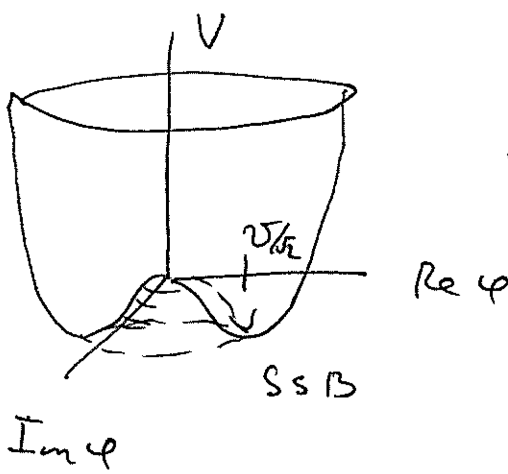
Last time | **The Higgs Mechanism** (U(1) model)

$$\boxed{\mathcal{L} = (D_\mu \varphi)^* (D^\mu \varphi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mu^2 \varphi^* \varphi - \lambda (\varphi^* \varphi)^2}$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, A_μ ~ real Abelian vector field

φ ~ scalar complex field



$$\frac{v}{\sqrt{2}} = \frac{\mu}{\sqrt{2\lambda}}$$

wrote $\varphi(x) = \dfrac{\rho(x) + v}{\sqrt{2}} e^{i\theta(x)}$

$$B_\mu = A_\mu - \frac{1}{g} \partial_\mu \theta(x)$$

$$\boxed{\begin{aligned}\mathcal{L} &= \frac{1}{2} \partial_\mu \rho \, \partial^\mu \rho - \mu^2 \rho^2 - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \frac{1}{2} g^2 v^2 B_\mu B^\mu \\ &+ \frac{1}{2} g^2 B_\mu B^\mu (2\rho v + \rho^2) - \lambda v \rho^3 - \frac{\lambda}{4} \rho^4\end{aligned}}$$

$m_\rho = \mu\sqrt{2}$, $m_B = g v$

no NG boson, as $\theta(x)$ got "eaten up" by B_μ.

B_μ has a mass $\Rightarrow$ Higgs phenomenon.

$\Rightarrow$ SSB of gauge symmetry ~ no Goldstone bosons

~ but get massive vector fields!

(e.g. Meissner effect in superconductivity when photon gets a "mass" and is screened in superconductor ~ P.W. Anderson, '58)

$\Rightarrow$ in particle physics this is known as the Higgs phenomenon. (P. Higgs, 1964).

$SU(2) \otimes U(1)$ Electroweak Theory.

history: Pauli postulated neutrino to explain β-decay (1930).

Fermi ('34) : to explain β-decay $n \to p e \bar{\nu}$

suggested an interaction term

$$\mathcal{L}_F = -\frac{G_F}{\sqrt{2}} [\bar{p}\, \gamma_\mu n][\bar{e}\, \gamma^\mu \nu] + h.c.$$

with $G_F = \frac{10^{-5}}{m_p^2}$. $\Rightarrow$ but as $[G_F] = \frac{1}{M^2}$ $\Rightarrow$ not renormalizable

$\Rightarrow$ theory may have (W, Z) vector bosons $\Rightarrow$ Glashow, Salam proposed a gauge theory ('61, '64)

$\Rightarrow$ problem with massive gauge fields:

the propagator is $-i \dfrac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M^2}}{k^2 - M^2 + i\epsilon}$ $\Rightarrow$ also non-renormalizable, as $\to$ const as $k^\mu \to \infty$ $\Rightarrow$

$\Rightarrow$ loops badly diverge...

$\Rightarrow$ Weinberg ('67) suggested using SSB to cure the problem

$\Rightarrow$ 1983 W, Z bosons discovered at CERN

$\Rightarrow$ Glashow – Weinberg – Salam model (1979 Nobel Prize in Physics)

$\Rightarrow$ define fermion fields of leptons:

e, μ, τ , ν_e, ν_μ, ν_τ — 1956: T.D. Lee & C.N. Yang, parity non-conservation; 1957: Wu ~ exp. observed $\Rightarrow$ ψ_L, ψ_R interact differently

$\Rightarrow$ define left & right handed ones $\psi_{L,R} = \dfrac{1 \mp \gamma_5}{2} \psi$

$\Rightarrow$ group leptons in left-handed weak-isospin doublets:

$$L_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \quad L_\mu = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \quad L_\tau = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} .$$

and in right-handed isospin singlets:

$$R_e = e_R , \quad R_\mu = \mu_R , \quad R_\tau = \tau_R$$

$\Rightarrow$ write the Lagrangian for the 3 generations (aka families) of leptons:

$$\boxed{\mathcal{L}_{free} = \bar{R}_e \, i\gamma\cdot\partial R_e + \bar{L}_e \, i\gamma\cdot\partial L_e + (\mu \,\&\, \tau\text{-terms})}$$

$\Rightarrow$ quantum #'s: $\vec{I}$ ~ weak isospin $\Rightarrow$

$\Rightarrow$ doublets have $I = \frac{1}{2}$, singlet has $I = 0$.

$\Rightarrow$ neutrinos have zero electric charge: $Q_{electric} = 0$

$\Rightarrow$ if we want to have $\boxed{Q = I_3 + \frac{Y}{2}}$ $\Rightarrow$
(Gell-Mann–Nishijima–type)

$\Rightarrow$ define weak hypercharge Y: neutrinos have $Q = 0$, $I_3 = +\frac{1}{2}$ $\Rightarrow$ $Y = -1$ $\Rightarrow$ all doublets L_e, L_μ, L_τ have $Y = -1$.

(check: electron has $Q = -1$ $\Rightarrow$ $-1 = -\frac{1}{2} - \frac{1}{2}$, OK)

~ the singlet: electron $Q = -1 = \underset{\overset{\parallel}{0}}{I_3} + \frac{Y}{2} \Rightarrow Y = -2$

$\Rightarrow$ iso-singlets have weak hypercharge $Y = -2$.
R_e, R_μ, R_τ

$\Rightarrow$ back to $\mathcal{L}_{free}$: it clearly has the following global symmetries:

$U(1)$: $L_e \to e^{-i\alpha_Y} L_e$, $R_e \to e^{-2i\alpha_Y} R_e$ (to have $\bar{L}_e \phi R_e$ term + required by anomaly cancellation to be the same α_Y)

$SU(2)$: $L_e \to e^{i\vec{\alpha}\cdot\frac{\vec{\tau}}{2}} L_e$, $\vec{\tau}$ ~ Pauli matrices

$\Rightarrow$ Gauge $U(1)$ symmetry first: introduce an abelian vector field $B_\mu(x)$ with field strength $f_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ & coupling to leptons of $g'/2$:

$$Y=-2 \Rightarrow \left(\partial_\mu - i\frac{g'}{2} Y B_\mu\right)$$

$$\mathcal{L} = \bar{R}_e\, i\gamma^\mu \Big(\partial_\mu \underbrace{+2}_{Y=-2} i\left(\frac{g'}{2}\right) B_\mu\Big) R_e + \bar{L}_e\, i\gamma^\mu \cdot$$

$$\cdot\Big(\partial_\mu \underbrace{+1}_{Y=-1} \cdot i\left(\frac{g'}{2}\right) B_\mu\Big) L_e - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} + (\mu, \tau).$$

$\Rightarrow$ Now let us gauge the $SU(2)$ symmetry: introduce a gauge field $\vec{W}_\mu = (W^1_\mu, W^2_\mu, W^3_\mu)$ with the field strength $F_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - ig[W_\mu, W_\nu]$

$W_\mu = \vec{W}_\mu \cdot \frac{\vec{\tau}}{2}$, g is the coupling of W_μ to itself & to the leptons:

$$\boxed{\begin{aligned}\mathcal{L} = \bar{R}_e\, i\gamma^\mu (\partial_\mu + i g' B_\mu) R_e + \bar{L}_e\, i\gamma^\mu \Big(\partial_\mu + i\frac{g'}{2} B_\mu - \\ - ig\, \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu\Big) L_e - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4} \vec{F}_{\mu\nu} \cdot \vec{F}^{\mu\nu} + (\mu, \tau).\end{aligned}}$$

now we have a Lagrangian for the leptons & 4 gauge fields $(B_\mu, \vec{W}_\mu)$, but so far everything is massless ($\Rightarrow$ bad.).

$\Rightarrow$ to give particles (especially $\vec{W}_\mu$'s) a mass need SSB mechanism

$\Rightarrow$ so far the Lagrangian is $SU(2) \otimes U(1)$ invariant

$\Rightarrow$ to break this symmetry introduce Higgs field: a weak isospin doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \begin{matrix} \Rightarrow Q = +1 \\ \Rightarrow Q = 0 \end{matrix} \quad \Rightarrow Q = I_3 + \frac{Y}{2}, \quad I_3 = \pm \frac{1}{2}$$

$\Rightarrow$ weak hypercharge is $Y = 1$.

$$\phi^+ = (\phi^-, \phi^{0+})$$

$\Rightarrow$ add Higgs field to the Lagrangian:

$$\mathcal{L}_{Higgs} = \left(\left[\partial_\mu - i \frac{g'}{2} B_\mu - i g \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu \right] \phi \right)^+ \left[\partial^\mu - i \frac{g'}{2} B^\mu - i g \frac{\vec{\tau}}{2} \cdot \vec{W}^\mu \right] \phi + \mu^2 \phi^+ \phi - \lambda (\phi^+ \phi)^2$$

Note that Higgs has $Y = +1 \Rightarrow \partial_\mu - i \frac{g'}{2} Y B_\mu = \partial_\mu - i \frac{g'}{2} B_\mu$

$\Rightarrow$ Higgs also couples to fermions (Yukawa coupling)

$$\mathcal{L}_{Higgs + leptons} = - G_e \left[\bar{L}_e \phi R_e + \bar{R}_e \phi^+ L_e \right] + (\mu, \tau\text{-terms})$$

as $L_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \Rightarrow \bar{L}_e \phi R_e = (\bar{\nu}_e \ \ \bar{e}_L) \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R$

$= \bar{\nu}_e \phi^+ e_R + \bar{e}_L \phi^0 e_R$ (matrices in isospin space).

$\Rightarrow$ the full Lagrangian is:

$$\boxed{\begin{aligned} \mathcal{L} &= \bar{R}_e \, i\gamma^\mu \left(\partial_\mu + i g' B_\mu\right) R_e + \bar{L}_e \, i\gamma^\mu \left(\partial_\mu + i\frac{g'}{2} B_\mu - i g \frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right) L_e \\ &+ (\mu,\tau\text{-terms}) - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4} \vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + \\ &+ \left[\left(\partial_\mu - i\frac{g'}{2} B_\mu - i g \frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\phi\right]^+ \left[\left(\partial^\mu - i\frac{g'}{2} B^\mu - i g \frac{\vec{\tau}}{2}\cdot\vec{W}^\mu\right)\phi\right] \\ &+ \mu^2 \phi^+\phi - \lambda(\phi^+\phi)^2 - G_e\left[\bar{L}_e \phi R_e + \bar{R}_e \phi^+ L_e\right] \end{aligned}}$$

$SU(2)_L \otimes U(1)_Y$ electroweak theory.

$\Rightarrow$ use the Higgs field to break the symmetry:

$$SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{EM}$$

$\Rightarrow \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ ~ has electric charge +1 $\Rightarrow$ if in vacuum have $\langle 0|Q|0\rangle \neq 0$ (SSB)

$\Rightarrow$ no charge symmetry $\Rightarrow$ no electric charge conservation $\Rightarrow$ don't want this

$\Rightarrow$ to conserve the electric charge require the vacuum of Higgs field to be at

$$\boxed{\langle 0|\phi|0\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}}.$$

$\Rightarrow \quad V(v) = -\mu^2 \frac{v^2}{2} + \frac{\lambda}{4} v^4 \Rightarrow$ minimize to get

$v = \mu/\sqrt{\lambda}$ $\Rightarrow$ write $\quad \phi(x) = e^{-i\frac{\vec{\tau}}{2}\cdot\vec{\Theta}(x)} \begin{pmatrix} 0 \\ \frac{v+\eta(x)}{\sqrt{2}} \end{pmatrix}$

with $\vec{\theta}, \eta$ ~ real fields.

Just like in the Abelian $U(1)$ case can absorb $\vec{\Theta}$ field into $\vec{W}_\mu$ by performing gauge rotation:

if $S(x) = e^{i\frac{\vec{\tau}}{2}\cdot\vec{\theta}(x)} \Rightarrow \phi \to \phi' = S\phi$, $L_{\substack{e\\ \mu\\ \tau}} \to L'_{\substack{e\\ \mu\\ \tau}} = S L_{\substack{e\\ \mu\\ \tau}}$

and $W_\mu \to W_\mu' = S W_\mu S^{-1} - \frac{i}{g}(\partial_\mu S) S^{-1}$. ($\vec{\theta}$ ~ would-be-Goldstone boson)

$\Rightarrow$ can drop primes to write (keep electrons only)

$$\begin{aligned} \mathcal{L} = {} & \bar{R}_e i\gamma^\mu(\partial_\mu + ig' B_\mu) R_e + \bar{L}_e i\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)L_e \\ & - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + \frac{1}{2}\left[\left(\partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\begin{pmatrix}0\\ v+\eta\end{pmatrix}\right]^\dagger \\ & \left[\left(\partial^\mu - i\frac{g'}{2}B^\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}^\mu\right)\begin{pmatrix}0\\ v+\eta\end{pmatrix}\right] + \frac{\mu^2}{2}(v+\eta)^2 - \frac{\lambda}{4}(v+\eta)^4 \\ & - G_e \frac{1}{\sqrt{2}}\left[(\bar{\nu}_e \; \bar{e}_L)\begin{pmatrix}0\\ v+\eta\end{pmatrix} e_R + h.c.\right]. \end{aligned}$$

in the potential

Start with η-particle: linear terms in η cancel as usual, as we are expanding around a minimum in η.