Elementary Particle Physics II (8802.02)

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grading: based on HWs ← once in 1-2 weeks

class notes: online

syllabus: online

textbook: T.-P. Cheng & L.-F. Li "Gauge Theory of Elementary Particle Physics"

more listed on-line

Exams: by request

Brief Review of First Semester

I will not review the parts of the 1^{st} semester which were QFT. Consider QFT as our main tool, but the subject will be particle physics.

~~Quark Model and Group Theory~~

~ defined <u>isospin</u> operator $\vec{I} \Rightarrow \vec{I}^2, I_z$ have eigenvalues $I(I+1)$ and $-I, \dots, I$ correspondingly

(p, n or π^+, π^0, π^-, etc.)

~ defined <u>baryon #</u> : # of Baryons (B)

~ <u>strangeness</u>: $\underbrace{K^+, K^0}_{S=+1}, \underbrace{\bar{K}^0, K^-}_{S=-1}$

$$Q = I_3 + \frac{Y}{2}$$

electric charge

Gell-Mann – Nishijima

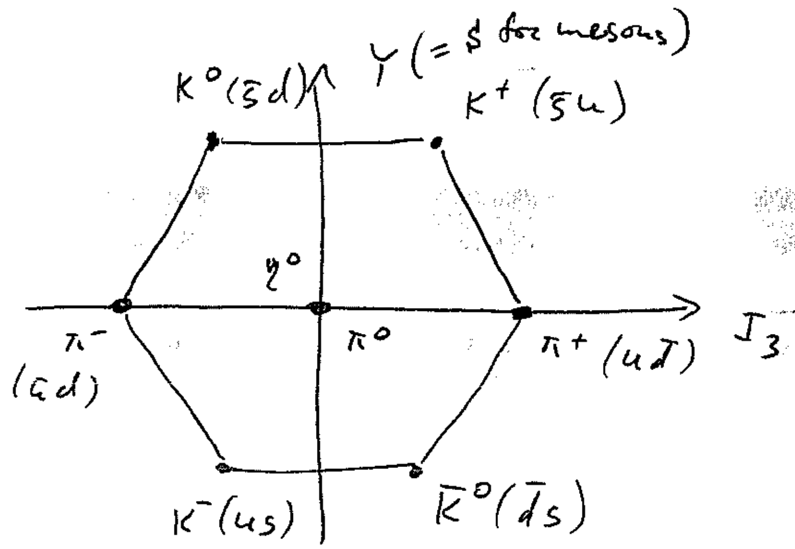
~ <u>hyper charge</u>: $Y \equiv B + S$

"Eight fold Way":

$$\pi^0 = \frac{\bar{u}u - \bar{d}d}{\sqrt{2}}$$

$$\eta^0 = \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s)$$

0^- mesons (pseudoscalar mesons)



Gell-Mann & Ne'eman ('61) Zweig

Quarks: u, d, s, c, b, t

have $Q = +2/3$ or $-1/3$, $B = +1/3$, u,d have $I = \frac{1}{2}, I_z = \pm\frac{1}{2}$

s-quark has $S = -1$.

~ another quantum #: color $i = 1, 2, 3$

$\Rightarrow u_i(x)$ ~ 3 colors of up quark.

mesons, baryons ~ always color-neutral!

$\Rightarrow$ quarks interact with each other by exchanging gluons ~ spin-1 non-Abelian gauge fields: A_μ^a, $a = 1, \dots, 8$ ~ gluon color $SU(3)_c$

quark fields: q^{if} (i ← color, f ← flavor).

$$\mathcal{L}_{QCD} = \bar{q}^{if}(i\gamma\cdot\partial - m_f)q^{if} - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + g\,\bar{q}^{jf}\gamma^\mu A^a_\mu (t^a)_{ji}\, q^{if}$$

with t^a the generators of group $SU(3)$ in the fundamental representation: $t^a = \frac{\lambda^a}{2}$

λ^a ~ Gell-Mann matrices

$[t^a, t^b] = i f^{abc} t^c$, f^{abc} ~ $SU(3)$ structure constants.

gluon field strength:

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

defining $A_\mu = \sum_{a=1}^{8} A_\mu^a t^a$ & defining the covariant derivative $D_\mu = \partial_\mu - i g A_\mu$ get

$$\boxed{\mathcal{L}_{QCD} = \bar{q}^f (i \gamma \cdot D - m_f) q^f - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}}$$

~ We studied group theory. In particular for SU(3) we showed that the following is true:

$$\boxed{3 \otimes \bar{3} = 1 \oplus 8}$$

$$\boxed{3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10}$$

$\Rightarrow$ for <u>mesons</u> made out of u, d, s quarks due to $3 \otimes \bar{3} = 1 \oplus 8$ get a flavor-octet. η' ~ singlet.

<u>Baryons</u>: $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10 \Rightarrow$ have an octet and a decuplet ~ all agrees with experiment.

~ also works for colors.

Quark-only Lagrangian: $N_f = 3$

$$\mathcal{L} = \bar{q}\,(i\gamma\cdot\partial - m)\,q \,, \quad q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \quad m = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

=> defined $\quad q_L = \dfrac{1-\gamma_5}{2}\,q \,, \quad q_R = \dfrac{\mathbb{1}+\gamma_5}{2}\,q$

$$\Rightarrow \mathcal{L}_{m=0} = \bar{q}_L\, i\gamma\cdot\partial\, q_L + \bar{q}_R\, i\gamma\cdot\partial\, q_R$$

=> $SU(3)_L \otimes SU(3)_R$ invariant ! $\quad q_L \to e^{i\vec{\alpha}_L\cdot\vec{t}}\, q_L$

chiral symmetry $\quad q_R \to e^{i\vec{\alpha}_R\cdot\vec{t}}\, q_R$

add mass but with $m = m_u = m_d = m_s$ =>

get $\mathcal{L} = \bar{q}_L\, i\gamma\cdot\partial\, q_L + \bar{q}_R\, i\gamma\cdot\partial\, q_R - m\,[\bar{q}_L q_R + \bar{q}_R q_L]$

=> $SU(3)_L \otimes SU(3)_R$ is broken down to $SU(3)$.

if $m_u \neq m_d \neq m_s$ $\;SU(3)$ is also broken:

$$\boxed{SU(3)_L \otimes SU(3)_R}$$

$\downarrow$ $\quad m_u = m_d = m_s \neq 0$ ⊕ SSB

$$\boxed{SU(3)}$$ ← still ≈ exists (Eightfold Way)

$\downarrow$ $\quad m_u \neq m_d \neq m_s \neq 0$

Nothing

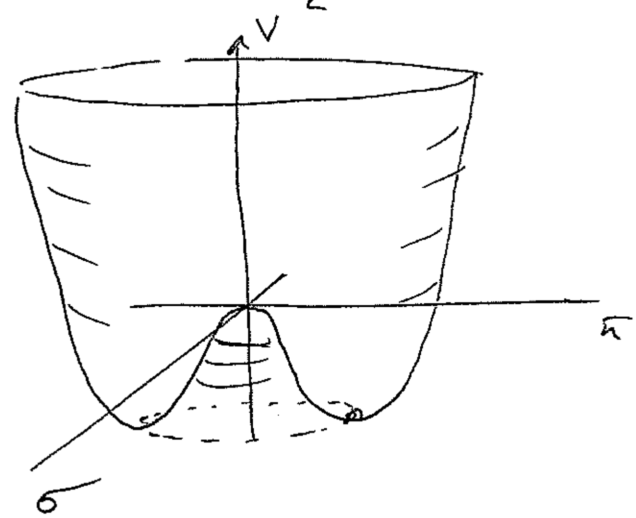
Spontaneous Symmetry Breaking (SSB)

~ symmetry manifest in $\mathcal{L}$, but not respected by ground state.

~ <u>Nambu - Goldstone theorem</u>: spontaneous breakdown of a continuous symm. $\Rightarrow$ massless spinless particles (Nambu-Goldstone bosons).

<u>Example</u>: Abelian σ - Model

$$\mathcal{L} = \frac{1}{2} \partial_\mu \sigma \, \partial^\mu \sigma + \frac{1}{2} \partial_\mu \pi \, \partial^\mu \pi + \frac{\mu^2}{2}(\sigma^2 + \pi^2) - \frac{\lambda}{4}(\sigma^2 + \pi^2)^2$$



at the minimum

$$\sigma^2 + \pi^2 = v^2 = \frac{\mu^2}{\lambda}$$

$\Downarrow$

pick vacuum at

$$\langle 0 | \sigma | 0 \rangle = v = \frac{\mu}{\sqrt{\lambda}}$$

$$\langle 0 | \pi | 0 \rangle = 0$$

$\Rightarrow$ exand near the vacuum: $\sigma = v + \sigma'$, π :

$$\mathcal{L} = \frac{1}{2} \partial_\mu \sigma' \partial^\mu \sigma' + \frac{1}{2} \partial_\mu \pi \, \partial^\mu \pi - \mu^2 \sigma'^2 - \lambda v \sigma' (\sigma'^2 + \pi^2) - \frac{\lambda}{4}(\sigma'^2 + \pi^2)^2$$

$\Rightarrow$ π is <u>massless</u> : Goldstone boson

σ' has mass $\sqrt{2}\mu$.

$U(1)$ symmetry broken spontaneously

Example: non-abelian σ-model:

$\pi \to \vec{\pi} = (\pi^1, \pi^2, \pi^3)$ ~ pion field.

$q^N = \begin{pmatrix} p \\ n \end{pmatrix}$ ~ fermions.

$SU(2)_L \otimes SU(2)_R$ symmetric

after SSB get: σ' has mass $\mu\sqrt{2}$

q^N have mass $g\,v$

$\vec{\pi}$ have mass $\emptyset$ (Goldstone bosons)

$SU(2)_L \otimes SU(2)_R$ spont. broken to $SU(2)$.

In QCD: pions (π^+, π^-, π^0) are Goldstone bosons of chiral SSB, $m_\pi = 0$ (as $m_u \neq m_d \neq 0$ not exact)

$v = \langle 0|\bar{q}\, q|0\rangle = -(230\,MeV)^3$.

The Electroweak Theory.

local vs global gauge symmetries:

$$\boxed{\mathcal{L}_{QED} = \bar{\psi}\left[i\gamma^\mu D_\mu - m\right]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$D_\mu = \partial_\mu - ig A_\mu \quad \Rightarrow \begin{cases} \psi \to e^{i\alpha(x)}\psi \\ A_\mu \to A_\mu + \frac{1}{g}\partial_\mu \alpha \end{cases}$$

local $U(1)$ symmetry (abelian)

non-abelian:

$$\mathcal{L} = \bar{\psi}\left[i\gamma^\mu D_\mu - m\right]\psi - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

$$D_\mu = \partial_\mu - ig A_\mu, \quad A_\mu = \sum_a t^a A_\mu^a, \quad F_{\mu\nu} = \frac{i}{g}\left[D_\mu, D_\nu\right] =$$

$$= \partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu\right].$$

$$\begin{cases} \psi \to S(x)\psi \\ A_\mu \to S(x) A_\mu S^{-1}(x) - \frac{i}{g}(\partial_\mu S) S^{-1} \end{cases}$$

$S(x)$ ~ unitary $N \times N$ matrix $\Rightarrow$ $SU(N)$ local symm.

Higgs mechanism: $\mathcal{L} = (D_\mu \varphi)^* (D^\mu \varphi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} +$
$U(1)$ model

$$+\mu^2 \varphi^* \varphi - \lambda(\varphi^* \varphi)^2$$

$\Rightarrow$ $U(1)$ gauge symm. $\Rightarrow$ pick a VEV: $\langle 0|\varphi|0\rangle = \frac{v}{\sqrt{2}} = \frac{\mu}{\sqrt{2\lambda}}$

$\Rightarrow$ write $\varphi = \frac{\rho'(x)}{\sqrt{2}} e^{i\theta(x)}$, $B_\mu(x) = A_\mu - \frac{1}{g}\partial_\mu \theta(x)$

& expand $\rho' = v + \rho$ around the VEV. One gets:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \rho \partial^\mu \rho - \mu^2 \rho^2 - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \frac{1}{2} g^2 v^2 B_\mu B^\mu$$
$$+ \frac{1}{2} g^2 B_\mu B^\mu (2v\rho + \rho^2) - \lambda v \rho^3 - \frac{\lambda}{4}\rho^4$$

ρ has mass $\sqrt{2}\mu$

B_μ ~ massive gauge field $m_B = gv$

(no Goldstone bosons) θ – would-be Goldstone boso

$SU(2)\otimes U(1)$ Electroweak theory: $\psi_{L,R} = \frac{1\mp\gamma_5}{2}\psi$

leptons

$Y=-1$ $L_e = \begin{pmatrix}\nu_e\\ e\end{pmatrix}_L$, $L_\mu = \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L$, $L_\tau = \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L$, $\boxed{Q = I_3 + \frac{Y}{2}}$

weak isospin (I_3), weak hypercharge (Y)

$Y=-2$ $R_e = e_R$, $R_\mu = \mu_R$, $R_\tau = \tau_R$.

gauge field: $\vec{W}_\mu = (W_\mu^1, W_\mu^2, W_\mu^3)$, $\vec{F}_{\mu\nu}$

B_μ with $f_{\mu\nu}$

Higgs field: $\phi = \begin{pmatrix}\phi^{(+)}\\ \phi^{(0)}\end{pmatrix}$, $Y=+1$, $\phi^\dagger = (\phi^{(-)}, \phi^{(0)*})$

$$\boxed{\begin{aligned}\mathcal{L}_{\text{leptons + gauge + Higgs}} &= \bar{R}_e i\gamma^\mu(\partial_\mu + i\underbrace{g'}_{-\frac{g'}{2}\cdot(-2)=Y} B_\mu)R_e + \bar{L}_e i\gamma^\mu(\partial_\mu + i\underbrace{\tfrac{g'}{2}}_{-\frac{g'}{2}\cdot(-1)=Y}B_\mu - \\ &- ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu)L_e + (\mu,\tau) - \frac{1}{4}f_{\mu\nu}f^{\mu\nu} - \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + \\ &+ \left[\left(\partial_\mu - i\overbrace{\tfrac{g'}{2}}^{-\frac{g'}{2}\cdot(+1)=Y}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\phi\right]^\dagger\left[\left(\partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\phi\right] \\ &+\mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 - G_e[\bar{L}_e\phi R_e + \text{c.c.}] - (\mu,\tau)\end{aligned}}$$

$Y = +1, +1, -2 = 0$

$SU(2)_L\otimes U(1)_Y$ symmetric

VEV (to break the $SU(2)_L\otimes U(1)_Y \to U(1)_{EM}$) $\langle 0|\phi|0\rangle = \begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}$, $\phi(x) = e^{-i\frac{\vec{\tau}}{2}\cdot\vec{\theta}(x)}\begin{pmatrix}0\\ \frac{v+\eta(x)}{\sqrt{2}}\end{pmatrix}$

$\vec{\tau} = (\tau^1, \tau^2, \tau^3)$ ~ Pauli matrices

Def.

$W^\pm$ bosons
$$W_\mu = \frac{1}{\sqrt{2}}(W_\mu^1 - iW_\mu^2)$$
$$W_\mu^\dagger = \frac{1}{\sqrt{2}}(W_\mu^1 + iW_\mu^2)$$

Z-boson
$$Z_\mu = -B_\mu\sin\theta_W + W_\mu^3\cos\theta_W$$

photon →
$$A_\mu = B_\mu\cos\theta_W + W_\mu^3\sin\theta_W$$

$\tan\theta_W = g'/g$ Weinberg angle

get $\mathcal{L} = \bar{e}\, i\gamma\cdot\partial e + \bar{\nu}_{e_L} i\gamma\cdot\partial \nu_{e_L} - \frac{G_e}{\sqrt{2}}(v+\eta)\bar{e}e -$

$$- \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + \frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta - \mu^2\eta^2 - \lambda v\eta^3 - \frac{\lambda}{4}\eta^4$$

$$+ \frac{g^2}{4}(v+\eta)^2 W_\mu^{+} W^{\mu} + \frac{g^2}{8\cos^2\theta_W}(v+\eta)^2 Z_\mu Z^\mu +$$

$$+ \frac{g}{2\cos\theta_W}\left[2\sin^2\theta_W\, \bar{e}_R\gamma\cdot Z e_R + (2\cdot\sin^2\theta_W - 1)\,\bar{e}_L\gamma\cdot Z e_L\right]$$

$$- e\,\bar{e}\gamma\cdot A e + \frac{g}{2\cos\theta_W}\bar{\nu}_{e_L}\gamma\cdot Z\,\nu_{e_L} + \frac{g}{\sqrt{2}}\left[\bar{\nu}_e\gamma\cdot W e_L + c.c.\right] +$$

$$+ (\mu, \tau)$$

$$M_W = \frac{gv}{2} \simeq 80.4 \text{ GeV}$$

$$M_Z = \frac{gv}{2\cos\theta_W} \simeq 91.2 \text{ GeV}$$

$$m_\gamma = 0$$

$$m_e = \frac{G_e v}{\sqrt{2}}, \quad m_\mu = \frac{G_\mu v}{\sqrt{2}}, \quad m_\tau = \frac{G_\tau v}{\sqrt{2}}$$

$m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0$ ($\geq .04$ eV in reality)

$\theta_W \simeq 30°$, $\frac{g^2}{4\pi} \simeq \frac{1}{30}$ ~ small

$M_H = \mu\sqrt{2} = v\sqrt{2\lambda}$, $v \simeq 246$ GeV

$m_H \simeq 125$ GeV

Quarks in EW theory:

$$\mathcal{L}_{\text{quarks + gauge}} = \bar{L}_u \, i\gamma^\mu \Big(\partial_\mu - i\frac{g'}{6} B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu \Big) L_u$$

$$\frac{g'}{6} = \frac{1}{2}\cdot\frac{1}{3}$$

$$+ \bar{R}_u \, i\gamma^\mu \Big(\partial_\mu - i\frac{2}{3}g' B_\mu \Big) R_u + \bar{R}_d \, i\gamma^\mu \Big(\partial_\mu + i\frac{g'}{3} B_\mu \Big) R_d$$

$$\frac{2}{3} = \frac{1}{2}\cdot\frac{4}{3}, \qquad \frac{1}{3} = \frac{1}{2}\cdot\left(-\tfrac{2}{3}\right)$$

+ 2 more generations.

$$2(Q - I_3) = Y$$

$$L_u = \begin{pmatrix} u \\ d' \end{pmatrix}_L, \quad L_c = \begin{pmatrix} c \\ s' \end{pmatrix}_L, \quad L_t = \begin{pmatrix} t \\ b' \end{pmatrix}_L \qquad 1/3$$

$$R_u = u_R \qquad R_c = c_R \qquad R_t = t_R \qquad 4/3$$

$$R_d = d_R \qquad R_s = s_R \qquad R_b = b_R \qquad -2/3$$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{\text{CKM matrix}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates — CKM matrix — mass eigenstates

Cabibbo - Kobayashi - Maskawa

(unitary)

quarks - Higgs coupling: $\tilde{\phi} = i\tau^2 \phi^*$, $Y = -1$ (Pauli matrix $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$)

$$\Rightarrow \text{write} \quad \mathcal{L} = -G_1 \left[\bar{L}_u \tilde{\phi} R_u + \bar{R}_u \tilde{\phi}^+ L_u \right] -$$

$$- G_2 \left[\bar{L}_u \phi R_d + \bar{R}_d \phi^+ L_u \right] + \text{other flavors } (c, t)$$

(all terms with $Y = 0$ ($U(1)$ inv) & $SU(2)$ inv.)

$\Rightarrow$ get quark masses.

(163)

$$\Rightarrow \mathcal{L}^{d,s \text{ part}}_{\text{quark+Higgs}} = -\frac{v}{\sqrt{2}}\Big\{ G_2\Big[\underline{\bar d d \cos\theta_c} + (\bar s_L d_R + \bar d_R s_L)\cdot$$

$$\sin\theta_c\Big] + G_3\Big[\underline{\bar s s \sin\theta_c} + (\bar d_L s_R + \bar s_R d_L)\cos\theta_c\Big] +$$

$$+ G_5\Big[\underline{-\bar d d \sin\theta_c} + (\bar s_L d_R + \bar d_R s_L)\cos\theta_c\Big] +$$

$$+ G_6\Big[\underline{\bar s s \cos\theta_c} - (\bar d_L s_R + \bar s_R d_L)\sin\theta_c\Big]\Big\} =$$

$$= -\bar d d \frac{v}{\sqrt{2}}\Big[G_2\cos\theta_c - G_5\sin\theta_c\Big] - \bar s s\frac{v}{\sqrt{2}}\Big[G_3\sin\theta_c +$$

$$+ G_6\cos\theta_c\Big] - \frac{v}{\sqrt{2}}(\bar s_L d_R + \bar d_R s_L)\underbrace{\Big[G_2\sin\theta_c + G_5\cos\theta_c\Big]}_{=0}$$

$$- \frac{v}{\sqrt{2}}(\bar d_L s_R + \bar s_R d_L)\underbrace{\Big[G_3\cos\theta_c - G_6\sin\theta_c\Big]}_{=0}$$

$\Rightarrow$ don't want $d \leftrightarrow s$ $\Rightarrow$ $G_5 = -G_2\tan\theta_c$

$G_6 = G_3\cot\theta_c$

$$\Rightarrow m_d = \frac{v}{\sqrt{2}}\Big[G_2\cos\theta_c - G_5\sin\theta_c\Big] = \boxed{\frac{v}{\sqrt{2}}\frac{G_2}{\cos\theta_c} = m_d}$$

$$m_s = \frac{v}{\sqrt{2}}\Big[G_3\sin\theta_c + G_6\cos\theta_c\Big] = \boxed{\frac{v}{\sqrt{2}}\frac{G_3}{\sin\theta_c} = m_s}$$

$\Rightarrow$ instead of unknown m_u, m_d, m_s, m_c have constants G_1, G_2, G_3, G_4 also unknown...

$\mathcal{L}_{\text{quark+Higgs}} = -\sum_f m_f \bar q^f q^f \, \underline{v+h(x)}$ ~ the net quarks + Higgs Lagrangian in unitary gauge

CKM matrix (absolute values)

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \simeq \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.225 & 0.973 & 0.041 \\ 0.009 & 0.041 & 0.999 \end{pmatrix}$$

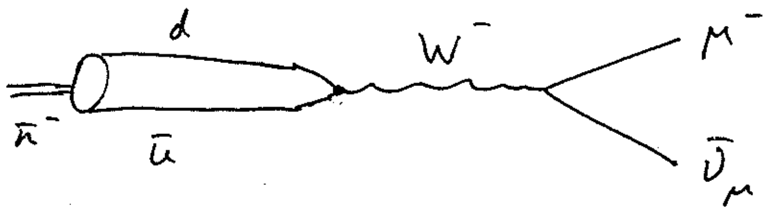
~ "almost" diagonal.

Why do we need d', s', b'? Look at $\mathcal{L}$:

$$g(\bar{u}_L\ \bar{d}'_L)\, i\gamma^\mu \underbrace{\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu}_{W_\mu} \begin{pmatrix} u_L \\ d'_L \end{pmatrix} \Rightarrow \text{has}$$

$$g\, \bar{u}_L\, \gamma\cdot W_\mu\, d'_L + g\, \bar{d}'_L\, \gamma\cdot W^+_\mu\, u_L$$

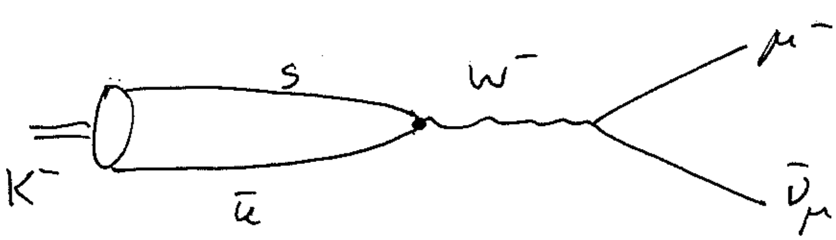
Experimentally one has the following leptonic decays:



$$\pi^- \to \mu^- \bar{\nu}_\mu$$

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

↑ Cabibbo angle



$$K^- \to \mu^- \bar{\nu}_\mu$$