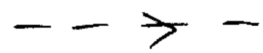
Last time

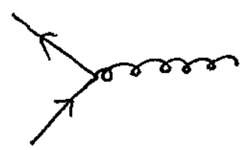
Feynman Rules in QCD (cont'd)

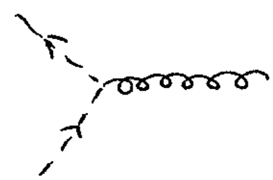
(i) Covariant (Lorenz) gauge ($\partial_\mu A^\mu = 0$)

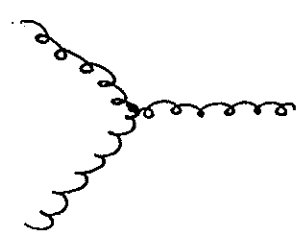
Quark Propagator 

Ghost Propagator 

Gluon Propagator 

Quark-Gluon Vertex 

Ghost-Gluon Vertex 

3-Gluon Vertex 

4-Gluon Vertex 

(ii) Light-cone gauge ($A^+ = 0$)

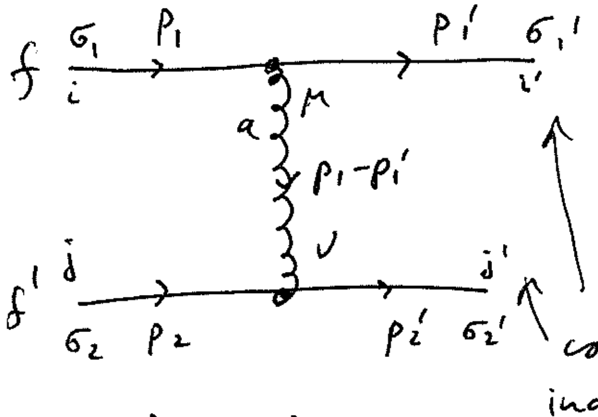
No ghosts, different propagator for gluons 
the rest of the rules are the same

Example of Using QCD Perturbation Theory:

Quark - Quark scattering, $q_f + q_{f'} \to q_f + q_{f'}$, $f \neq f'$

~ quarks have different flavors

Only one diagram (massless quarks):



$$iM = \bar{u}_{\sigma_1'}(p_1')\gamma^\mu u_{\sigma_1}(p_1)\ \bar{u}_{\sigma_2'}(p_2')\gamma^\nu u_{\sigma_2}(p_2) \cdot ig(t^a)_{i'i}\ ig(t^a)_{j'j}\ \frac{-i g_{\mu\nu}}{(p_1-p_1')^2 + i\varepsilon}$$

Mandelstam $t = (p_1 - p_1')^2$.

$$\Rightarrow \underbrace{\frac{1}{N_c^2}}_{\text{color averaging}}\ \frac{1}{4}\sum_{\substack{\sigma_1,\sigma_1' \\ \sigma_2,\sigma_2'}} |M|^2 = \frac{1}{4} g^4\ \frac{1}{t^2} \cdot \underbrace{\underbrace{tr[t^b t^a]}_{=\frac{1}{2}\delta^{ab}}\ \underbrace{tr[t^b t^a]}_{=\frac{1}{2}\delta^{ab}} \cdot \frac{1}{N_c^2}}_{\text{color factor } = \frac{1}{4}(N_c^2-1)}$$

spin averaging

$$\cdot\ tr[\not{p}_1' \gamma^\mu \not{p}_1 \gamma^\alpha] \cdot tr[\not{p}_2' \gamma_\mu \not{p}_2 \gamma_\alpha] =$$

$$= \frac{1}{4} g^4 \frac{1}{t^2} \cdot \frac{1}{4N_c^2}(N_c^2-1)\ 4\left[p_1'^\mu p_1^\alpha + p_1'^\alpha p_1^\mu - g^{\mu\alpha} p_1 \cdot p_1'\right].$$

↑ # colors, $N_c = 3$

$$\cdot 4\left[p_{2\mu}' p_{2\alpha} + p_{2\alpha}' p_{2\mu} - g_{\mu\alpha}\, p_2 \cdot p_2'\right] = \frac{g^4}{t^2}\frac{(N_c^2-1)}{N_c^2} \cdot \Big[2 p_1 \cdot p_2\, p_1' \cdot p_2'$$

$$+ 2 p_1' \cdot p_2\, p_2' \cdot p_1 + \cancel{4\, p_1 \cdot p_1'\, p_2 \cdot p_2'} - \cancel{4\, p_1 \cdot p_1'\, p_2 \cdot p_2'}\Big] = \frac{2g^4}{t^2}\frac{(N_c^2-1)}{N_c^2}$$

$$\cdot \left[p_1 \cdot p_2\, p_1' \cdot p_2' + p_1' \cdot p_2\, p_2' \cdot p_1\right]$$

assume quarks to be massless:

$$s = (p_1 + p_2)^2 = 2 p_1 \cdot p_2 \quad , \quad p_1 + p_2 = p_1' + p_2'$$

energy-momentum conservation

$$\Rightarrow s = (p_1' + p_2')^2 = 2 p_1' \cdot p_2'$$

$$\Rightarrow p_1 \cdot p_2 \; p_1' \cdot p_2' = \frac{1}{4} s^2$$

$$\left.\begin{aligned} u &= (p_1 - p_2')^2 = -2 p_1 \cdot p_2' \\ u &= (p_2 - p_1')^2 = -2 p_2 \cdot p_1' \end{aligned}\right\} \Rightarrow p_1 \cdot p_2' \; p_2 \cdot p_1' = \frac{u^2}{4}$$

$$\Rightarrow \boxed{\langle |M|^2 \rangle = \frac{g^4}{2t^2} \frac{(N_c^2 - 1)}{N_c^2} [s^2 + u^2]} = \frac{N_c^2 - 1}{4 N_c^2} \otimes \text{QED term}$$

$$d\sigma = \frac{1}{2E_1 \, 2E_2 \underbrace{|\vec{v}_1 - \vec{v}_2|}_{=2 \text{ (massless quarks, CMS frame)}}} \frac{d^3 p_1'}{2E_1' (2\pi)^3} \frac{d^3 p_2'}{(2\pi)^3 \, 2E_2'} \cdot (2\pi)^4 \delta^4 (p_1 + p_2 - p_1' - p_2')$$

$$\cdot \langle |M|^2 \rangle$$

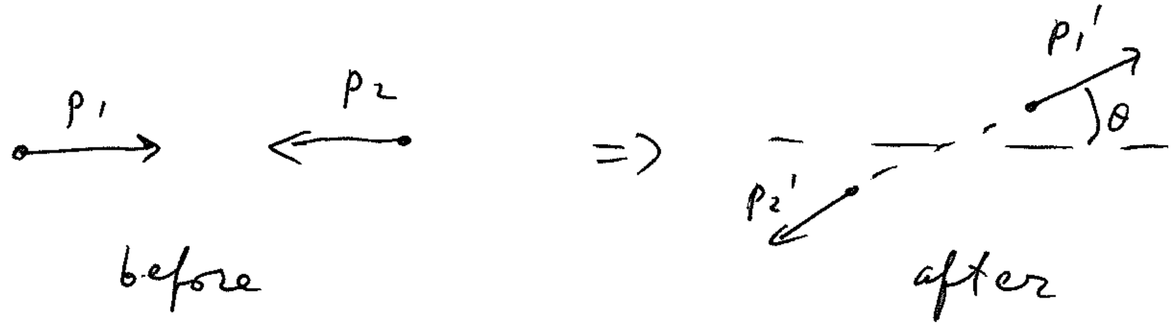
$$= \frac{1}{8 E_1 E_2} \int \frac{\overbrace{d^3 p_1'}^{= p_1'^2 \, dp_1' \, d\cos\theta \, d\varphi}}{2E_1' (2\pi)^3} \frac{1}{2E_2'} 2\pi \, \delta (E_1 + E_2 - E_1' - E_2')$$

$$\cdot \langle |M|^2 \rangle = \frac{1}{8 E_1 E_2} \frac{1}{4 (2\pi)^3} \int dE_1' \cdot \frac{E_1'}{E_2'} \delta (E_1 + E_2 - E_1' - E_2') \cdot$$

$$\cdot \langle |M|^2 \rangle \cdot 2\pi \, d\cos\theta$$

where $E_2' = E_1'$ (CMS frame) & $M = M(\theta)$ only $\Rightarrow$

$$\Rightarrow d\sigma = \frac{1}{8 E_1^2} \frac{\pi}{16 \pi^2} \cdot d\cos\theta \, \langle |M|^2 \rangle$$



$$t = (p_1 - p_1')^2 \underset{\substack{\uparrow \\ \text{massless} \\ \text{quarks}}}{=} -2 p_1 \cdot p_1' = -2\left(E_1 E_1' - \vec{p}_1 \cdot \vec{p}_1'\right) =$$

$$= -2 E_1^2 (1 - \cos\theta) \leq 0$$

note, $\boxed{t \leq 0}$

$$dt = -2 E_1^2 \sin\theta \, d\theta = 2 E_1^2 \, d\cos\theta$$

$$\Rightarrow d\cos\theta = \frac{dt}{2E_1^2}$$

$$d\sigma = \frac{1}{8E_1^2} \frac{1}{16\pi} \cdot \frac{dt}{2E_1^2} \langle |M|^2 \rangle$$

$$s = (p_1 + p_2)^2 = 2 p_1 \cdot p_2 = 2\left(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2\right) = 4 E_1^2$$

$$\Rightarrow E_1^2 = s/4 \Rightarrow d\sigma = \frac{1}{2s} \frac{1}{16 \cdot \pi} 2 \frac{dt}{s} \langle |M|^2 \rangle$$

$$\Rightarrow \frac{d\sigma}{dt} = \frac{1}{16\pi s^2} \langle |M|^2 \rangle = \frac{1}{16\pi s^2} \frac{g^4}{2t^2} \frac{N_c^2 - 1}{N_c^2} (s^2 + u^2)$$

$$= \Big| \; \alpha_s = \frac{g^2}{4\pi} \quad = \frac{\alpha_s^2 \pi}{s^2} \frac{1}{t^2} \frac{N_c^2 - 1}{2N_c^2} (s^2 + u^2)$$

$$\Rightarrow \boxed{\frac{d\sigma^{f+f' \to f+f'}}{dt} = \frac{\pi \alpha_s^2}{s^2} \frac{N_c^2 - 1}{2N_c^2} \frac{s^2 + u^2}{t^2}}$$