Last time | Flavor SU(2) and SU(3) Symmetries

(cont'd)

$N_f = 2 \qquad m_u = m_d = 0 \qquad \mathcal{L} = \bar{q}\, i \not{\partial}\, q$

$$q(x) = \begin{pmatrix} u(x) \\ d(x) \end{pmatrix}$$

Write $q = q_L + q_R = \frac{\mathbb{1} - \gamma_5}{2} q + \frac{\mathbb{1} + \gamma_5}{2} q$

$$\Rightarrow \mathcal{L} = \bar{q}_L i \not{\partial} q_L + \bar{q}_R i \not{\partial} q_R$$

$$\left\{ \begin{array}{l} q_L \to e^{i \vec{\alpha}_L \cdot \frac{\vec{\sigma}}{2}} q_L \\ q_R \to e^{i \vec{\alpha}_R \cdot \frac{\vec{\sigma}}{2}} q_R \end{array} \right. \Rightarrow SU(2)_L \otimes SU(2)_R \text{ flavor symmetry}$$

put $m_u = m_d \neq 0 \Rightarrow$

$$\mathcal{L} = \bar{q}_L i \not{\partial} q_L + \bar{q}_R i \not{\partial} q_R - m \left[\bar{q}_L q_R + \bar{q}_R q_L \right]$$

$\Rightarrow$ need $\vec{\alpha}_R = \vec{\alpha}_L \Rightarrow SU(2)_L \otimes SU(2)_R$

is broken down to $SU(2)$.

$$\boxed{SU(2)_L \otimes SU(2)_R} \qquad m_u = m_d = 0$$

$$\downarrow$$

$$\boxed{SU(2)} \qquad m_u = m_d \neq 0$$

$$\downarrow$$

$$\text{Nothing} \qquad m_u \neq m_d \neq 0$$

Conserved currents of $SU(2)_L \otimes SU(2)_R$:

$$j_L^{i\mu} = \bar{q}_L \gamma^\mu \frac{\sigma^i}{2} q_L \qquad j_R^{i\mu} = \bar{q}_R \gamma^\mu \frac{\sigma^i}{2} q_R$$

or $\boxed{j_\mu^i = \bar{q} \gamma_\mu \frac{\sigma^i}{2} q}$ $\qquad$ $\boxed{j_{5\mu}^i = \bar{q} \gamma_\mu \gamma_5 \frac{\sigma^i}{2} q}$

vector isospin current $\qquad$ axial vector isospin current

charges $\qquad Q_{L,R}^i = \int d^3x \; j_{L,R}^{i0}(\vec{x}, t)$

(Def) $\Rightarrow$ $\partial_\mu j_L^{i\mu} = 0$ where $\boxed{j_L^{i\mu} = \bar{q}_L \gamma^\mu \frac{\sigma^i}{2} q_L}$

~ left-handed isospin current.

(Def) Similarly define $\boxed{j_R^{i\mu} = \bar{q}_R \gamma^\mu \frac{\sigma^i}{2} q_R}$ ~ right-handed isospin current.

$\partial_\mu j_R^{i\mu} = 0$

Alternatively can define

(Def) $j_\mu^i = j_{L\mu}^i + j_{R,\mu}^i \quad = \bar{q} \gamma_\mu \frac{\sigma^i}{2} q$ ~ vector isospin current

$j_{5\mu}^i = j_{R\mu}^i - j_{L\mu}^i \quad = \bar{q} \gamma_\mu \gamma_5 \frac{\sigma^i}{2} q$ ~ axial vector isospin current

Define charges: $\boxed{Q_{L,R}^i(t) = \int d^3x \; j_{L,R\,0}^i(\vec{x}, t)}$

$$\frac{dQ_L^i(t)}{dt} = \int d^3x \; \frac{\partial j_{L0}^i(\vec{x},t)}{\partial t} = \int d^3x \left[\underbrace{\partial_\mu j_L^{i\mu}}_{=0 \text{ (conserved current)}} - \vec{\nabla}\cdot\vec{j}_L^{\,i}\right]$$

$$= -\int d^3x \; \vec{\nabla}\cdot\vec{j}_L^{\,i} = 0$$

surface term

$\Rightarrow$ charges are conserved!

$\Rightarrow$ the charges are generators of $SU(2)_L \otimes SU(2)_R$!

One can show that they form the chiral $SU(2)_L \otimes SU(2)_R$ algebra:

$$\boxed{\begin{aligned} [Q_L^i, Q_L^j] &= i\,\varepsilon_{ijk}\, Q_L^k \\ [Q_R^i, Q_R^j] &= i\,\varepsilon_{ijk}\, Q_R^k \\ [Q_L^i, Q_R^j] &= 0 \end{aligned}}$$

$\leftarrow SU(2)_L$

$\leftarrow SU(2)_R$

~ commute with each other.

Let's show how Q_L^i generate $SU(2)_L$ transformations.

Let's calculate $[\,Q_L^i(t),\ q_L(t,\vec{x})\,]$:

flavor index 1,2

$$[\,Q_L^i(t),\ q_{L\,\alpha}^{\,a}(\vec{x},t)\,] = \int d^3x' \left[\, \bar{q}_L \gamma_0 \frac{\sigma^i}{2} q_L(\vec{x}',t), \right.$$

spinor index 1,2,3,4

$$\left. q_{L\,\alpha}^{\,a}(\vec{x},t)\right] = \int d^3x' \left[\, \underbrace{\bar{q}^{\,b}_{L\,\beta}(\vec{x}',t)\,(\gamma^0)_{\beta\delta}}_{q^{+\,b}_{L\,\delta}(\vec{x}',t) \,=\, q^{+\,b}_{\delta'}\left(\frac{1-\gamma_5}{2}\right)_{\delta'\delta}} \left(\frac{\sigma^i}{2}\right)_{bc} \underbrace{q^{\,c}_{L\,\delta}(\vec{x}',t)}_{=\left(\frac{1-\gamma_5}{2}\right)_{\delta\delta''} q^{c}_{\delta''}}, \right.$$

$$\left. q_{L\,\alpha}^{\,a}(\vec{x},t)\right] =$$

$$= \int d^3x' \cdot \left(\frac{1-\gamma_5}{2}\right)_{\delta'\delta} \left(\frac{1-\gamma_5}{2}\right)_{\delta\delta''} \left(\frac{1-\gamma_5}{2}\right)_{\alpha\alpha'} \cdot \left(\frac{\sigma^i}{2}\right)_{bc} \cdot$$

$$\cdot \left[\, q^{+\,b}_{\delta'}(\vec{x}',t)\, q^{c}_{\delta''}(\vec{x}',t),\ q^{a}_{\alpha'}(\vec{x},t) \,\right]$$

$\Rightarrow$ use the anti-commutation relations

$$\{\, q^a_\alpha(\vec{x},t),\ q^{+\,b}_\beta(\vec{x}',t) \,\} = \delta^{ab}\delta_{\alpha\beta}\,\delta(\vec{x}-\vec{x}').$$

$$[\,Q_L^i(t),\ q_{L\,\alpha}^{\,a}(\vec{x},t)\,] = \left(\frac{1-\gamma_5}{2}\right)_{\delta'\delta''} \left(\frac{1-\gamma_5}{2}\right)_{\alpha\alpha'} \left(\frac{\sigma^i}{2}\right)_{bc} \cdot \int d^3x' \cdot$$

$$\cdot(-)\,\{\, q^a_{\alpha'}(\vec{x},t),\ q^{+\,b}_{\delta'}(\vec{x}',t) \,\}\, q^c_{\delta''}(\vec{x}',t) = -\left(\frac{1-\gamma_5}{2}\right)_{\delta'\delta''} \cdot$$

$$\cdot \left(\frac{1-\gamma_5}{2}\right)_{\alpha\alpha'} \left(\frac{\sigma^i}{2}\right)_{bc} \delta^{ab}\,\delta_{\alpha'\delta'}\, q^c_{\delta''}(\vec{x},t) = -\left(\frac{\sigma^i}{2}\right)_{ac} \cdot$$

$$\cdot \left(\frac{1-\gamma_5}{2}\right)_{\alpha\delta''} q^c_{\delta''}(\vec{x},t) = -\left(\frac{\sigma^i}{2}\right)_{ac} q^c_{L\,\alpha}(\vec{x},t)$$

$\Rightarrow$ get $\left[Q_L^{\,i}(t),\, q_L(\vec{x},t) \right] = -\frac{\sigma^i}{2}\, q_L(\vec{x},t)$

$\Rightarrow$ can show that

$$e^{-i\vec{\alpha}_L\cdot\vec{Q}_L(t)}\, q_L(\vec{x},t)\, e^{i\vec{\alpha}_L\cdot\vec{Q}_L(t)} = e^{i\vec{\alpha}_L\cdot\frac{\vec{\sigma}}{2}}\, q_L(\vec{x},t)$$

$\Rightarrow$ Q_L's generate transformations of $SU(2)_L$

$\Rightarrow$ Q_R's —"— of $SU(2)_R$ (can show similarly).

c.f. $\hat{O}(t) = e^{i\hat{H}t}\,\hat{O}(0)\,e^{-i\hat{H}t} = e^{t\frac{\partial}{\partial t'}}\,\hat{O}(t')\big|_{t'=0}$ $\Rightarrow$ $\hat{H}$ generates time translations

bring back the strange quark $\Rightarrow$ how can perform the same decomposition and for $m_u = m_d = m_s = 0$ have $SU(3)_R \otimes SU(3)_L$ chiral symmetry.

$$\mathcal{L} = \bar{q}_L\, i\gamma\cdot\partial\, q_L + \bar{q}_R\, i\gamma\cdot\partial\, q_R$$

$\Rightarrow$ invariant under $q_L \to e^{i\vec{\alpha}_L\cdot\vec{t}}\, q_L$, $q_R \to e^{i\vec{\alpha}_R\cdot\vec{t}}\, q_R$

$t^a = \frac{\lambda^a}{2}$ ~ generators of $SU(3)$.

Problem: $SU(3)_L \otimes SU(3)_R$ would imply twice as many degenerate multiplets of hadrons: 8 0^- mesons should come in together with 8 0^+ mesons, etc.

$\Rightarrow$ this does not happen in nature. Why?

$\Rightarrow$ you may say: well, as $m_u, m_d, m_s \neq 0$

$\Rightarrow SU(3)_R \otimes SU(3)_L$ is broken.

But then one would not have any multiplets at all, one would not have the Eight fold Way, etc...

$\Rightarrow$ OK, you would say, we have $SU(3)$ flavor if $m_u = m_d = m_s \neq 0$.

$\Rightarrow$ But as $m_u \neq m_d \neq m_s$ $SU(3)$ flavor is just as broken as $SU(3)_L \otimes SU(3)_R$.

NB Fact of the matter is both $SU(3)_L \otimes SU(3)_R$ and $SU(3)$ are broken "slightly" in the Lagrangian. The real symmetry breaking is $SU(3)_L \otimes SU(3)_R \to SU(3)$ is done through <u>spontaneous symmetry breaking</u>. (SSB)

SSB has <u>nothing</u> to do with quark masses!

$\Rightarrow$ SSB would also explain why the masses of hadrons are so much higher than the current masses of quarks they are made of.

($\left(m_p = 938 \text{ MeV}, \; 2m_u + m_d < 30 \text{ MeV} \right)$

$$\frac{2m_u + m_d}{m_p} \simeq 3\% .$$

$$\boxed{SU(3)_L \otimes SU(3)_R} \xrightarrow{m_u = m_d = m_s \neq 0} \boxed{SU(3)} \xrightarrow{m_u \neq m_d \neq m_s \neq 0} \text{Nothing}$$

equally broken

$\Rightarrow$ $SU(3)$ works (the Eight fold way) approximately

$\Rightarrow$ $SU(3)_L \otimes SU(3)_R$ <u>does not work!</u> (no 0^+ meson octet, no $(\frac{1}{2})^-$ baryon octet, ...)

Solution:

$$\boxed{SU(3)_L \otimes SU(3)_R} \xrightarrow{m_u = m_d = m_s \neq 0 \;\oplus\; \text{SSB}} \boxed{SU(3)} \xrightarrow{m_u \neq m_d \neq m_s \neq 0} \text{Nothing}$$

SSB makes this breaking much worse than that

SSB (Spontaneous (chiral) Symmetry Breaking)