Last time | $SU(2) \otimes U(1)$ Electroweak Theory (cont'd)

lepton doublets

$$L_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad L_\mu = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \quad L_\tau = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

right-handed singlets

$$R_e = e_R, \quad R_\mu = \mu_R, \quad R_\tau = \tau_R$$

$$\boxed{Q = I_3 + \frac{Y}{2}}$$

← weak hypercharge

↑ weak isospin

$\mathcal{L}$ for L & R has global $SU(2) \otimes U(1)$ symmetry

$\Rightarrow$ gauge it.

All particles still massless $\Rightarrow$ need SSB to give masses $\Rightarrow$ introduce Higgs field (weak iso-doublet)

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \phi^\dagger = \begin{pmatrix} \phi^- & \phi^{0*} \end{pmatrix}$$

Higgs couples to gauge fields through covariant derivatives, and has a Yukawa coupling to leptons.

The full Lagrangian is

$$\boxed{\begin{aligned}\mathcal{L} = {} & \bar{R}_e\, i\gamma^\mu(\partial_\mu + ig' B_\mu) R_e + \bar{L}_e\, i\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}B_\mu - ig\frac{\vec{\sigma}}{2}\cdot\vec{W}_\mu\right) \cdot \\ & \cdot L_e - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} - G_e\left[\bar{L}_e \phi R_e + \bar{R}_e \phi^\dagger L_e\right] \\ & + (\mu, \tau\text{ - terms}) + \left[\left(\partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\vec{\sigma}}{2}\cdot\vec{W}_\mu\right)\phi\right]^\dagger \cdot \\ & \cdot \left[\left(\partial^\mu - i\frac{g'}{2}B^\mu - ig\frac{\vec{\sigma}}{2}\cdot\vec{W}^\mu\right)\phi\right] + \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2\end{aligned}}$$

$SU(2)_L \otimes U(1)_Y$ Electroweak Theory as $L_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \Rightarrow \bar{L}_e \phi R_e = (\bar{\nu}_e \ \bar{e}_L)\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R$

$= \bar{\nu}_e \phi^+ e_R + \bar{e}_L \phi^0 e_R$ (matrices in isospin space).

$\Rightarrow$ the full Lagrangian is:

$$\boxed{\begin{aligned} \mathcal{L} &= \bar{R}_e i\gamma^\mu(\partial_\mu + ig' B_\mu) R_e + \bar{L}_e i\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)L_e \\ &+(\mu,\tau\text{-terms}) - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + \\ &+\left[\left(\partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\phi\right]^\dagger\left[\left(\partial^\mu - i\frac{g'}{2}B^\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}^\mu\right)\phi\right] \\ &+\mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 - G_e\left[\bar{L}_e \phi R_e + \bar{R}_e \phi^\dagger L_e\right] \end{aligned}}$$

$SU(2)_L \otimes U(1)_Y$ electroweak theory.

$\Rightarrow$ use the Higgs field to break the symmetry:

$$SU(2)_L \otimes U(1)_Y \to U(1)_{EM}$$

$\Rightarrow \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ ~ has electric charge +1 $\Rightarrow$ if in vacuum have $\langle 0|Q|0\rangle \neq 0$ (SSB)

$\Rightarrow$ no charge symmetry $\Rightarrow$ no electric charge conservation $\Rightarrow$ don't want this

$\Rightarrow$ to conserve the electric charge require the vacuum of Higgs field to be at

$$\boxed{\langle 0|\phi|0\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}}.$$

$\Rightarrow \quad V(v) = -\mu^2 \frac{v^2}{2} + \frac{\lambda}{4} v^4 \Rightarrow$ minimize to get

$v = \mu/\sqrt{\lambda}$ $\Rightarrow$ write $\quad \phi(x) = e^{-i\frac{\vec{\tau}}{2}\cdot\vec{\theta}(x)} \begin{pmatrix} 0 \\ \frac{v+\eta(x)}{\sqrt{2}} \end{pmatrix}$

with $\vec{\theta}, \eta$ ~ real fields.

Just like in the Abelian $U(1)$ case can absorb $\vec{\theta}$ field into $\vec{W}_\mu$ by performing gauge rotation:

if $S(x) = e^{i\frac{\vec{\tau}}{2}\cdot\vec{\theta}(x)} \Rightarrow \phi \to \phi' = S\phi, \; L_{e,\mu,\tau} \to L'_{e,\mu,\tau} = S L_{e,\mu,\tau}$

and $W_\mu \to W_\mu' = S W_\mu S^{-1} - \frac{i}{g}(\partial_\mu S) S^{-1}$. ($\vec{\theta}$ ~ would-be-Goldstone boson)

$\Rightarrow$ can drop primes to write (keep electrons only)

$$\boxed{\begin{aligned} \mathcal{L} = {} & \bar{R}_e i\gamma^\mu(\partial_\mu + ig' B_\mu) R_e + \bar{L}_e i\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)L_e \\ & - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + \frac{1}{2}\left[\left(\partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\begin{pmatrix} 0 \\ v+\eta\end{pmatrix}\right]^\dagger \\ & \left[\left(\partial^\mu - i\frac{g'}{2}B^\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}^\mu\right)\begin{pmatrix} 0 \\ v+\eta\end{pmatrix}\right] + \frac{\mu^2}{2}(v+\eta)^2 - \frac{\lambda}{4}(v+\eta)^4 \\ & - G_e \frac{1}{\sqrt{2}}\left[\left(\bar{\nu}_e \; \bar{e}_L\right)\begin{pmatrix} 0 \\ v+\eta\end{pmatrix} e_R + h.c.\right]. \end{aligned}}$$

"unitary gauge"

in the potential

Start with η-particle: linear terms in η cancel as usual, as we are expanding around a minimum in η.

$O(\eta^2)$: $\frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta + \eta^2\left(\frac{\mu^2}{2} - \frac{6}{4}\lambda v^2\right) = \frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta - \mu^2\eta^2$

$\Rightarrow$ η is a massive particle with mass $\sqrt{2}\,\mu$.

"the Higgs particle"

Now, look at leptons: e, ν: get a term

$$-G_e \frac{v}{\sqrt{2}}\left[\bar{e}_L e_R + \bar{e}_R e_L\right] = -\frac{G_e v}{\sqrt{2}}\bar{e}e$$

$\Rightarrow$ electron has a mass $m_e = \frac{G_e v}{\sqrt{2}}$

(ibid for μ, τ : $m_\ell = \frac{G_\ell v}{\sqrt{2}}$, $\ell = e, \mu, \tau$)

note that $m_\nu = 0$ in the Standard Model

(not true in nature, more on this later)

$\Rightarrow$ Gauge bosons also get a mass: get a term

$$+\frac{1}{2}\left[-\left[i\frac{g'}{2}B_\mu + ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right]\begin{pmatrix}0\\v\end{pmatrix}\right]^\dagger\left[\left(-i\frac{g'}{2}B^\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}^\mu\right)\begin{pmatrix}0\\v\end{pmatrix}\right].$$

Consider one factor:

$$\left(\frac{g'}{2}B_\mu + \frac{g}{2}\vec{\tau}\cdot\vec{W}_\mu\right)\begin{pmatrix}0\\v\end{pmatrix} = \begin{pmatrix}\frac{g'}{2}B_\mu + \frac{g}{2}W_\mu^3 & \frac{g}{2}(W_\mu^1 - iW_\mu^2)\\ \frac{g}{2}(W_\mu^1 + iW_\mu^2) & \frac{g'}{2}B_\mu - \frac{g}{2}W_\mu^3\end{pmatrix}\begin{pmatrix}0\\v\end{pmatrix}$$

Define $$W_\mu^{(\pm)} \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^{(2)}\right) \quad (W^\pm \text{ bosons})$$

(also $W_\mu^{(+)} = W_\mu$, $W_\mu^{(-)} = W_\mu^+$)

$$Z_\mu \equiv \frac{-g' B_\mu + g W_\mu^3}{\sqrt{g^2+g'^2}} \quad (Z\text{-boson})$$

$$A_\mu \equiv \frac{g B_\mu + g' W_\mu^3}{\sqrt{g^2+g'^2}} \quad (\text{the photon})$$

Equivalently: $$\begin{cases} Z_\mu = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W \\ A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W \end{cases}$$

with $\tan\theta_W = g'/g$ (Weinberg angle)

$$\Rightarrow \begin{cases} B_\mu = A_\mu \cos\theta_W - Z_\mu \sin\theta_W \\ W_\mu^3 = A_\mu \sin\theta_W + Z_\mu \cos\theta_W \end{cases}$$

$\Rightarrow$ the term in the Lagrangian is $+\frac{v^2}{2}\left|\frac{g}{\sqrt{2}} W_\mu^{(+)}\right|^2 +$

$$+ \frac{v^2}{2}\cdot\frac{1}{4}(g^2+g'^2)\,|Z_\mu|^2 = + \frac{g^2 v^2}{4} \underbrace{W_\mu^{(+)} W^{\mu(-)}}_{W_\mu^+ W^\mu} + \frac{v^2}{8}(g^2+g'^2) Z_\mu Z^\mu$$

$W_\mu^+ W^\mu$ ~ complex vector field

$$\underbrace{\text{vector field}}_{\frac{1}{2}\left(W_\mu^1 W^{1\mu} + W_\mu^2 W^{2\mu}\right)}$$

$$\Rightarrow \quad M_W = \frac{g v}{2} \qquad M_Z = \frac{\sqrt{g^2+g'^2}}{2} v.$$

$\Rightarrow$ W, Z bosons get a mass

$\Rightarrow$ A_μ (photon) remains massless!