Homework Set No. 5, Physics 880.02

Deadline – Friday, March 13, 2009

1. (15 pts) In the $SU(2)_L \otimes U(1)_Y$ Electroweak theory the Lagrangian for the gauge fields is

$$\mathcal{L}_{gauge} \,=\, -\frac{1}{4}\, f_{\mu\nu}\, f^{\mu\nu} - \frac{1}{4}\, \vec{F}_{\mu\nu} \cdot \vec{F}^{\mu\nu}.$$

It involves an Abelian gauge field B_μ with the field strength tensor

$$f_{\mu\nu} \,=\, \partial_\mu B_\nu - \partial_\nu B_\mu$$

and a non-Abelian field $\vec{W}_\mu = (W_\mu^1, W_\mu^2, W_\mu^3)$ with the field strength tensor

$$F^a_{\mu\nu} \,=\, \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\, \epsilon^{abc}\, W^b_\mu\, W^c_\nu.$$

Rewrite the Lagrangian $\mathcal{L}_{gauge}$ in terms of the $W^\pm$, Z, and photon fields

$$W_\mu \,=\, \frac{1}{\sqrt{2}}\,(W^1_\mu - i\, W^2_\mu)$$
$$W^\dagger_\mu \,=\, \frac{1}{\sqrt{2}}\,(W^1_\mu + i\, W^2_\mu)$$
$$Z_\mu = -B_\mu\, \sin\theta_W + W^3_\mu\, \cos\theta_W$$
$$A_\mu = B_\mu\, \cos\theta_W + W^3_\mu\, \sin\theta_W.$$

Simplify $\mathcal{L}_{gauge}$ and separate the kinetic terms of the fields W_μ, Z_μ, A_μ from the interaction terms. List all the possible interaction vertices for $W^\pm$, Z, and the photon. (You do not have to spell out the Feynman rules for the vertices.)

2. A dedicated effort is underway by experimentalists at the B factories (BABAR collaboration at SLAC in the US, and Belle collaboration at KEK in Japan) to precisely measure some of the lesser known CKM matrix elements. One way to do this is to study semileptonic decays of B mesons

$$B \to D + l + \mu \quad \text{or} \quad B \to \pi + l + \nu$$

where we use the following notation:

leptons l are l^+ or l^- $\quad (l = e, \mu, \tau)$

neutrinos ν denote ν_l or $\bar{\nu}_l$

pions π are $\pi^\pm$ or π^0

B mesons B are $B^\pm$ or B^0 or $\bar{B}_0$

D mesons D are $D^\pm$ or D^0 or $\bar{D}_0$

a. (5 pts) Consider the semileptonic decay of the B^- meson into one of the D mesons plus $l + \nu$. Draw the relevant Feynman diagram for the decay process, determine the correct final state particles (i.e. find which D meson, l and ν would B^- meson decay into) and the correct intermediate state W boson. Which of the CKM matrix elements V_{xy} can be probed this way? You may wish to look up the quark content of the B and D mesons at the Particle Data Group website http://pdg.lbl.gov/ .

b. (5 pts) Repeat **a** for the semileptonic decay of $\bar{B}^0$ into a D plus $l + \nu$.

c. (5 pts) Repeat **a** for the semileptonic decay of $\bar{B}^0$ into a pion plus $l + \nu$.

d. (5 pts) Repeat **a** for the semileptonic decay of B^0 into a pion plus $l + \nu$.

3. (15 pts) Work through the part of the discussion entitled *W decay* in Cheng and Li concerning Eqs. (12.128) - (12.132). Reproduce the steps of the calculation to derive the W^- decay rate into e^- and $\bar{\nu}_e$ given by Eq. (12.132).