Last time: started from EW Lagrangian:

$$\boxed{\begin{aligned}\mathcal{L}^{EW} &= \bar{R}_e i\gamma^\mu(\partial_\mu + ig' B_\mu)R_e + \bar{L}_e i\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)L_e + (\mu,\tau)\\ &-\frac{1}{4}f_{\mu\nu}f^{\mu\nu} - \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + \left[\left(\partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\phi\right]^+\left[\left(\partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu\right)\phi\right]\\ &+\mu^2\phi^+\phi - \lambda(\phi^+\phi)^2 - G_E\left[\bar{L}_e\phi R_e + R_e^+\phi^+ L_e\right] + (\mu,\tau)\end{aligned}}$$

SSB of $SU(2)_L \otimes U(1)_Y$ is accomplished by Higgs VEV $\langle 0|\phi|0\rangle = \begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix} \Rightarrow$ writing $\phi_{(x)} = \begin{pmatrix}0\\ \frac{v+\eta(x)}{\sqrt{2}}\end{pmatrix}$ we expanded $\mathcal{L}^{EW}$ around the VEV.

We defined
$$\begin{cases} W_\mu = \frac{1}{\sqrt{2}}(W_\mu^1 - iW_\mu^2) = W_\mu^{(+)}\\ W_\mu^+ = \frac{1}{\sqrt{2}}(W_\mu^1 + iW_\mu^2) = W_\mu^{(-)}\\ Z_\mu = -B_\mu\sin\theta_W + W_\mu^3\cos\theta_W\\ A_\mu = B_\mu\cos\theta_W + W_\mu^3\sin\theta_W\end{cases}$$

with $\tan\theta_W = g'/g$ ~ Weinberg angle. We got:

$$\boxed{\begin{aligned}\mathcal{L}^{EW} &= \bar{e}\,i\gamma\cdot\partial e + \bar{\nu}_e i\gamma\cdot\partial\nu_e - \frac{G_e}{\sqrt{2}}(v+\eta)\bar{e}e - \frac{1}{4}f_{\mu\nu}f^{\mu\nu} - \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu}\\ &+\frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta - \mu^2\eta^2 - \lambda v\eta^3 - \frac{\lambda}{4}\eta^4 + \frac{g^2}{4}(v+\eta)^2 W_\mu^+W^\mu + \frac{g^2}{8\cos^2\theta_W}(v+\eta)^2 Z_\mu Z^\mu\\ &+\frac{g}{2\cos\theta_W}\left[2\sin^2\theta_W\,\bar{e}_R\gamma\cdot Z e_R + (2\sin^2\theta_W - 1)\bar{e}_L\gamma\cdot Z e_L\right] - e\,\bar{e}\gamma\cdot A e\\ &+\frac{g}{2\cos\theta_W}\bar{\nu}_e\gamma\cdot Z\nu_e - \frac{g}{\sqrt{2}}\left[\bar{\nu}_e\gamma\cdot W e_L + \bar{e}_L\gamma\cdot W^+\nu_e\right] + (\mu,\tau)\end{aligned}}$$

$\Rightarrow$ we got massive gauge bosons $W^{\pm}$, Z:

$$M_W = \frac{g v}{2} \qquad M_Z = \frac{g v}{2\cos\theta_W} \; ; \; m_\gamma = 0$$ as needed!

$\Rightarrow$ leptons: $m_e = \frac{G_e v}{\sqrt{2}}$ $\quad m_\mu = \frac{G_\mu v}{\sqrt{2}}$ $\quad m_\tau = \frac{G_\tau v}{\sqrt{2}}$

neutrinos have no mass: $m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0$

(not true, there is a neutrino with $m \gtrsim 0.04\,eV$).

$$\boxed{\begin{aligned} M_{W^\pm} &= 80.398 \pm 0.025 \text{ GeV} \\ M_Z &= 91.1876 \pm 0.0021 \text{ GeV} \end{aligned}}$$

$$M_W = \frac{g v}{2} \; ; \; M_Z = \frac{\sqrt{g^2 + g'^2}}{2} v \quad \Rightarrow$$

$$\Rightarrow \quad \frac{M_W}{M_Z} = \frac{g}{\sqrt{g^2 + g'^2}} = \cos\theta_W \Rightarrow \cos\theta_W = \frac{M_W}{M_Z}$$

$$\Rightarrow \sin^2\theta_W \simeq 0.23120 \pm 0.00015$$ (involves quantum corrections)

$$\Rightarrow \sin^2\theta_W \simeq \frac{1}{4} \Rightarrow \sin\theta_W \simeq \frac{1}{2} \Rightarrow \theta_W \simeq 30^\circ$$

What about g? $\quad g = \frac{e}{\sin\theta_W} \Rightarrow \frac{g^2}{4\pi} = \frac{\alpha_{EM}}{\sin^2\theta_W}$, $\alpha_{EM} = e^2/4\pi$

$$\Rightarrow \frac{g^2}{4\pi} \simeq \frac{1/137}{0.23} \simeq .03 \approx \frac{1}{30} \quad \Rightarrow \frac{g^2}{4\pi} \simeq \frac{1}{30}$$

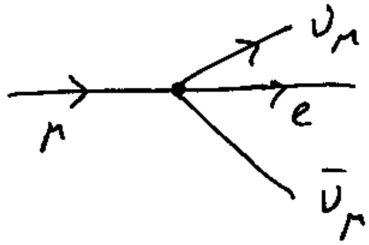
~ very small still, even though it is not as small as α_{EM}.

$\Rightarrow$ What about the old Fermi constant G_F?

Consider this purely leptonic decay:

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu \, .$$

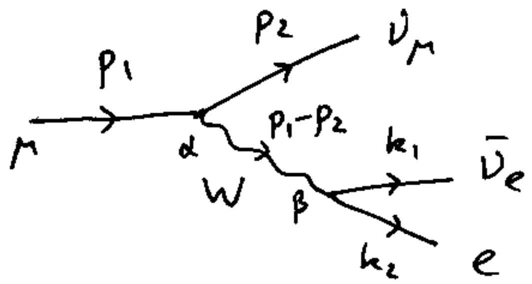
In Fermi theory this decay would be given by the following "effective" vertex:



with $\mathcal{L} = \frac{G_F}{\sqrt{2}} \bar{\psi}_e \gamma_\mu (1-\gamma_5) \psi_{\nu_e} \; \bar{\psi}_{\nu_\mu} \gamma^\mu (1-\gamma_5) \psi_\mu$

+ hermitean conjugate

Let's derive this from the Electroweak Lagrangian we wrote. At the lowest order the process is given by the diagram:



$\Rightarrow$ need W - lepton coupling

$$-\frac{g}{\sqrt{2}}\left[\bar{\nu}_e\,\gamma\cdot W\,e_L + \bar{e}_L\,\gamma\cdot W^+\,\nu_e\right]$$

+ same for μ, ν_μ.

$$-\frac{g}{\sqrt{2}}\left[\bar{\nu}_e\,\gamma\cdot W e_L + \bar{e}_L\,\gamma\cdot W^+\nu_e\right] = -\frac{g}{\sqrt{2}}\left[\bar{\nu}_e\,\gamma\cdot W\,\frac{1-\gamma_5}{2}\,e + \right.$$

$$\left. + \bar{e}\,\frac{1+\gamma_5}{2}\,\gamma\cdot W^+\,\nu_e\right] = -\frac{g}{2\sqrt{2}}\left[\bar{\nu}_e\,\gamma\cdot W\,(1-\gamma_5)\,e + \right.$$

$$\left. + \bar{e}\,\gamma\cdot W^+\,(1-\gamma_5)\,\nu_e\right].$$

$\Rightarrow$ the diagram is

$$\left(\frac{ig}{2\sqrt{2}}\right)^2\left[\bar{u}_{\nu_\mu}(p_2)\,\gamma_\alpha\,(1-\gamma_5)\,u_\mu(p_1)\right]\underbrace{\frac{(-i)\left[g^{\alpha\beta}-\frac{(p_1-p_2)^\alpha(p_1-p_2)^\beta}{M_W^2}\right]}{(p_1-p_2)^2-M_W^2+i\varepsilon}}_{\approx\,\frac{i}{M_W^2}\,g^{\alpha\beta}}\cdot$$

$$\cdot\left[\bar{u}_e(k_2)\,\gamma_\beta\,(1-\gamma_5)\,v_{\nu_e}(k_1)\right]$$

if $|p_1-p_2| \ll M_W$ (low energy)

$$= -i\,\frac{g^2}{8M_W^2}\left[\bar{u}_{\nu_\mu}(p_2)\,\gamma_\alpha\,(1-\gamma_5)\,u_\mu(p_1)\right]\left[\bar{u}_e(k_2)\,\gamma^\alpha\,(1-\gamma_5)\,v_{\nu_e}(k_1)\right].$$

$\Rightarrow$ looks just like the term in Fermi theory

$\Rightarrow$ equate the prefactors:

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}} \Rightarrow G_F = \underbrace{\frac{g^2}{4\pi}}_{.03} \cdot \frac{\pi}{\sqrt{2}} \frac{1}{M_W^2} \simeq 8.5 \cdot 10^{-6}\,\text{GeV}^{-2}$$

$$\approx 10^{-5}\,\text{GeV}^{-2}$$

as advertised.

$\Rightarrow$ What about Higgs and related parameters?

$$M_W = \frac{g v}{2} \Rightarrow v = \frac{2}{g} M_W \Rightarrow \text{as } g \simeq 0.63$$

$\Rightarrow$ get $\boxed{v \simeq 289\ \text{GeV}}$

The Higgs mass $m_H = \mu\sqrt{2} = v\sqrt{2\lambda}$

v is rather large, $\lambda \sim$ dimensionless

λ can't be very big ~ certainly less than Plank mass $m_{Planck} \simeq 10^{19}$ GeV (gravity is important) or can't hit Landau pole...

~ conversely if λ is too small $\xRightarrow{\text{(for fixed } v)}$ quantum effects become important, eliminating SSB

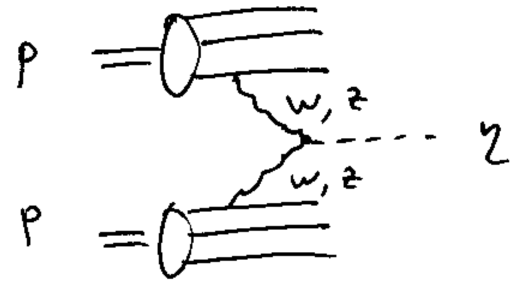
~ Most likely Higgs mass range:

$$\boxed{m_{Higgs} = 129^{+74}_{-49}\ \text{GeV}}$$

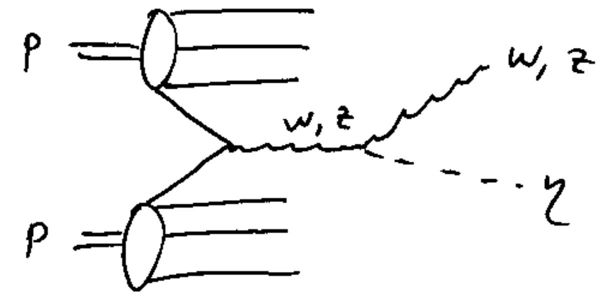
LEP, Tevatron data.

Higgs boson will be searched for at LHC.

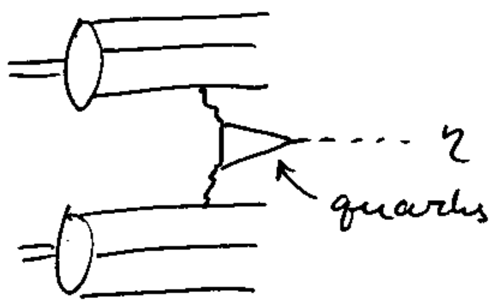
Possible discovery processes are:



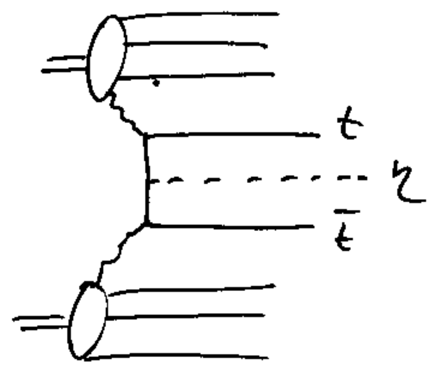
Weak boson fusion



associated Drell-Yan process



gluon fusion



associated top pair ...

Quarks in the Electroweak Theory.

Quarks also form left-handed doublets under weak isospin:

$$L_u = \begin{pmatrix} u \\ d' \end{pmatrix}_L \qquad L_c = \begin{pmatrix} c \\ s' \end{pmatrix}_L \qquad L_t = \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

$$R_u = u_R \qquad R_c = c_R \qquad R_t = t_R$$

$$R_d = d_R \qquad R_s = s_R \qquad R_b = b_R$$