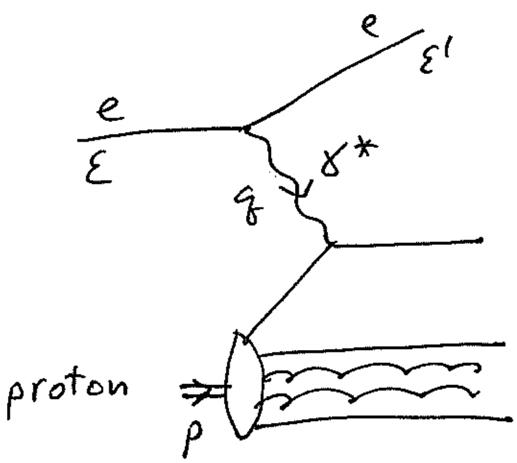
Last time | Parton Model and DIS

DIS (cont'd)



$$e + p \to e + X$$

$$\boxed{Q^2 = -q^2}$$ photon virtuality

$$\boxed{x = \frac{Q^2}{2p\cdot q}}$$ Bjorken x

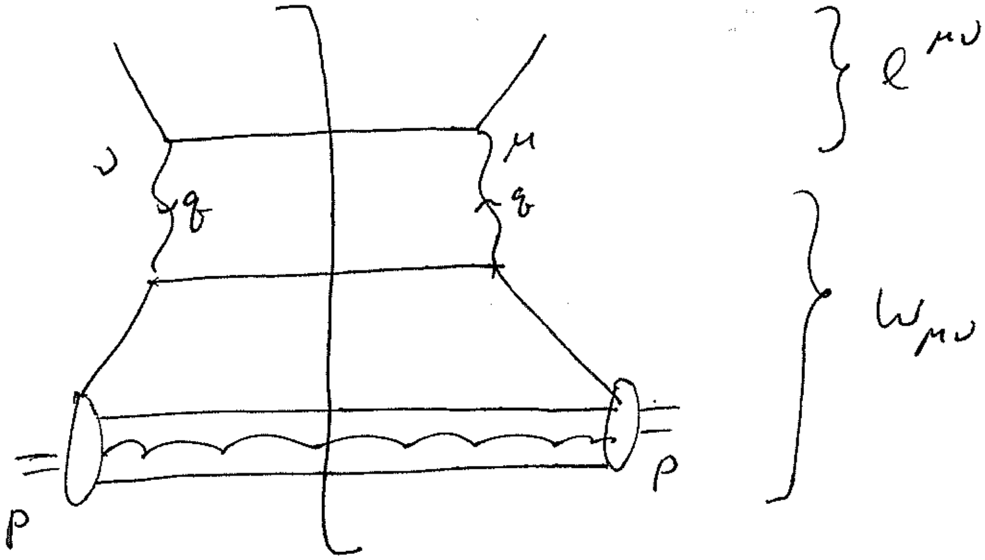
$$\boxed{\frac{d\sigma}{d^3k'} = \frac{\alpha_{EM}^2}{Q^4\,\varepsilon\,\varepsilon'}\, l_{\mu\nu} W^{\mu\nu}}$$ ← rest frame of the proton

$$l_{\mu\nu} = 2\left(k_\mu k_\nu' + k_\nu k_\mu' - g_{\mu\nu}\, k\cdot k'\right)$$ leptonic tensor (put $m_e = 0$)

$$W_{\mu\nu} = \frac{1}{4\pi m_p}\int d^4x\, e^{iq\cdot x}\langle p|\, j_\mu(x)\, j_\nu(0)\,|p\rangle$$

hadronic tensor

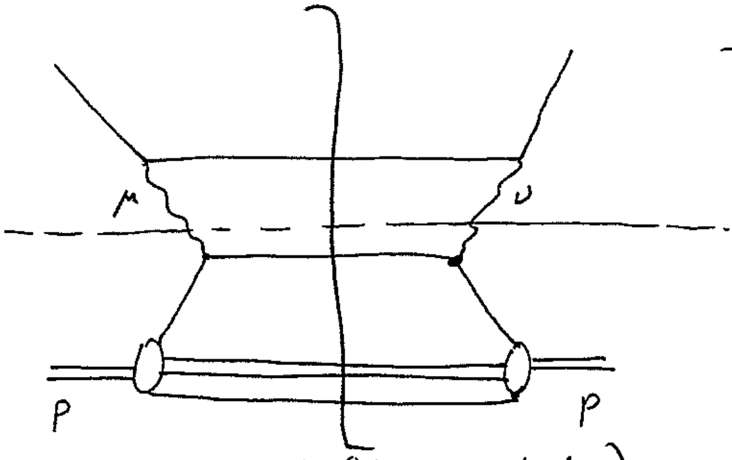
$$j_\mu(x) = \sum_f e_f\, \bar{q}_f(x)\,\gamma_\mu\, q_f(x)$$ ~ EM current


ν
μ
q
q
p
p
ℓ^{μν}
W_{μν}

$$\boxed{W_{\mu\nu} = \frac{1}{4\pi m_p} \int d^4x \, e^{iq\cdot x} \langle p | j_\mu(x) \, j_\nu(0) | p \rangle}$$

(averaged over σ)

$\leftarrow$ proton polarization



$\Big\}$ $l_{\mu\nu}$ (EM part)

$\Big\}$ $W_{\mu\nu}$ (hadronic part)

cut (final state)

$$W_{\mu\nu}(p,q) = A(x,Q^2)\, p_\mu p_\nu + B(x,Q^2)\, q_\mu q_\nu + C(x,Q^2)\, g_{\mu\nu} +$$

$$+ D(x,Q^2)\,(p_\mu q_\nu + p_\nu q_\mu) + E(x,Q^2)\,(p_\mu q_\nu - p_\nu q_\mu) +$$

$$+ F(x,Q^2)\, \varepsilon_{\mu\nu\rho\sigma}\, p^\rho q^\sigma$$

$F = 0$ in $\gamma^* p$, $\gamma^* A$ (F comes from γ_5's, appears in ν DIS).

(1) $q_\mu W^{\mu\nu} = 0$ (current conservation)

$q_\mu \to \partial_\mu \Rightarrow \partial_\mu j^\mu(x) = 0$

$$A\, p_\nu\, (p\cdot q) + B\, q_\nu \cdot q^2 + C\, q_\nu + D\,(p\cdot q\, q_\nu + q^2 p_\nu)$$

$$+ E\,(p\cdot q\; q_\nu - q^2 p_\nu) = 0$$

(2) $q_\nu W^{\mu\nu} = 0 = A\, p_\mu\,(p\cdot q) + B\, q^2 q_\mu + C q_\mu + D(p\cdot q\; q_\mu +$

$$+ q^2 p_\mu) + E\,(p_\mu q^2 - p\cdot q\; q_\mu) = 0$$

$(1) - (2) = 0 \quad \Rightarrow \quad (E = 0.)$

p_μ and q_μ are independent $\Rightarrow$

$$0 = A\, p\cdot q + D q^2 \qquad 0 = B q^2 + C + D\, p\cdot q$$

$$D = -A \frac{p\cdot q}{q^2} \qquad B = -\frac{1}{q^2} C + A \left(\frac{p\cdot q}{q^2}\right)^2$$

$$W_{\mu\nu} = A\left[p_\mu p_\nu - \frac{p\cdot q}{q^2}(p_\mu q_\nu + p_\nu q_\mu) + \left(\frac{p\cdot q}{q^2}\right)^2 q_\mu q_\nu\right] + C\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right]$$

Usually one writes

$$W_{\mu\nu} = -W_1(x, Q^2)\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right] + \frac{W_2(x, Q^2)}{m_p^2} \cdot \left[p_\mu p_\nu - \frac{p\cdot q}{q^2}(p_\mu q_\nu + p_\nu q_\mu) + \left(\frac{p\cdot q}{q^2}\right)^2 q_\mu q_\nu\right]$$

W_1 & W_2 are <u>structure</u> <u>functions</u> (Def.)

Using $q_\mu \ell^{\mu\nu} = q_\nu \ell^{\mu\nu} = 0$ yields

$$\ell_{\mu\nu} W^{\mu\nu} = -W_1 (-4 \underbrace{k\cdot k'}_{2\varepsilon\varepsilon' \sin^2\frac{\theta}{2}}) + \frac{2W_2}{m_p^2} \underbrace{\left[2 p\cdot k\; p\cdot k' - m^2 k\cdot k'\right]}_{2m^2\varepsilon\varepsilon' - 2m^2\varepsilon\varepsilon'\sin^2\frac{\theta}{2}}$$

$$2m^2 \varepsilon\varepsilon' - 2m^2 \varepsilon\varepsilon' \sin^2\frac{\theta}{2} = 2m^2 \varepsilon\varepsilon' \cos^2\frac{\theta}{2}$$

$$q_\mu l^{\mu\nu} = (k-k')_\mu \, 2\left(k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu}\, k\cdot k'\right) =$$

$$= 2\left(k^2 k'^\nu + \cancel{k^\nu\, k\cdot k'} - \cancel{k^\nu\, k\cdot k'} - \cancel{k'\cdot k\, k'^\nu} - k^\nu k'^2\right.$$

$$\left. + \cancel{k'^\nu\, k\cdot k'}\right) = 2\left(k^2 k'^\nu - k'^2 k^\nu\right) \simeq 0 \quad \text{as } k^2 \simeq k'^2 \simeq 0$$

(neglect electron's mass), $q_\nu l^{\mu\nu} = 0$ (similar)

$$\Rightarrow l_{\mu\nu} W^{\mu\nu} = l^{\mu\nu}\left[-W_1\left(g_{\mu\nu} - \frac{\cancelto{0}{q_\mu q_\nu}}{q^2}\right) + \frac{W_2}{m_p^2}\right.$$

$$\left.\left(p_\mu p_\nu - \frac{p\cdot q}{q^2}\left(p_\mu \cancelto{0}{q_\nu} + p_\nu \cancelto{0}{q_\mu}\right) + \left(\frac{p\cdot q}{q^2}\right)^2 \cancelto{0}{q_\mu q_\nu}\right)\right]$$

$$= -l^\mu{}_\mu W_1 + \frac{W_2}{m_p^2}\, p_\mu p_\nu l^{\mu\nu} = \left|\begin{array}{l} \text{as } l^{\mu\nu} = 2(k^\mu k'^\nu + \\ + k^\nu k'^\mu - g^{\mu\nu} k\cdot k') \end{array}\right.$$

$$= -2\left(2k\cdot k' - \overset{= g^\mu{}_\mu}{4}\, k\cdot k'\right) W_1 + \frac{W_2}{m_p^2}\, 2\left(2p\cdot k\; p\cdot k' - p^2\, k\cdot k'\right)$$

$$= 4\, k\cdot k'\, W_1 + 2\,\frac{W_2}{m_p^2}\left(2p\cdot k\; p\cdot k' - m_p^2\, k\cdot k'\right)$$

Remember: $k^\mu = (\varepsilon, 0, 0, \varepsilon)$, $k'^\mu = (\varepsilon', \varepsilon'\sin\theta, 0, \varepsilon'\cos\theta)$

$p^\mu = (m_p, \vec{0})$

$$\Rightarrow k\cdot k' = 2\varepsilon\varepsilon' \sin^2(\theta/2)\ ; \quad p\cdot k = m_p\,\varepsilon\,,\ p\cdot k' = \varepsilon' m_p$$

$$l_{\mu\nu} W^{\mu\nu} = 4\varepsilon\varepsilon'\left[2W_1 \sin^2\frac{\theta}{2} + W_2\cos^2\frac{\theta}{2}\right]$$

$$\boxed{\frac{d\sigma}{d^3k'} = \frac{4\alpha_{EM}^2}{Q^4}\left[2W_1 \sin^2\frac{\theta}{2} + W_2\cos^2\frac{\theta}{2}\right]}$$

By varying the angle θ can separate W_1 & W_2 contributions in experiments. $\Rightarrow$ Rosenbluth separation

Usually one defines $F_1(x,Q^2) = m_p W_1(x,Q^2)$, $F_2(x,Q^2) = \nu\, W_2(x,Q^2)$

The Parton Model.

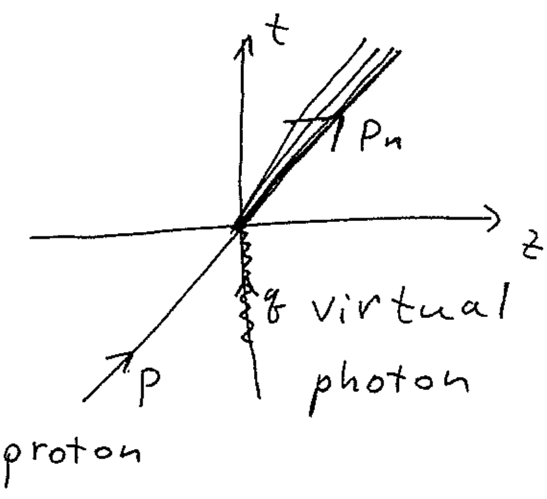
Sterman ch 14, Peskin 17.5
Y.K. & Levin, ch. 2.2

Go to Infinite Momentum Frame:

$$p_\mu \approx \Big(\underset{0}{p+\frac{m^2}{2p}},\ \underset{1}{0},\ \underset{2}{0},\ \underset{3}{p}\Big),$$

$\underline{q} = (q^1, q^2) \sim$ 2d vector in transverse plane

$$q_\mu = \Big(\underset{0}{q_0},\ \underset{1,2}{\underline{q}},\ \underset{3}{0}\Big),$$



Q^2 and x are 2 invariants
large, $Q \gg \Lambda_{QCD}$

$p \cdot q = m\nu = q_0 \cdot p$

$\Rightarrow q_0 = \dfrac{m\nu}{p} \sim$ small as p goes large, $p \gg Q$

$\Rightarrow$ as $\nu = \dfrac{Q^2}{2m_p x}$ $\Rightarrow q_0 = \dfrac{Q^2}{2xp}$

$\Rightarrow Q^2 = -q^2 = \underline{q}^2$

$\Rightarrow q_0 \ll Q$ since $xp \gg Q$.

$$F_1(x, Q^2) = m_p W_1(x, Q^2)$$

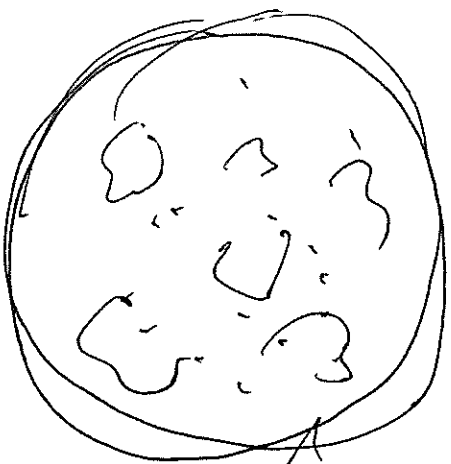
$$F_2(x, Q^2) = \nu W_2(x, Q^2) = \frac{Q^2}{2 m_p x} W_2(x, Q^2)$$

$$\frac{d\sigma}{d^3k'} = \frac{4\alpha_{EM}^2}{Q^4}\left[2 \cdot \frac{1}{m_p} F_1(x, Q^2) \sin^2\frac{\theta}{2} + \frac{2 m_p x}{Q^2} F_2(x, Q^2) \cdot \cos^2(\theta/2) \right]$$

F_1, F_2 are dimensionless

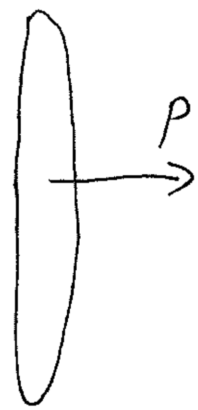
proton at rest

infinite momentum frame (IMF)



ultra-
-boost

$\longrightarrow$



quantum fluctuations

time scale

$$\tau_{fluct.} \simeq \frac{1}{\Lambda_{QCD}}$$ ← only scale in the problem

$$t_{fluct.} = \gamma \tau_{fluct} = \gamma \frac{1}{\Lambda_{QCD}}$$

$$\gamma = \frac{p}{m_p}$$

$$\Rightarrow t_{fluct} = \frac{p}{m_p} \frac{1}{\Lambda_{QCD}}$$

$\frac{p}{m_p} \simeq 7000$ at LHC
($\sqrt{s} = 14$ TeV)

$\Rightarrow$ fluctuations live much longer in IMF $\Rightarrow$
$\Rightarrow$ can probe them

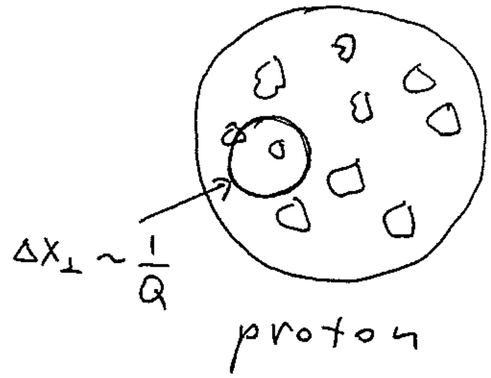
$\Rightarrow$ IMF "freezes" time for quantum fluctuations, allowing us to measure them.

$Q^2 = \not{q}^2 \Rightarrow$ photon acts like a <u>microscope</u>

in <u>transverse</u> plane:

$\Delta x_\perp \cdot q_\perp \sim 1 \qquad (\hbar = 1)$

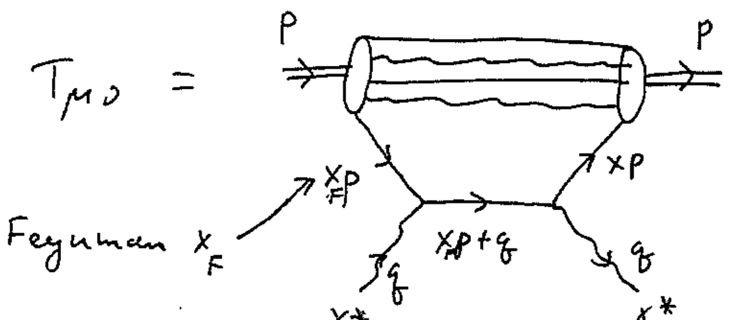
$\Delta x_\perp \sim \frac{1}{q_\perp} \sim \frac{1}{Q}$



large Q ~ resolve just 1 quark

Define $\boxed{T_{\mu\nu} = \frac{1}{4\pi m_p} \int d^4x \, e^{iq\cdot x} \, \frac{1}{2} \sum_\sigma \langle p, \sigma | T j_\mu(x) j_\nu(0) | p, \sigma \rangle}$

$W_{\mu\nu} = 2 \, \text{Im} \, (i \, T_{\mu\nu})$ (optical theorem)

$T_{\mu\nu} =$ 

"Forward Amplitude"

Def Feynman-x: the fraction of proton's longitudinal momentum carried by struck quark

typical interaction time in proton's rest frame is $\frac{1}{\Lambda_{QCD}} \Rightarrow$ boost to get $\frac{p}{m} \frac{1}{\Lambda} \equiv \tau_\Lambda$

int. time of DIS is $\tau_{DIS} \simeq \frac{1}{q^0}$, where

$q^0 \simeq \frac{Q^2}{2x_B p}$ ($= \frac{m_\nu}{p}$ ← as $m_\nu = \frac{Q^2}{2x}$) is struck quark's velocity: $\tau_{DIS} \simeq \frac{2x_B p}{Q^2}$.

time-ordered product: (denoted T)

$$\boxed{T\, j_\mu(x)\, j_\nu(y) \equiv \Theta(x^0-y^0)\, j_\mu(x)\, j_\nu(y) + \Theta(y^0-x^0)\, j_\nu(y)\, j_\mu(x)}$$

note: currents do not commute with each other in general $\Rightarrow$ not a trivial object.

$$2\,\mathrm{Im}\left(i\,T_{\mu\nu}\right) = 2\,\mathrm{Im}\Big[i \cdot \frac{1}{4\pi m_p} \int d^4x\, e^{iq\cdot x} \langle p|\, \Theta(x^0)\, \underbrace{j_\mu(x)}_{= e^{ip\cdot x} j_\mu(0) e^{-ip\cdot x}} \underset{\sum_n |n\rangle\langle n|}{\wedge} j_\nu(0) +$$

$$+ \Theta(-x^0)\, j_\nu(0)\, j_\mu(x)\, |p\rangle \Big] = 2\cdot \frac{1}{4\pi m_p} \sum_n \mathrm{Re}\Big\{ \int d^4x\, e^{iq\cdot x + ip\cdot x - ip_n\cdot x}$$

$$\cdot\, \Theta(x^0)\, \langle p| j_\mu(0)|n\rangle\langle n| j_\nu(0)|p\rangle + \int d^4x\, e^{iq\cdot x + ip_n\cdot x - ip\cdot x}\, \Theta(-x^0)$$

$$\cdot \langle p| j_\nu(0)|n\rangle\langle n| j_\mu(0)|p\rangle \Big\} = \sum_n 2\,\frac{1}{4\pi m_p} \Big\{ (2\pi)^3\, \delta(\vec q + \vec p - \vec p_n) \cdot \mathrm{Re}$$

$$\left(\frac{-1}{i(q^0+p^0-p_n^0+i\varepsilon)} \langle p| j_\mu(0)|n\rangle\langle n| j_\nu(0)|p\rangle \right) + (2\pi)^3\, \delta(\vec q + \vec p_n - \vec p) \cdot$$

$$\cdot\, \mathrm{Re}\left(\frac{1}{i(q^0+p_n^0-p^0-i\varepsilon)} \langle p| j_\nu(0)|n\rangle\langle n| j_\mu(0)|p\rangle \right) \Big\}$$

$\propto \delta(q^0+p_n^0-p^0)$ $= 0$ not physical $\Rightarrow$ drop (after including $\delta(q^0+p_n^0-p^0)$)

$$= 2\,\frac{1}{4\pi m_p} \sum_n (2\pi)^3\, \delta(\vec q + \vec p - \vec p_n)\, (-) \left(\mathrm{Im}\, \frac{1}{q^0+p^0-p_n^0+i\varepsilon} \right) \cdot \langle p| j_\mu(0)|n\rangle$$

$$\langle n| j_\nu(0)|p\rangle = \left|\ \text{as } \mathrm{Im}\,\frac{1}{x+i\varepsilon} = -\pi\,\delta(x)\right. = \frac{1}{4\pi m_p} \sum_n (2\pi)^4\, \delta^4(q+p-p_n)$$

$$\cdot \langle p| j_\mu(0)|n\rangle\langle n| j_\nu(0)|p\rangle = W_{\mu\nu} \quad \text{as desired.}$$

(can prove that $\langle p| j_\mu(0)|n\rangle\langle n| j_\nu(0)|p\rangle$ is real)