Introduction

This course will cover Standard Model of Particle Physics. The Standard Model comprises strong and electroweak interactions.

(i) Electroweak interactions:

leptons (spin 1/2 particles)

$e \quad \mu \quad \tau$ ~ electron, muon, tau

$\nu_e \quad \nu_\mu \quad \nu_\tau$ ~ — , — - neutrinos

interactions between them is mediated by gauge bosons:

W^+, W^-, Z ~ massive spin-1 particles

γ ~ photon ~ massless — , —

(ii) Strong interactions:

quarks (spin 1/2) have 6 flavors:

$u \quad c \quad t$ u-up, d-down, s-strange,

$d \quad s \quad b$ c-charm, b-bottom, t-top quarks also have 3 colors, such that each quark of a given flavor comes in in 3 diff. colors.

Quarks interact by exchanging gluons:

g ~ gluon ~ spin-1 massless particle.

there are 8 gluon colors

Quarks & gluons combine into bound states like mesons ($q\bar{q}$) & baryons (qqq)

$$\underset{\pi^{\pm},\ \pi^{0},\ K,\ \rho,\ \omega,\ \dots}{\text{mesons} \downarrow} \qquad \underset{p,\ n,\ \Omega,\ \Sigma^{\pm},\ \Sigma^{0},\ \Lambda^{0},\ \Xi,\ \dots}{\text{baryons} \downarrow}$$

(+) Higgs boson (spin-0) ~ yet to be discovered.

The Standard Model does not include gravity: the "fundamental" interactions:

$$\underbrace{\text{strong} \quad \text{electric} \quad \text{weak}}_{\text{Standard Model}} \quad \text{gravity}$$

Standard Model depends on 16 (!) external parameters (quark $\overset{6}{\text{masses}}$, lepton $\overset{3}{\text{masses}}$, $\underset{3}{\text{couplings}}$, CKM $\underset{4}{\text{matrix}}$).

⇒ however SM is surprisingly <u>robust</u>: it described everything we know about strong & electroweak interactions up until 2003, when neutrino masses were discovered, indicating that there is physics beyond SM.

⇒ theories beyond SM have been proposed ever since the construction of SM, and include technicolor, supersymmetry, etc. (no exp. evidence yet)
⇒ a complete "theory of everything" should indeed incorporate (quantum) gravity ~ string theory is a possibility

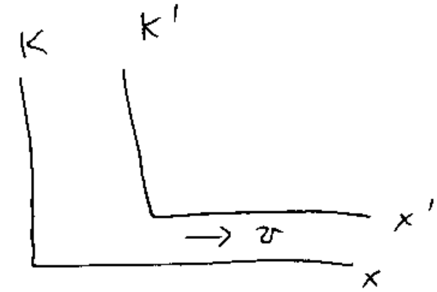
not covered in this class

⇒ Nowdays a lot of SM physics is considered "nuclear physics", while beyond SM physics is labelled "particle physics".

$\Rightarrow$ Theoretical language of SM is Quantum Field Theory (QM + special relativity). Hence knowledge of QFT is needed for the course. We will start by reviewing some QFT material.

Brief Review of Quantum Field Theory

4-vectors, notations



defining $x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$

write Lorentz transformation

as
$$\begin{pmatrix} x'^0 \\ x'^1 \\ x'^2 \\ x'^3 \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}, \quad \beta = v/c, \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}.$$

Definition: a 4-vector $A^\mu = \begin{pmatrix} A^0 \\ A^1 \\ A^2 \\ A^3 \end{pmatrix}$ is an object

which under Lorentz transformation transforms

as
$$\begin{pmatrix} A'^0 \\ A'^1 \\ A'^2 \\ A'^3 \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A^0 \\ A^1 \\ A^2 \\ A^3 \end{pmatrix}.$$
(example: x^μ is a contravariant 4-vector)

$\Rightarrow$ A^μ is a <u>contravariant</u> vector: $A'^\mu = \dfrac{\partial x'^\mu}{\partial x^\nu} A^\nu$

B_μ is a <u>covariant</u> vector: $B'_\mu = \dfrac{\partial x^\nu}{\partial x'^\mu} B_\nu$

$\Rightarrow$ $\dfrac{\partial \varphi}{\partial x^\mu} \equiv \partial_\mu \varphi$ with φ scalar field is a covariant vector as $\frac{\partial \varphi}{\partial x'^\mu} = \frac{\partial x^\nu}{\partial x'^\mu} \frac{\partial \varphi}{\partial x^\nu}$.

Tensors: $A^\mu B^\nu$ ~ contravariant, $A_\mu B_\nu$ ~ covariant (rank 2), can have higher ranks.

Def. Scalar (inner) product of 2 vectors is $A_\mu B^\mu$. (assume summation).

It is Lorentz-invariant: $A'_\mu B'^\mu = \frac{\partial x^\alpha}{\partial x'^\mu} A_\alpha \frac{\partial x'^\mu}{\partial x^\beta} B^\beta$

$= \frac{\partial x^\alpha}{\partial x^\beta} A_\alpha B^\beta = \delta^\alpha_\beta A_\alpha B^\beta = A_\alpha B^\alpha$.

Def.
The interval $ds^2 \left(= dx_\mu dx^\mu\right) = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2$.

It's a Lorentz-invariant too.

Def. The metric tensor $g_{\mu\nu}$ is defined by

$$\boxed{ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu}$$

In our Minkowski space (throughout the course we'll use this notation) $g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

$dx_\mu dx^\mu$ ~ also a Lorentz-scalar $\Rightarrow dx_\mu = g_{\mu\nu} dx^\nu$

$\Rightarrow g_{\mu\nu}$ lowers & raises indeces!

Example: $x^\mu = (ct, \vec{x}) \Rightarrow x_\mu = g_{\mu\nu} x^\nu = (ct, -\vec{x})$
contravariant covariant

In general $A_\mu = g_{\mu\nu} A^\nu$, $A^\mu = g^{\mu\nu} A_\nu$

where $g^{\mu\nu}$ is defined by requiring that

$g^{\mu\alpha} g_{\alpha\nu} = \delta^\mu{}_\nu$: if that is true $\Rightarrow$ start

with $A_\mu = g_{\mu\nu} A^\nu \Rightarrow g^{\alpha\mu} A_\mu = g^{\alpha\mu} g_{\mu\nu} A^\nu =$

$= \delta^\alpha{}_\nu A^\nu = A^\alpha \Rightarrow A^\alpha = g^{\alpha\mu} A_\mu$.

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \text{too.} \quad (= g_{\mu\nu})$$

Def. $\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$, $\partial^\mu \equiv \frac{\partial}{\partial x_\mu} \Rightarrow \partial_\mu \varphi$ is a covariant vector,

$\partial^\mu \varphi$ is a contravariant vector. (check!)

$\partial_\mu A^\mu$ is a Lorentz-invariant.

$\partial_\mu \partial^\mu = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \vec{\nabla}^2$ is also Lorentz-invariant.

Examples: other important 4-vectors are

$p^\mu = \begin{pmatrix} \mathcal{E}/c \\ \vec{p} \end{pmatrix}$, $p_\mu = \begin{pmatrix} \mathcal{E}/c \\ -\vec{p} \end{pmatrix} \Rightarrow p_\mu p^\mu = \left(\frac{\mathcal{E}}{c}\right)^2 - \vec{p}^{\,2} = m^2 c^2$.

$A^\mu = (\Phi, \vec{A})$ in E & M, Φ ~ electric potential,

$\vec{A}$ - vector potential.

$J^\mu = (c\rho, \vec{J})$ with ρ the charge density, $\vec{J}$ the current density.

Notation: from now on $\boxed{c = 1}$ and $\boxed{\hbar = 1}$

"natural units".

$\Rightarrow$ mass, momentum, energy are measured in the same units (eV, keV, MeV, GeV, ...)

$$1\,eV = 1.6 \times 10^{-19}\,J.$$

distances, time are measured in femto-meters aka fermis (fm):

$$1\,fm \simeq 5\,GeV^{-1}$$

$1\,GeV = 10^9\,eV$, $\underset{\text{-meter}}{1\ \text{femto}} := 10^{-15}\,m. = 1\,fm.$

proton's mass $m_p = 0{,}938\ GeV \simeq 1\,GeV$

electron's mass $m_e = 0{,}511\ MeV \simeq 0.5 \times 10^{-3}\ GeV$

Free Scalar Field (real)

$\varphi(x^\mu) = \varphi(x^0, \vec{x})$ ~ a function of space-time points x^μ

Classical Field Theory.

in Classical Mechanics one has point particles $i = 1, \dots, N$ with the Lagrangian $L(q_i, \dot{q}_i, t)$ and the action $S = \int dt \, L(q_i, \dot{q}_i, t)$

q_i ~ degrees of freedom (e.g. particle coordinates)

$\dot{q}_i = \frac{dq_i}{dt}$ ~ generalized velocities

Now, instead of discrete point particles we have a field $\varphi(\vec{x}, t)$ =>

Classical Mechanics		Classical Field Theory
q_i	$\longrightarrow$	$\varphi(x^0, \vec{x})$
i	$\longrightarrow$	$\vec{x}, t$
$\dot{q}_i$	$\longrightarrow$	$\partial_\mu \varphi$, $\mu = 0, 1, 2, 3$
$L(q_i, \dot{q}_i, t)$	$\longrightarrow$	$\int d^3x \, \mathcal{L}(\varphi, \partial_\mu \varphi)$

$\mathcal{L}$ is <u>Lagrangian</u> <u>density</u>. (usually called the lagrangian)

The action is $S = \int dt\, L = \underbrace{\int dt\, d^3x}_{d^4x} \mathcal{L}(\varphi, \partial_\mu \varphi)$ (remember $c=1$)

S is a Lorentz-scalar (better be, physics is Lorentz-invariant)

What about $d^4x = dx^0 dx^1 dx^2 dx^3$? Remember that

$x'^\mu = \Lambda^\mu{}_\nu x^\nu$ with $\Lambda^\mu{}_\nu = \dfrac{\partial x'^\mu}{\partial x^\nu}$ a matrix of L. tr.

$\Rightarrow d^4x' = \underbrace{\det \Lambda}_{\text{Jacobian}} \cdot d^4x$

Now, $\det \Lambda = \det \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \gamma^2(1-\beta^2) = 1$
(true in general)

$\Rightarrow d^4x' = d^4x \Rightarrow d^4x$ is a Lorentz-scalar

$\Rightarrow$ $\mathcal{L}$ is <u>a Lorentz-scalar</u>!

Just like in classical mechanics, in classical field theory dynamics is given by the <u>least action principle</u>: field φ is determined by requiring that S is stationary with respect to small perturbations around φ: $S[\varphi + \delta\varphi] = S[\varphi] + o(\delta\varphi^2)$.

$$0 = \delta S = \int d^4x \left[\frac{\delta \mathcal{L}}{\delta \varphi} \delta\varphi + \frac{\delta \mathcal{L}}{\delta(\partial_\mu \varphi)} \delta(\partial_\mu \varphi) \right] =$$

$$= \left(\text{as } \delta \partial_\mu \varphi = \partial_\mu \delta\varphi \Rightarrow \text{parts} \right) = \int d^4x \left[\frac{\delta \mathcal{L}}{\delta\varphi} \delta\varphi - \right.$$

$$\left. - \partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu \varphi)} \delta\varphi \right] + \underbrace{\text{surface term}}_{= 0}$$

$$\Rightarrow 0 = \int d^4x \, \delta\varphi \left[\frac{\delta \mathcal{L}}{\delta\varphi} - \partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu \varphi)} \right] \text{ for any } \delta\varphi$$

$$\Rightarrow \boxed{\frac{\delta \mathcal{L}}{\delta\varphi} - \partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu\varphi)} = 0}$$

Euler - Lagrange equations (aka equations of motion) for field φ. (EOM)

Now, $\varphi(x)$ is a scalar field $\Rightarrow$ it is Lorentz-inv., which means that : $\varphi(x) \to \varphi'(x') = \varphi(x)$

$$\Rightarrow \text{as } x'^\mu = \Lambda^\mu{}_\nu x^\nu \Rightarrow x' = \Lambda \cdot x \Rightarrow \varphi'(x) = \varphi(\Lambda^{-1} x).$$

Lagrangian density for massive scalar field:

$$\boxed{\mathcal{L} = \frac{1}{2} \partial_\mu \varphi \, \partial^\mu \varphi - \frac{m^2}{2} \varphi^2}$$

EOM: $\displaystyle \frac{\delta \mathcal{L}}{\delta\varphi} = -m^2 \varphi \; ; \; \frac{\delta \mathcal{L}}{\delta(\partial_\mu \varphi)} = \partial^\mu \varphi \Rightarrow$

$$\Rightarrow \partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu \varphi)} = \partial_\mu \partial^\mu \varphi \Rightarrow -m^2 \varphi - \partial_\mu \partial^\mu \varphi = 0$$

$$\Rightarrow \boxed{\left[\partial_\mu \partial^\mu + m^2\right]\varphi = 0}$$ Klein-Gordon equation

or $$\left[\Box + m^2\right]\varphi = 0$$

To solve K-G equation write $\varphi(x) = \int d^4k \, e^{-ik\cdot x}\tilde{\varphi}(k)$

with $k\cdot x = k_\mu x^\mu = k^0 x^0 - \vec{k}\cdot\vec{x}$.

$$\left[\Box + m^2\right]\varphi = \int d^4k \, \tilde{\varphi}(k)\left(\Box + m^2\right)e^{-ik\cdot x} = \int d^4k \, \tilde{\varphi}(k)\cdot$$

$$\cdot\left[-k^2 + m^2\right] = 0 \quad \text{with } k^2 = k_\mu k^\mu = (k^0)^2 - (\vec{k})^2.$$

$$\Rightarrow \left[k^2 - m^2\right]\tilde{\varphi} = 0 \Rightarrow \text{as } \tilde{\varphi} \neq 0 \Rightarrow k^2 = m^2 \quad \text{or}$$

$$E_k^2 - \vec{k}^2 = m^2 \Rightarrow \mathcal{E}_k = \pm\sqrt{\vec{k}^2 + m^2} \Rightarrow \text{define } \mathcal{E}_k = \sqrt{\vec{k}^2 + m^2}$$

$$\Rightarrow \varphi(x) = \int \frac{d^3k}{(2\pi)^3 \, 2\mathcal{E}_k}\left[a_{\vec{k}}\, e^{-i\mathcal{E}_k t + i\vec{k}\cdot\vec{x}} + a^*_{\vec{k}}\, e^{i\mathcal{E}_k t - i\vec{k}\cdot\vec{x}}\right]$$

most general solution.

Canonical Quantization

In your QM class you must have seen that if we treat K-G equation as the equation for a single-particle wave function $\varphi(x)$ (just like