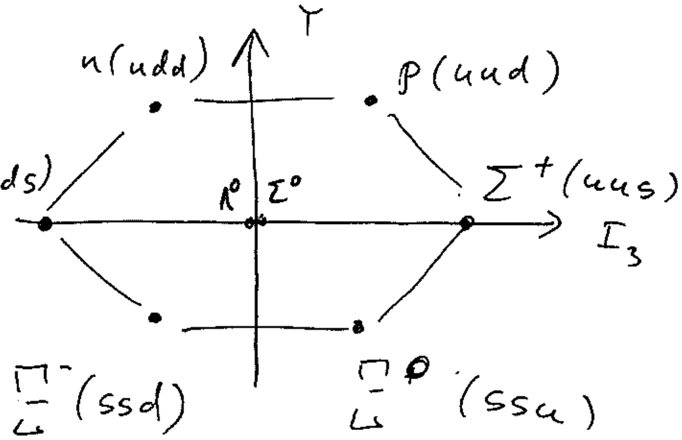
$K^0 = \bar{s}d \Rightarrow Y = 1,$

$I_3 = \frac{1}{2}$, etc.

baryons: $p \sim uud$, $n \sim udd$, $\Sigma^+ = uus$, $\Sigma^- = dds$,

$\Sigma^0 = s\,\frac{ud+du}{\sqrt{2}}$, $\Xi^0 \sim ssu$, $\Xi^- = ssd$, $\Lambda^0 \sim s\,\frac{ud-du}{\sqrt{2}}$

$\Rightarrow$



e.g. $p\,(uud) \Rightarrow$

$Y = B + S = +\frac{1}{3}+\frac{1}{3}+\frac{1}{3} = +1$

$I_3 = +\frac{1}{2}+\frac{1}{2}-\frac{1}{2} = \frac{1}{2}.$

etc.

$\Rightarrow$ in reality there are 6 quark flavors:

$$u, d, s, \underset{\substack{\uparrow\\ \text{charm}}}{c}, \underset{\substack{\uparrow\\ \text{bottom}}}{b}, \underset{\nwarrow\,\text{top}}{t}$$

c, b, t ~ heaviest, do not contribute to the lightest hadrons considered so far.

$\frac{3}{2}^{+}$ baryons form a decuplet:

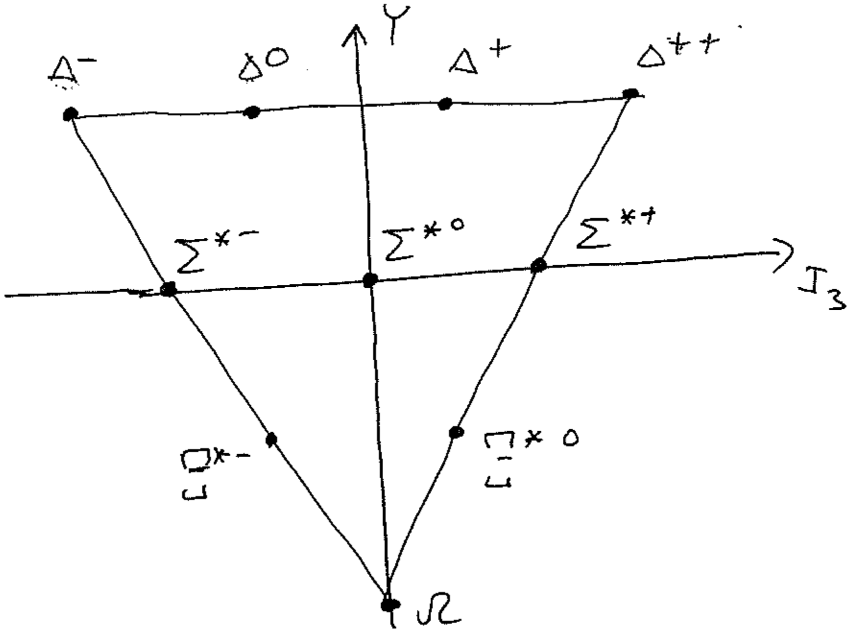
$\Delta^{++} = uuu, \Delta^{+} \sim uud,$

$\Delta^{0} \sim udd, \Delta^{-} - ddd$

$\Sigma^{*+} \sim suu, \Sigma^{*0} \sim sud,$

$\Sigma^{*-} \sim sdd, \Xi^{*0} \sim ssu,$

$\Xi^{*-} \sim ssd, \Omega^{-} - sss.$



$\Rightarrow$ seems OK? but let's look at $\Delta^{++}$ for instance: it has spin-$3/2$ $\Rightarrow$ the spin state is

$|\underset{uuu}{\uparrow\uparrow\uparrow}\rangle = |\uparrow\rangle \otimes |\uparrow\rangle \otimes |\uparrow\rangle$ $\Rightarrow$ symmetric

the isospin state is $|\uparrow\uparrow\uparrow\rangle$ too ~ also symmetric!

What happens to Pauli principle ~ the full wave function $\Psi = \Psi_{spin} \otimes \Psi_{isospin}$ has to be anti-symmetric! (Fermi-Dirac statistic)

$\Psi_{spatial}$ is symmetric too (ground state for uuu)

$\Rightarrow$ the way out is to postulate a new quantum number called color (Greenberg, Han, Nambu '64-'66)

there~~am~~ are 3 colors: $i = 1, 2, 3$ $\Rightarrow$ $u_i(x)$ ~ up quark w.f.