Last time | We finished deriving the Electroweak Lagrangian:

$$\boxed{\begin{aligned}\mathcal{L}_{EW} &= \bar{e}\, i\not{\partial} e + \bar{\nu}_e i\not{\partial} \nu_e - \frac{G_e}{\sqrt{2}}(v+\eta)\,\bar{e}e - \frac{1}{4}f_{\mu\nu}f^{\mu\nu} - \frac{1}{4}\vec{F}_{\mu\nu}\vec{F}^{\mu\nu} \\ &+ \frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta - \mu^2\eta^2 - \lambda v\eta^3 - \frac{\lambda}{4}\eta^4 + \frac{g^2}{4}(v+\eta)^2 W_\mu^+ W^\mu \\ &+ \frac{g^2}{8\cos^2\theta_W}(v+\eta)^2 Z_\mu Z^\mu + \frac{g}{2\cos\theta_W}\Big[2\sin^2\theta_W\, \bar{e}_R\gamma\cdot Z e_R + \\ &+ (2\sin^2\theta_W - 1)\,\bar{e}_L\gamma\cdot Z e_L\Big] - e\,\bar{e}\gamma\cdot A e + \frac{g}{2\cos\theta_W}\bar{\nu}_e\gamma\cdot Z\nu_e \\ &+ \frac{g}{\sqrt{2}}\Big[\bar{\nu}_e\gamma\cdot W e_L + \bar{e}_L\gamma\cdot W^+\nu_e\Big] + (\mu,\tau\text{-terms}).\end{aligned}}$$

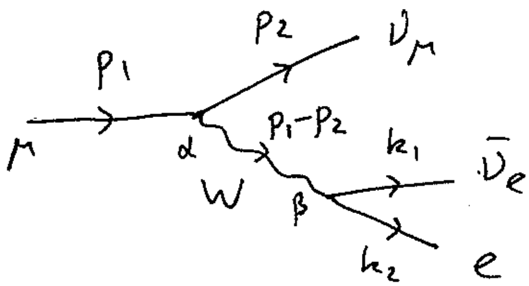
$$M_W = 80.4\ \text{GeV}, \quad M_Z \simeq 91.2\ \text{GeV}$$

$$\frac{M_W}{M_Z} = \cos\theta_W, \quad \sin^2\theta_W = 0.22$$

$$\frac{g^2}{4\pi} = \frac{1}{30}$$ ~ weak coupling

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \Rightarrow G_F \simeq 10^{-5}\,\text{GeV}^{-2}$$

Let's derive this from the Electroweak Lagrangian we wrote. At the lowest order the process is given by the diagram:



$\Rightarrow$ need W - lepton coupling

$$\frac{g}{\sqrt{2}}\left[\bar{\nu}_e \gamma\cdot W e_L + \bar{e}_L \gamma\cdot W^+ \nu_e\right]$$

+ same for μ, ν_μ.

$$\frac{g}{\sqrt{2}}\left[\bar{\nu}_e \gamma\cdot W e_L + \bar{e}_L \gamma\cdot W^+ \nu_e\right] = \frac{g}{\sqrt{2}}\left[\bar{\nu}_e \gamma\cdot W \frac{1-\gamma_5}{2} e + \right.$$

$$\left. + \bar{e}\,\frac{1+\gamma_5}{2}\,\gamma\cdot W^+ \nu_e\right] = \frac{g}{2\sqrt{2}}\left[\bar{\nu}_e \gamma\cdot W (1-\gamma_5) e + \right.$$

$$\left. + \bar{e}\,\gamma\cdot W^+ (1-\gamma_5)\nu_e\right].$$

$\Rightarrow$ the diagram is

$$\left(\frac{ig}{2\sqrt{2}}\right)^2 \left[\bar{u}_{\nu_\mu}(p_2)\,\gamma_\alpha (1-\gamma_5)\, u_\mu(p_1)\right] \underbrace{\frac{(-i)\left[g^{\alpha\beta} - \frac{(p_1-p_2)^\alpha (p_1-p_2)^\beta}{M_W^2}\right]}{(p_1-p_2)^2 - M_W^2 + i\varepsilon}}_{\approx \frac{i}{M_W^2} g^{\alpha\beta}}.$$

$$\cdot \left[\bar{u}_e(k_2)\,\gamma_\beta (1-\gamma_5)\, v_{\nu_e}(k_1)\right]$$

if $|p_1-p_2| \ll M_W$ (low energy)

$$= -i\,\frac{g^2}{8 M_W^2}\left[\bar{u}_{\nu_\mu}(p_2)\,\gamma_\alpha (1-\gamma_5)\, u_\mu(p_1)\right]\left[\bar{u}_e(k_2)\,\gamma^\alpha (1-\gamma_5)\, v_{\nu_e}(k_1)\right].$$

$\Rightarrow$ looks just like the term in Fermi theory

$\Rightarrow$ equate the prefactors:

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}} \Rightarrow G_F = \underbrace{\frac{g^2}{4\pi}}_{.03} \cdot \frac{\pi}{\sqrt{2}} \frac{1}{M_W^2} \simeq 8.5 \cdot 10^{-6}\ \text{GeV}^{-2}$$

$$\approx 10^{-5}\ \text{GeV}^{-2}$$

as advertised.

$\Rightarrow$ What about Higgs and related parameters?

$M_W = \frac{g v}{2} \Rightarrow v = \frac{2}{g} M_W \Rightarrow$ as $g \simeq 0.63$

$\Rightarrow$ get $v \approx 289$ GeV $\quad$ ($v \simeq 246$ GeV PDG value)

The Higgs mass is $m_H = \mu\sqrt{2} = v\sqrt{2\lambda}$

$$\boxed{m_{Higgs} = 125.09 \pm \underset{\text{Stat.}}{0.21} \pm \underset{\text{syst.}}{0.11}\ \text{GeV}}$$

$\Rightarrow \lambda \approx \frac{1}{2}\left(\frac{m_H}{v}\right)^2 \approx 0.09$ ~ fairly small, $\boxed{\lambda \simeq 0.1}$

about right at the scale of ~1 TeV.

Quarks in the Electroweak Theory.

Quarks also form left-handed doublets under weak isospin:

$$L_u = \begin{pmatrix} u \\ d' \end{pmatrix}_L \qquad L_c = \begin{pmatrix} c \\ s' \end{pmatrix}_L \qquad L_t = \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

$$R_u = u_R \qquad R_c = c_R \qquad R_t = t_R$$

$$R_d = d_R \qquad R_s = s_R \qquad R_b = b_R$$

$$L_u = \begin{pmatrix} u \\ d' \end{pmatrix}_L$$ a doublet $\Rightarrow I_3 = \frac{\pm 1}{2} \Rightarrow Q = I_3 + \frac{Y}{2}$

$\Rightarrow Y = 2(Q - I_3) \Rightarrow$ for u have $Q = +\frac{2}{3}$, $I_3 = +\frac{1}{2} \Rightarrow$

$\Rightarrow Y = 2\left(\frac{2}{3} - \frac{1}{2}\right) = \frac{1}{3}$; for d' have $Q = -\frac{1}{3}$, $I_3 = -\frac{1}{2} \Rightarrow$

$\Rightarrow Y = 2\left(-\frac{1}{3} + \frac{1}{2}\right) = \frac{1}{3} \Rightarrow Y = \frac{1}{3}$ for the doublet!

Singlets: $R_u = u_R$ has $Q = +\frac{2}{3}$, $I_3 = 0 \Rightarrow Y = \frac{4}{3}$

$R_d = u_d$ has $Q = -1/3$, $I_3 = 0 \Rightarrow Y = -2/3$

(same for other quark generations / families)

$\Rightarrow$ We have <u>defined</u> the quark <u>weak</u> eigenstates d', s', b' by:

$$\underbrace{\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}}_{\substack{\text{weak} \\ \text{eigenstates}}} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{\substack{\text{CKM} \\ \text{matrix}}} \underbrace{\begin{pmatrix} d \\ s \\ b \end{pmatrix}}_{\substack{\text{quarks in QCD} \\ \text{(mass eigenstates)}}}$$

CKM = $\underbrace{\overset{1963}{\text{Cabibbo}}}_{\text{? No prize?}}$ - $\underbrace{\overset{1973}{\text{Kobayashi - Maskawa}}}_{\text{Nobel Prize '08}}$ matrix

CKM matrix is unitary: $V^\dagger V = V V^\dagger = \mathbb{1}$.

(Logic: our mass matrix for quarks is diagonal, but there is no reason for the EW interaction one to be diagonal too.)

Let's write down the Lagrangian:

$$\mathcal{L}_{\text{quarks + gauge}} = \bar{L}_u \, i\gamma^\mu \Big(\partial_\mu - i\frac{g'}{2}\underbrace{Y}_{=1/3} B_\mu - i g\,\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\Big) L_u$$

$$+ \bar{R}_u \, i\gamma^\mu \Big(\partial_\mu - i\frac{g'}{2}\underbrace{Y}_{=\frac{4}{3}} B_\mu\Big) R_u + \bar{R}_d \, i\gamma^\mu \Big(\partial_\mu - i\frac{g'}{2} B_\mu \underbrace{Y}_{=-2/3}\Big) R_d$$

+ other 2 generations.

$$\Rightarrow \boxed{\begin{aligned} \mathcal{L}_{\text{quarks + gauge}} &= \bar{L}_u \, i\gamma^\mu \Big(\partial_\mu - i\frac{g'}{6} B_\mu - i g\,\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\Big) L_u \\ &+ \bar{R}_u \, i\gamma^\mu \Big(\partial_\mu - i\,\frac{2}{3} g' B_\mu\Big) R_u + \bar{R}_d \, i\gamma^\mu \Big(\partial_\mu + i\,\frac{1}{3} g' B_\mu\Big) R_d \\ &+ \text{2 more generations.} \end{aligned}}$$

Need to couple quarks to the Higgs: (don't have to, but it would be nice $\Uparrow$ for $SU(2)_L \times U(1)_Y$)

$$\phi = \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix}.$$

We may write a term like $\bar{L}_u \phi R_u$ and $\bar{L}_u \phi R_d$.

However the VEV is $\langle 0|\phi|0\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \Rightarrow$

$\Rightarrow$ near the Higgs VEV get

$$\bar{L}_u \phi R_u = (\bar{u}_L\ \bar{d}_L') \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} u_R = \bar{d}_L' u_R \frac{v}{\sqrt{2}}$$ ~ no mass for u ...

$Y=-1/3$, $Y=+1$, $Y=+4/3$ $\Rightarrow$ not $U(1)_Y$ invariant too...

~ like neutrinos, u would not get a mass... ?

(same for c, t quarks).

$\Rightarrow$ to give quarks mass define $\tilde{\phi}(x) \equiv i\tau^2 \phi^*$

for the VEV: $\langle 0|\tilde{\phi}|0\rangle = \underbrace{i\tau^2}_{=\begin{pmatrix} 0 & -i \\ i & 0\end{pmatrix}} \cdot \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} =$

$= \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$. ~ have the VEV $\neq 0$ on top now

Under $SU(2)_L$ gauge transform: $\phi \to e^{i\frac{\vec{\alpha}\cdot\vec{\tau}}{2}}\phi$

$$\Rightarrow \tilde{\phi} \to i\tau^2 \left(e^{i\frac{\vec{\alpha}\cdot\vec{\tau}}{2}}\phi\right)^* = i\tau^2 e^{-i\frac{\vec{\alpha}\cdot\vec{\tau}^*}{2}} \phi^* =$$

$(\tau^2)^2 = \mathbb{1}$

$$= \underbrace{\tau^2 e^{-i\frac{\vec{\alpha}\cdot\vec{\tau}^*}{2}} \tau_2}_{e^{i\frac{\vec{\alpha}\cdot\vec{\tau}}{2}}} \tilde{\phi}$$

this is true because: $\tau^2(-\tau^{1*})\tau^2 = \begin{pmatrix} 0 & -i \\ i & 0\end{pmatrix}$.

$$\begin{pmatrix} 0 & -1 \\ -1 & 0\end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0\end{pmatrix} = \begin{pmatrix} 0 & -i \\ i & 0\end{pmatrix}\begin{pmatrix} -i & 0 \\ 0 & i\end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix} = \tau^1$$

Similarly $\tau^2(-\tau^{2*})\tau^2 = \tau^2$ (obvious) and

$\tau^2(-\tau^{3*})\tau^3 = \tau^3$ $\Rightarrow$ equ is true $\left(\begin{array}{l}\tau^2(-\vec{\tau}^*)\tau^2 = \vec{\tau},\ (\tau^2)^2=\mathbb{1} \\ \Rightarrow \text{sandwich } (\tau^2)^2\text{'s in}\end{array}\right)$

$\Rightarrow$ under $SU(2)_L$ have $\tilde{\phi} \to e^{i\frac{\vec{\alpha}\cdot\vec{\tau}}{2}}\tilde{\phi}$

$\Rightarrow$ transforms just like ϕ !

$\Rightarrow$ can write $\bar{L}_u \tilde{\phi} R_u$ ~ $SU(2)_L$ invariant!

near VEV: $\bar{L}_u \tilde{\phi} R_u = (\bar{u}_L \ \bar{d}'_L)\begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} u_R = \frac{v}{\sqrt{2}} \bar{u}_L u_R$

$\langle 0|\tilde{\phi}|0\rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$ $\Rightarrow$ may give u-quark a mass!

terms like $\bar{L}_u \phi R_d = (\bar{u}_L \ \bar{d}'_L)\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} d_R = \frac{v}{\sqrt{2}} \bar{d}'_L d_R$

can give d-quark mass (and s, b quarks too).

$\Rightarrow$ also need to check weak hypercharge:

ϕ has $Y=+1$ $\Rightarrow$ $\tilde{\phi}$ has $Y=-1$ $\Rightarrow$ $\underbrace{\bar{L}_u}_{Y=-1/3} \underbrace{\tilde{\phi}}_{Y=-1} \underbrace{R_u}_{Y=4/3}$ $\Rightarrow$ net $Y=0$

$\underbrace{\bar{L}_u}_{Y=-1/3} \underbrace{\phi}_{Y=+1} R_d \Rightarrow Y=-2/3$ $\Rightarrow$ net $Y=0$ ~ both work!

To write quarks + Higgs couplings let's limit ourselves to 2 generations: $L_u, L_c, R_u, R_d, R_c, R_s$.

First write all possible terms:

$$\mathcal{L}_{\text{quarks+Higgs}} = -G_1[\bar{L}_u\tilde{\phi}R_u + \bar{R}_u\tilde{\phi}^+L_u] - G_2[\bar{L}_u\phi R_d + \bar{R}_d\phi^+L_u] - G_3[\bar{L}_u\phi R_s + \bar{R}_s\phi^+L_u] - G_4[\bar{L}_c\tilde{\phi}R_c + \bar{R}_c\tilde{\phi}^+L_c]$$

$$- G_5 \left[\bar{L}_c \phi R_d + \bar{R}_d \phi^\dagger L_c \right] - G_6 \left[\bar{L}_c \phi R_s + \bar{R}_s \phi^\dagger L_c \right]$$

$$- G_7 \left[\bar{L}_u \tilde{\phi} R_c + \bar{R}_c \tilde{\phi}^\dagger L_u \right] - G_8 \left[\bar{L}_c \tilde{\phi} R_u + \bar{R}_u \tilde{\phi}^\dagger L_c \right].$$

Plug in $\langle 0|\phi|0\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$, $\langle 0|\tilde{\phi}|0\rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$:

$$\mathcal{L}^{\text{2 generations}}_{\text{quark + Higgs}} = -\frac{v}{\sqrt{2}} \Big\{ G_1 \bar{u} u + G_2 (\bar{d}'_L d_R + \bar{d}_R d'_L) + G_3 (\bar{d}'_L s_R +$$

$$+ \bar{s}_R d'_L) + G_4 \bar{c} c + G_5 (\bar{s}'_L d_R + \bar{d}_R s'_L) + G_6 (\bar{s}'_L s_R + \bar{s}_R s'_L)$$

$$+ G_7 (\bar{u}_L c_R + \bar{c}_R u_L) + G_8 (\bar{c}_L u_R + \bar{u}_R c_L) \Big\}$$

$\Rightarrow$ first of all we see $m_u = \frac{G_1 v}{\sqrt{2}}$ $m_c = \frac{G_4 v}{\sqrt{2}}$.

$\Rightarrow$ can't have $u \to c$ & vice versa $\Rightarrow \boxed{G_7 = G_8 = 0}$.

$\Rightarrow$ Left with d, s quarks: for those write:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$\theta_c \sim$ Cabibbo angle, in CKM matrix $V_{ud} \simeq \cos\theta_c \simeq V_{cs}$,

$V_{us} \simeq \sin\theta_c \simeq -V_{cd}$

$\theta_c \simeq 13^\circ$ small mixing.

$\Rightarrow d' = d\cos\theta_c + s\sin\theta_c$

$s' = -d\sin\theta_c + s\cos\theta_c$

$$\Rightarrow \mathcal{L}^{d,s\ \text{part}}_{\text{quark + Higgs}} = -\frac{v}{\sqrt{2}}\Big\{ G_2\Big[\underline{\bar{d}d\cos\theta_c} + (\bar{s}_L d_R + \bar{d}_R s_L)\cdot$$

$$\sin\theta_c\Big] + G_3\Big[\underline{\bar{s}s\sin\theta_c} + (\bar{d}_L s_R + \bar{s}_R d_L)\cos\theta_c\Big] +$$

$$+ G_5\Big[\underline{-\bar{d}d\sin\theta_c} + (\bar{s}_L d_R + \bar{d}_R s_L)\cos\theta_c\Big] +$$

$$+ G_6\Big[\underline{\bar{s}s\cos\theta_c} - (\bar{d}_L s_R + \bar{s}_R d_L)\sin\theta_c\Big]\Big\} =$$

$$= -\bar{d}d\frac{v}{\sqrt{2}}\Big[G_2\cos\theta_c - G_5\sin\theta_c\Big] - \bar{s}s\frac{v}{\sqrt{2}}\Big[G_3\sin\theta_c +$$

$$+ G_6\cos\theta_c\Big] - \frac{v}{\sqrt{2}}(\bar{s}_L d_R + \bar{d}_R s_L)\underbrace{\Big[G_2\sin\theta_c + G_5\cos\theta_c\Big]}_{=0}$$

$$- \frac{v}{\sqrt{2}}(\bar{d}_L s_R + \bar{s}_R d_L)\underbrace{\Big[G_3\cos\theta_c - G_6\sin\theta_c\Big]}_{=0}$$

$\Rightarrow$ don't want $d \leftrightarrow s$ $\Rightarrow$ $G_5 = -G_2\tan\theta_c$

$$G_6 = G_3\cot\theta_c$$

$$\Rightarrow m_d = \frac{v}{\sqrt{2}}\Big[G_2\cos\theta_c - G_5\sin\theta_c\Big] = \boxed{\frac{v}{\sqrt{2}}\frac{G_2}{\cos\theta_c} = m_d}$$

$$m_s = \frac{v}{\sqrt{2}}\Big[G_3\sin\theta_c + G_6\cos\theta_c\Big] = \boxed{\frac{v}{\sqrt{2}}\frac{G_3}{\sin\theta_c} = m_s}$$

$\Rightarrow$ instead of unknown m_u, m_d, m_s, m_c have constants G_1, G_2, G_3, G_4 also unknown...

$$\mathcal{L}_{\text{quark+Higgs}} = -\sum m_f \bar{q}^f q^f \; v + h(x)$$ ~ the net quarks + Higgs Lagrangian

CKM matrix (absolute values)

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \simeq \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.225 & 0.973 & 0.041 \\ 0.009 & 0.041 & 0.999 \end{pmatrix}$$

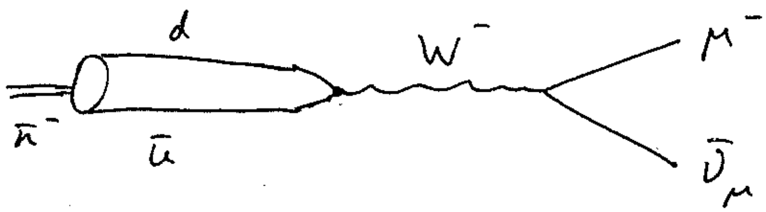
~ "almost" diagonal.

Why do we need d', s', b'? Look at $\mathcal{L}$:

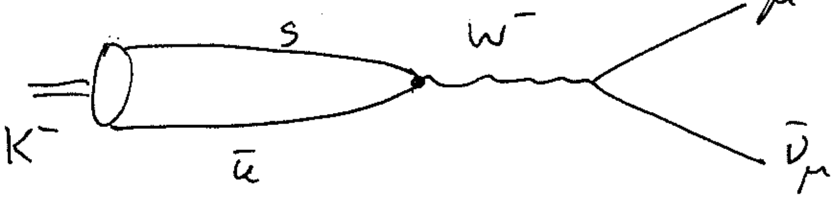
$$g(\bar u_L \; \bar d'_L)\, i\gamma^\mu \underbrace{\frac{\vec\tau}{2}\cdot\vec W_\mu}_{W_\mu} \begin{pmatrix} u_L \\ d'_L \end{pmatrix} \Rightarrow \text{has}$$

$$g\, \bar u_L\, \gamma\cdot W_\mu\, d'_L + g\, \bar d'_L\, \gamma\cdot W_\mu^+ u_L$$

Experimentally one has the following decays:



$\pi^- \to \mu^- \bar\nu_\mu$



$K^- \to \mu^- \bar\nu_\mu$