Last time Started with the full Lagrangian for the leptons, Higgs and gauge bosons W_μ^i, B_μ.

Chose the Higgs VEV to be

$$\langle \Psi_0 | \phi | \Psi_0 \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v = \mu/\sqrt{\lambda}$$

We wrote $\phi(x) = e^{-i\frac{\vec{\sigma}}{2}\cdot\vec{\theta}(x)} \begin{pmatrix} 0 \\ \frac{v+\eta(x)}{\sqrt{2}} \end{pmatrix}$ and

did a gauge transform into unitary gauge:

$$W_\mu \to W'_\mu = S W_\mu S^{-1} - \frac{i}{g}(\partial_\mu S) S^{-1}$$

$$L_{e,\mu,\tau} \to S L_{e,\mu,\tau} = L'_{e,\mu,\tau}$$

$$\phi \to \phi' = S\phi$$

with $S = e^{i\frac{\vec{\sigma}}{2}\cdot\vec{\theta}(x)}$. We got

$$\boxed{\begin{aligned}\mathcal{L} &= \bar{R}_e i\gamma^\mu(\partial_\mu + ig' B_\mu)R_e + \bar{L}_e i\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}B_\mu - ig\frac{\vec{\sigma}}{2}\cdot\vec{W}_\mu\right)L_e \\ &- \frac{G_e}{\sqrt{2}}\left[(\bar{\nu}_{eL}\ \bar{e}_L)\begin{pmatrix}0\\ v+\eta\end{pmatrix}e_R + c.c.\right] + (\mu,\tau\text{-terms}) \\ &- \frac{1}{4}f_{\mu\nu}f^{\mu\nu} - \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + \frac{1}{2}\left[\left(\partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\vec{\sigma}}{2}\cdot\vec{W}_\mu\right)\begin{pmatrix}0\\ v+\eta\end{pmatrix}\right]^\dagger \\ &\cdot\left[\left(\partial^\mu - i\frac{g'}{2}B^\mu - ig\frac{\vec{\sigma}}{2}\cdot\vec{W}^\mu\right)\begin{pmatrix}0\\ v+\eta\end{pmatrix}\right] + \frac{\mu^2}{2}(v+\eta)^2 - \frac{\lambda}{4}(v+\eta)^4.\end{aligned}}$$

We obtained the following masses:

$$m_{e,\mu,\tau} = \frac{G_{e,\mu,\tau}\, v}{\sqrt{2}}$$

$$m_\nu = 0$$

$$M_W = \frac{g v}{2} \qquad M_Z = \frac{\sqrt{g^2+g'^2}}{2}\, v$$

Def. $$W_\mu^{(\pm)} = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right), \quad W_\mu^{(+)} = W_\mu$$

$$W_\mu^{(-)} = W_\mu^+$$

$$\begin{cases} Z_\mu = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W \\ A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W \end{cases}$$

$$\tan\theta_W = g'/g$$ Weinberg angle

$O(\eta^2)$: $\frac{1}{2}\partial_\mu \eta \partial^\mu \eta + \eta^2\left(\frac{\mu^2}{2} - \frac{6}{4}\lambda v^2\right) = \frac{1}{2}\partial_\mu \eta \partial^\mu \eta - \mu^2 \eta^2$

$\Rightarrow$ η is a massive particle with mass $\sqrt{2}\,\mu$.

"the Higgs particle"

Now, look at leptons: e, ν: get a term

$$- G_e \frac{v}{\sqrt{2}} \left[\bar{e}_L e_R + \bar{e}_R e_L \right] = - \frac{G_e v}{\sqrt{2}} \bar{e} e$$

$\Rightarrow$ electron has a mass $m_e = \frac{G_e v}{\sqrt{2}}$

(ibid for μ, τ : $m_\ell = \frac{G_\ell v}{\sqrt{2}}$, $\ell = e, \mu, \tau$)

note that $m_\nu = 0$ in the Standard Model

(not true in nature, more on this later)

$\Rightarrow$ Gauge bosons also get a mass: get a term

$$+ \frac{1}{2}\left[\left[-i \frac{g'}{2} B_\mu + i g \frac{\vec{\tau}}{2}\cdot \vec{W}_\mu\right]\begin{pmatrix}0\\ v\end{pmatrix}\right]^\dagger \left[\left(-i \frac{g'}{2} B^\mu - i g \frac{\vec{\tau}}{2}\cdot \vec{W}^\mu\right)\begin{pmatrix}0\\ v\end{pmatrix}\right].$$

Consider one factor:

$$\left(\frac{g'}{2} B_\mu + \frac{g}{2}\vec{\tau}\cdot\vec{W}_\mu\right)\begin{pmatrix}0\\ v\end{pmatrix} = \begin{pmatrix} \frac{g'}{2}B_\mu + \frac{g}{2}W_\mu^3 & \frac{g}{2}(W_\mu^1 - i W_\mu^2) \\ \frac{g}{2}(W_\mu^1 + i W_\mu^2) & \frac{g'}{2}B_\mu - \frac{g}{2}W_\mu^3 \end{pmatrix}\begin{pmatrix}0\\ v\end{pmatrix}$$

Define $$W_\mu^{(\pm)} \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^{(2)}\right) \quad (W^\pm \text{ bosons})$$

(also $W_\mu^{(+)} = W_\mu$, $W_\mu^{(-)} = W_\mu^+$)

$$Z_\mu \equiv \frac{-g' B_\mu + g W_\mu^3}{\sqrt{g^2+g'^2}} \quad (Z\text{-boson})$$

$$A_\mu \equiv \frac{g B_\mu + g' W_\mu^3}{\sqrt{g^2+g'^2}} \quad (\text{the photon})$$

Equivalently: $$\begin{cases} Z_\mu = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W \\ A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W \end{cases}$$

with $\tan\theta_W = g'/g$ (Weinberg angle)

$$\Rightarrow \begin{cases} B_\mu = A_\mu \cos\theta_W - Z_\mu \sin\theta_W \\ W_\mu^3 = A_\mu \sin\theta_W + Z_\mu \cos\theta_W \end{cases}$$

$\Rightarrow$ the term in the Lagrangian is $+\frac{v^2}{2}\left|\frac{g}{\sqrt{2}} W_\mu^{(+)}\right|^2 +$

$$+ \frac{v^2}{2}\cdot\frac{1}{4}(g^2+g'^2)\,|Z_\mu|^2 = + \frac{g^2 v^2}{4} \underbrace{W_\mu^{(+)} W^{\mu(-)}}_{W_\mu^+ W^\mu} + \frac{v^2}{8}(g^2+g'^2) Z_\mu Z^\mu$$

$\sim$ complex $\underbrace{\text{vector field}}_{\text{"}\frac{1}{2}\text{"}(W_\mu^1 W^{1\mu} + W_\mu^2 W^{2\mu})}$

$$\Rightarrow \quad M_W = \frac{g v}{2} \qquad M_Z = \frac{\sqrt{g^2+g'^2}}{2}\, v .$$

$\Rightarrow$ W, Z bosons get a mass

$\Rightarrow$ A_μ (photon) remains massless!

The rest of the Lagrangian:

$$\bar{R}_e\, i\gamma^\mu(\partial_\mu + ig' B_\mu) R_e = \bar{e}_R\, i\gamma\cdot\partial\, e_R - g'\,\bar{e}_R\gamma^\mu e_R \cdot$$

$$\cdot (A_\mu\cos\theta_W - Z_\mu\sin\theta_W) = \bar{e}_R\, i\gamma\cdot\partial\, e_R - \underbrace{g'\cos\theta_W}_{\text{QED coupling} \;=\; e\,(>0)} \cdot$$

$$\cdot\, \bar{e}_R\gamma\cdot A\, e_R + \underbrace{g'\sin\theta_W}_{=\; g\frac{\sin^2\theta_W}{\cos\theta_W}}\, \bar{e}_R\gamma\cdot Z\, e_R$$

$$e = g'\cos\theta_W = g\sin\theta_W$$

think of ν_e as $\nu_{eL} = \frac{1-\gamma_5}{2}\nu_e$

$$\bar{L}_e\, i\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)L_e = \bar{\nu}_e\, i\gamma\cdot\partial\,\nu_e +$$

$$+\, \bar{e}_L\, i\gamma\cdot\partial\, e_L + \frac{1}{2}(\bar{\nu}_e\;\; \bar{e}_L)\gamma^\mu \begin{pmatrix} -g'B_\mu + gW_\mu^3 & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & -g'B_\mu - gW_\mu^3 \end{pmatrix}\begin{pmatrix}\nu_e \\ e_L\end{pmatrix}$$

$$= \bar{\nu}_e\, i\gamma\cdot\partial\,\nu_e + \bar{e}_L\, i\gamma\cdot\partial\, e_L + \frac{1}{2}(\bar{\nu}_e\;\; \bar{e}_L)\cdot\gamma^\mu \cdot$$

$$\cdot \begin{pmatrix} -g\tan\theta_W(\cancel{A_\mu\cos\theta_W} - Z_\mu\sin\theta_W) + g(\cancel{A_\mu\sin\theta_W} + Z_\mu\cos\theta_W) & \sqrt{2}gW_\mu \\ \sqrt{2}\,g\,W_\mu^+ & -g\tan\theta_W(A_\mu\cos\theta_W - Z_\mu\sin\theta_W) - g(A_\mu\sin\theta_W + Z_\mu\cos\theta_W) \end{pmatrix}$$

$$\cdot\begin{pmatrix}\nu_e \\ e_L\end{pmatrix} = \bar{\nu}_e\, i\gamma\cdot\partial\,\nu_e + \bar{e}_L\, i\gamma\cdot\partial\, e_L + \frac{1}{2}(\bar{\nu}_e\;\; \bar{e}_L)\cdot\gamma^\mu$$

$$\cdot\begin{pmatrix} g Z_\mu \frac{1}{\cos\theta_W} & \sqrt{2}\,g\,W_\mu \\ \sqrt{2}\,g\,W_\mu^+ & -2g\sin\theta_W A_\mu + g Z_\mu \frac{2\sin^2\theta_W - 1}{\cos\theta_W}\end{pmatrix}\begin{pmatrix}\nu_e \\ e_L\end{pmatrix}$$

$$= \bar{\nu}_e \, i\gamma\cdot\partial \, \nu_e + \bar{e}_L \, i\gamma\cdot\partial \, e_L + \frac{g}{\sqrt{2}} \left[\bar{\nu}_e \, \gamma\cdot W e_L + \bar{e}_L \gamma\cdot W^{+} \nu_e \right] \qquad (152)$$

$$+ \frac{g}{2\cos\theta_W} \bar{\nu}_e \, \gamma\cdot Z \, \nu_e - \underbrace{g\sin\theta_W}_{e} \, \bar{e}_L \, \gamma\cdot A e_L + \frac{g}{2\cos\theta_W} \cdot$$

$$\cdot (2\sin^2\theta_W - 1) \, \bar{e}_L \, \gamma\cdot Z \, e_L$$

$\Rightarrow$ note that the photon does not couple to neutrinos

$\Rightarrow$ it is indeed charge-neutral! (for electric charge)

Putting these two terms together get:

$$\bar{R}_e \, i\gamma^\mu (\partial_\mu + i g' B_\mu) R_e + \bar{L}_e \, i\gamma^\mu \left(\partial_\mu + i\frac{g'}{2} B_\mu - i g \frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right) L_e$$

$$= \bar{e} \, i\gamma\cdot\partial e + \bar{\nu}_e \, i\gamma\cdot\partial \, \nu_e - e\,\bar{e}\,\gamma\cdot A e + g\frac{\sin^2\theta_W}{\cos\theta_W} \bar{e}_R \gamma\cdot Z e_R$$

$$+ \frac{g}{2\cos\theta_W} \bar{\nu}_e \, \gamma\cdot Z \, \nu_e + \frac{g}{2\cos\theta_W} (2\sin^2\theta_W - 1) \, \bar{e}_L \, \gamma\cdot Z \, e_L +$$

$$+ \frac{g}{\sqrt{2}} \left[\bar{\nu}_e \, \gamma\cdot W e_L + \bar{e}_L \, \gamma\cdot W^{+} \nu_e \right].$$

$\Rightarrow$ in principle need to re-write $-\frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4} \vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu}$

in terms of fields A_μ, Z_μ, W_μ ... let's assume we did

$$\Rightarrow \frac{1}{2}\left[\left(\partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\begin{pmatrix}0\\ v+\eta(x)\end{pmatrix}\right]^{+}\left[\left(\partial_\mu - i\frac{g'}{2}B_\mu -\right.\right.$$

$$\left.\left. - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\begin{pmatrix}0\\ v+\eta(x)\end{pmatrix}\right] = \frac{1}{2}\begin{pmatrix}0 & v+\eta\end{pmatrix}\left(\overleftarrow{\partial}_\mu + i\frac{g'}{2}B_\mu + ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right).$$

$$\left| \left(\partial_\mu - i\,\frac{g'}{2} B_\mu - i g\,\frac{\vec{\tau}}{2}\cdot \vec{W}_\mu\right)\begin{pmatrix} 0 \\ v+\eta \end{pmatrix}\right|^2 = \frac{1}{2}\,\partial_\mu \eta\,\partial^\mu \eta$$

$$+ \frac{g^2}{4}(v+\eta)^2\, W_\mu^+ W^\mu + \underbrace{\frac{g^2+g'^2}{8}}(v+\eta)^2\, Z_\mu Z^\mu$$

$$\frac{1}{8} g^2\left(1+\frac{\sin^2\theta_W}{\cos^2\theta_W}\right) = \frac{g^2}{8\cos^2\theta_W}$$

Note that terms linear in η cancel!

$\Rightarrow$ Combining everything we write:

$$\boxed{\begin{aligned}
\mathcal{L}_{EW} &= \bar{e}\, i\gamma\cdot\partial e + \bar{\nu}_e\, i\gamma\cdot\partial \nu_e - \frac{G_e}{\sqrt{2}}(v+\eta)\bar{e} e - \\
&- \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + \frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta - \mu^2\eta^2 - \lambda v \eta^3 \\
&- \frac{\lambda}{4}\eta^4 + \frac{g^2}{4}(v+\eta)^2 W_\mu^+ W^\mu + \frac{g^2}{8\cos^2\theta_W}(v+\eta)^2 Z_\mu Z^\mu \\
&+ \frac{g}{2\cos\theta_W}\left[2\sin^2\theta_W\,\bar{e}_R\gamma\cdot Z e_R + (2\sin^2\theta_W - 1)\bar{e}_L\gamma\cdot Z e_L\right] \\
&- e\,\bar{e}\gamma\cdot A e + \frac{g}{2\cos\theta_W}\bar{\nu}_e\gamma\cdot Z\nu_e + \frac{g}{\sqrt{2}}\left[\bar{\nu}_e\gamma\cdot W e_L + \right. \\
&\left. + \bar{e}_L\gamma\cdot W^+\nu_e\right]
\end{aligned}}$$

+ (μ,τ terms)

The Electroweak Lagrangian.

$M_W = 80.385 \pm 0.015$ GeV

$M_Z = 91.1876 \pm 0.0021$ GeV

$M_W = \frac{g v}{2}$; $M_Z = \frac{\sqrt{g^2+g'^2}}{2} v$ $\Rightarrow$

$$\Rightarrow \frac{M_W}{M_Z} = \frac{g}{\sqrt{g^2+g'^2}} = \cos\theta_W \Rightarrow \cos\theta_W = \frac{M_W}{M_Z}$$

$$\Rightarrow \sin^2\theta_W \simeq 0.2223 \pm 0.0021$$ (involves quantum corrections)

$$\Rightarrow \sin^2\theta_W \simeq \frac{1}{4} \Rightarrow \sin\theta_W \simeq \frac{1}{2} \Rightarrow \theta_W \simeq 30^\circ$$

What about g? $g = \frac{e}{\sin\theta_W} \Rightarrow \frac{g^2}{4\pi} = \frac{\alpha_{EM}}{\sin^2\theta_W}$, $\alpha_{EM} = e^2/4\pi$

$$\Rightarrow \frac{g^2}{4\pi} \simeq \frac{1/137}{0.22} \simeq .03 \approx \frac{1}{30} \Rightarrow \frac{g^2}{4\pi} \simeq \frac{1}{30}$$

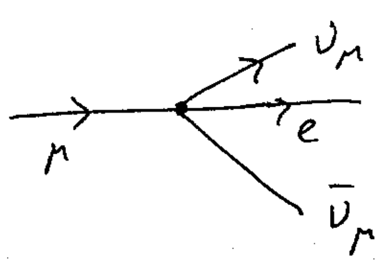
~ very small still, even though it is not as small as α_{EM}.

$\Rightarrow$ What about the old Fermi constant G_F?

Consider this purely leptonic decay:

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

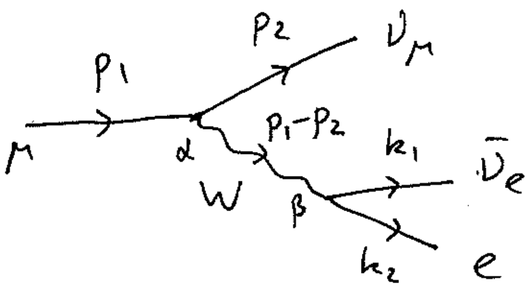
In Fermi theory this decay would be given by the following "effective" vertex:



with $\mathcal{L} = -\frac{G_F}{\sqrt{2}} \bar{\psi}_e \gamma_\mu (1-\gamma_5) \psi_{\nu_e} \bar{\psi}_{\nu_\mu} \gamma^\mu (1-\gamma_5) \psi_\mu$

+ hermitean conjugate

Let's derive this from the Electroweak Lagrangian we wrote. At the lowest order the process is given by the diagram:



$\Rightarrow$ need W - lepton coupling

$$\frac{g}{\sqrt{2}}\left[\bar{\nu}_e\, \gamma\cdot W e_L + \bar{e}_L \gamma\cdot W^+ \nu_e\right]$$

+ same for μ, ν_μ.

$$\frac{g}{\sqrt{2}}\left[\bar{\nu}_e\, \gamma\cdot W e_L + \bar{e}_L\, \gamma\cdot W^+ \nu_e\right] = \frac{g}{\sqrt{2}}\left[\bar{\nu}_e \gamma\cdot W\, \frac{1-\gamma_5}{2} e + \right.$$

$$\left. + \bar{e}\, \frac{1+\gamma_5}{2}\, \gamma\cdot W^+ \nu_e\right] = \frac{g}{2\sqrt{2}}\left[\bar{\nu}_e \gamma\cdot W (1-\gamma_5) e + \right.$$

$$\left. + \bar{e}\, \gamma\cdot W^+ (1-\gamma_5)\nu_e\right].$$

$\Rightarrow$ the diagram is

$$\left(\frac{ig}{2\sqrt{2}}\right)^2 \left[\bar{u}_{\nu_\mu}(p_2)\, \gamma_\alpha (1-\gamma_5)\, u_\mu(p_1)\right] \underbrace{\frac{(-i)\left[g^{\alpha\beta} - \frac{(p_1-p_2)^\alpha (p_1-p_2)^\beta}{M_W^2}\right]}{(p_1-p_2)^2 - M_W^2 + i\varepsilon}}_{\approx \frac{i}{M_W^2} g^{\alpha\beta}} \cdot$$

$$\cdot \left[\bar{u}_e(k_2)\, \gamma_\beta (1-\gamma_5)\, v_{\nu_e}(k_1)\right]$$

if $|p_1 - p_2| \ll M_W$ (low energy)

$$= -i\, \frac{g^2}{8 M_W^2}\left[\bar{u}_{\nu_\mu}(p_2)\, \gamma_\alpha (1-\gamma_5)\, u_\mu(p_1)\right]\left[\bar{u}_e(k_2)\, \gamma^\alpha (1-\gamma_5)\, v_{\nu_e}(k_1)\right]$$

$\Rightarrow$ looks just like the term in Fermi theory

$\Rightarrow$ equate the prefactors:

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}} \Rightarrow G_F = \underbrace{\frac{g^2}{4\pi}}_{.03} \cdot \frac{\pi}{\sqrt{2}} \frac{1}{M_W^2} \simeq 8.5 \cdot 10^{-6}\,\mathrm{GeV}^{-2}$$

$$\approx 10^{-5}\,\mathrm{GeV}^{-2}$$

as advertised.

$\Rightarrow$ What about Higgs and related parameters?

$M_W = \frac{g v}{2} \Rightarrow v = \frac{2}{g} M_W \Rightarrow$ as $g \simeq 0.63$

$\Rightarrow$ get $v \approx 289$ GeV ($v \simeq 246$ GeV PDG value)

The Higgs mass is $m_H = \mu\sqrt{2} = v\sqrt{2\lambda}$

$$\boxed{m_{Higgs} = 125.09 \underset{\text{stat.}}{\pm 0.21} \underset{\text{syst.}}{\pm 0.11}\ \mathrm{GeV}}$$

$\Rightarrow \lambda \approx \frac{1}{2}\left(\frac{m_H}{v}\right)^2 \approx 0.09$ ~ fairly small; $\boxed{\lambda \simeq 0.1}$

about right at the scale of ~1 TeV.