Homework Set No. 3, Physics 880.02

Deadline – Tuesday, May 19, 2009

1. Let us study "DIS" on a single quark of mass m. Virtual photon interacts with the quark through the electromagnetic current

$$j_\mu = e_f \, \bar{q} \gamma_\mu q$$

with e_f the electric charge of the quark. Define, for $q^0 > 0$ and $Q^2 = -q^2 > 0$,

$$W_{\mu\nu} = \int d^4x \, e^{iq\cdot x} \, \frac{1}{2} \sum_\sigma \langle p, \sigma | \, j_\mu(x) \, j_\nu(0) \, | p, \sigma \rangle \tag{1}$$

and

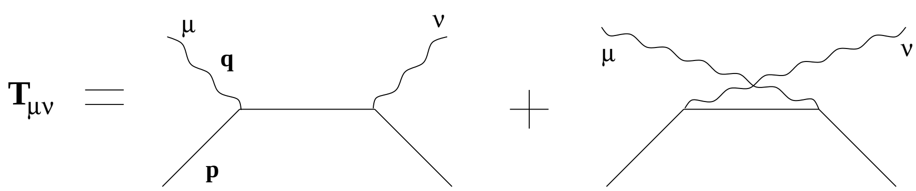
$$T_{\mu\nu} = \int d^4x \, e^{iq\cdot x} \, \frac{1}{2} \sum_\sigma \langle p, \sigma | \, \mathrm{T} \, j_\mu(x) \, j_\nu(0) \, | p, \sigma \rangle \tag{2}$$

where $|p, \sigma\rangle$ represents a quark with 4-momentum p and polarization σ.

a. (10 pts) Show that

$$T_{\mu\nu} = e_f^2 \, \frac{1}{2} \sum_\sigma \bar{u}_\sigma(p) \left[\gamma_\nu \frac{i}{\gamma \cdot (p+q) - m} \gamma_\mu + \gamma_\mu \frac{i}{\gamma \cdot (p-q) - m} \gamma_\nu \right] u_\sigma(p). \tag{3}$$

The following diagram may come in handy:



b. (10 pts) Use the relationship (for any matrix Γ)

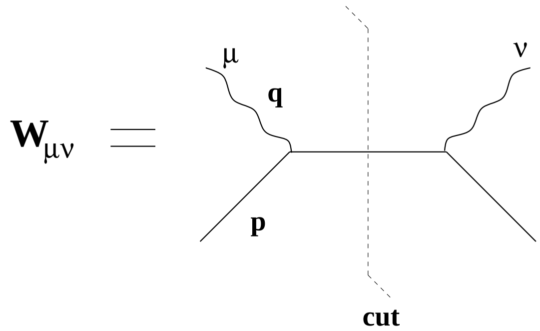
$$\sum_\sigma \bar{u}_\sigma(p) \, \Gamma \, u_\sigma(p) = \mathrm{tr} \left[(\gamma \cdot p + m) \, \Gamma \right]$$

to evaluate $T_{\mu\nu}$ in the form (for $Q^2 = -q^2 \gg m^2$)

$$T_{\mu\nu} = -T_1 \left[g_{\mu\nu} - \frac{q_\mu \, q_\nu}{q^2} \right] + \frac{T_2}{m^2} \left[p_\mu \, p_\nu - \frac{p \cdot q}{q^2} \, (p_\mu \, q_\nu + p_\nu \, q_\mu) + \left(\frac{p \cdot q}{q^2} \right)^2 q_\mu \, q_\nu \right].$$

Find T_1 and T_2. How are they related to each other?

c. (10 pts) Evaluate $W_{\mu\nu}$ using Eq. (1) above, or, equivalently, from the following diagram:



(In order to stick to the same notation as in Eq. (3), there is no need to put in the initial state phase space factors.)

d. (10 pts) Using the results of parts **b** and **c** show that

$$2\,\mathrm{Im}\,[i\,T_{\mu\nu}] \;=\; W_{\mu\nu}.$$

Also show that $W_{\mu\nu}$ is obtained from $T_{\mu\nu}$ in Eq. (3) by replacing the quark propagator in the first term

$$\frac{i\,[\gamma\cdot(p+q)+m]}{(p+q)^2-m^2+i\epsilon} \;\longrightarrow\; (2\pi)\,[\gamma\cdot(p+q)+m]\,\delta^{(+)}[(p+q)^2-m^2]$$

and doing a similar replacement for the quark propagator in the second term in Eq. (3).