Last time: worked out interactions of $W^\pm$, Z & γ with quarks. We got:

(i) Charged current ($W^\pm$ bosons coupling)

$$\boxed{\begin{aligned}\mathcal{L}_{c.c.} = \frac{g}{2\sqrt{2}}\Big\{ &\bar{u}\,\gamma\cdot W(1-\gamma_5)\left[V_{ud}\, d + V_{us}\, s + V_{ub}\, b\right] + \\ &+ \bar{c}\,\gamma\cdot W(1-\gamma_5)\left[V_{cd}\, d + V_{cs}\, s + V_{cb}\, b\right] + \bar{t}\,\gamma\cdot W(1-\gamma_5) \\ &\left[V_{td}\, d + V_{ts}\, s + V_{tb}\, b\right] + h.c.\Big\}\end{aligned}}$$

(ii) Neutral current: (Z-bosons & photons coupling)

$$\boxed{\begin{aligned}\mathcal{L}_{n.c.} = \frac{g}{4\cos\theta_W}\Big\{ &\bar{U}\,\gamma\cdot Z\left[(1-\gamma_5)\left(1-\tfrac{4}{3}\sin^2\theta_W\right) - (1+\gamma_5)\tfrac{4}{3}\sin^2\theta_W\right] U \\ &- \bar{D}\,\gamma\cdot Z\left[(1-\gamma_5)\left(1-\tfrac{2}{3}\sin^2\theta_W\right) - (1+\gamma_5)\tfrac{2}{3}\sin^2\theta_W\right] D + \sum_f e_f\, \bar{q}_f\, \gamma\cdot A\, q_f\Big\}\end{aligned}}$$

where we have defined

$$U = \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad D = \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad e_f = e\begin{cases} 2/3 & \text{for } u, c, t \\ -1/3 & \text{for } d, s, b \end{cases}$$

$q_f = u, d, s, c, b, t$ for different index f values.

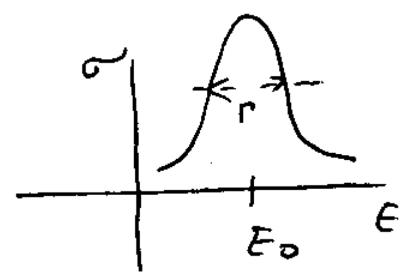
$$d\Gamma = \frac{1}{2m} \prod_{i=1}^{N} \frac{d^3 p_i}{(2\pi)^3\, 2\mathcal{E}_i} \, |M(m; p_1, \dots, p_N)|^2 (2\pi)^4 \delta(k - \sum_{j=1}^{N} p_j)$$

~ decay rate

$$\Gamma = \frac{\#\ \text{decays per unit time}}{\#\ \text{of particles}}$$

Breit-Wigner formula for scattering amplitude:

$$f(E) \sim \frac{1}{E - E_0 + i\,\Gamma/2}$$



$$\Rightarrow \sigma(E) \propto |f(E)|^2 \sim \frac{1}{(E-E_0)^2 + \frac{\Gamma^2}{4}}$$

Width of resonance peak!

⇒ NB in calculating $|M|^2$ sum over spins, etc in the final state, average over them in the initial state.

Decay of the Z-boson.

⇒ let's calculate decay rate of the Z-boson

⇒ the Z-boson interaction Lagrangian is:

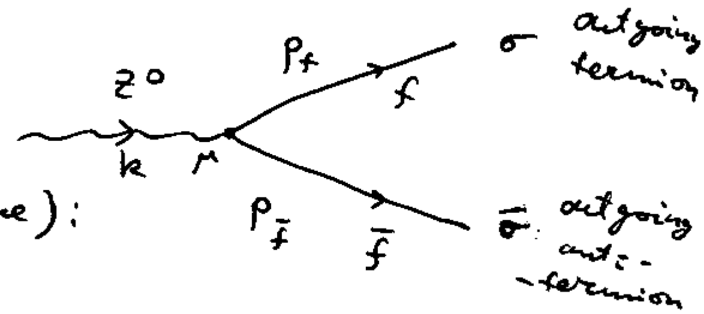
$$\mathcal{L}_Z = \frac{g}{4\cos\theta_W} \Big\{ \bar{\nu}_e \gamma \cdot Z (1-\gamma_5) \nu_e + 2\sin^2\theta_W\, \bar{e}\, \gamma\cdot Z (1+\gamma_5) e + (2\sin^2\theta_W - 1)\, \bar{e}\, \gamma \cdot Z (1-\gamma_5) \overset{+(\mu,\tau)}{e} + \bar{u}\, \gamma \cdot Z \left[(1-\gamma_5)\left(1 - \tfrac{4}{3}\sin^2\theta_W\right) - (1+\gamma_5) \cdot \tfrac{4}{3}\sin^2\theta_W \right] u - \bar{D}\, \gamma\cdot Z \left[(1-\gamma_5)\left(1 - \tfrac{2}{3}\sin^2\theta_W\right) - (1+\gamma_5)\tfrac{2}{3}\sin^2\theta_W \right] D \Big\}$$

$Z \to W^+W^-$, $Z \to \gamma Z$, $Z \to W^+W^- Z, \dots$ are all prohibited by energy conservation! $M_Z \simeq 91\,\text{GeV} < 2M_W = 2 \cdot 80\,\text{GeV} \dots$

For each fermion species the Lagrangian looks like: $\mathcal{L} = \bar{\Psi}\, \gamma \cdot Z \left[a_f (1-\gamma_5) - b_f (1+\gamma_5) \right] \Psi$

with a_f, b_f coefficients being species-dependent.

Consider Z-decay:



Amplitude (in Z^0 rest frame):

$$M = \underbrace{-i}_{\text{vertex}} \; \bar{u}_\sigma(p_f) \underbrace{\gamma^\mu \left[a_f (1-\gamma_5) - b_f (1+\gamma_5) \right]}_{\text{vertex}} v_{\bar{\sigma}}(p_{\bar{f}}) \cdot \varepsilon_\mu^\lambda(k)$$

Need to find:

$$\frac{1}{3} \sum_{\lambda,\sigma,\bar{\sigma}} |M|^2 = \frac{1}{3} \sum_{\lambda,\sigma,\bar{\sigma}} \bar{u}_\sigma \, \gamma \cdot \varepsilon^\lambda \left[a_f (1-\gamma_5) - b_f (1+\gamma_5) \right]$$

$\uparrow$ average over initial polarizations; $\leftarrow$ sum over spins

$$\cdot\, v_{\bar{\sigma}} \quad \underbrace{\bar{v}_{\bar{\sigma}} \gamma^0}_{v^+ \underbrace{\gamma^0\gamma^0}_{1}} \left[a_f^* (1-\underbrace{\gamma_5^+}_{=\gamma_5}) - b_f^* (1+\underbrace{\gamma_5^+}_{=\gamma_5}) \right] \gamma^+ \cdot \varepsilon^{\lambda *} \gamma^0 u_\sigma$$

$\Rightarrow$ as $(\gamma^0)^+ = \gamma^0$, $(\gamma^i)^+ = -\gamma^i$, $i = 1, 2, 3$

$$\Rightarrow \gamma^0 \gamma_\mu^+ \gamma^0 = \begin{cases} \gamma^0 & \text{if } \mu = 0 \\ \gamma^i & \text{if } \mu = i \end{cases} \Rightarrow \gamma^0 (\gamma^\mu)^+ \gamma^0 = \gamma^\mu$$

as $\{\gamma^\mu, \gamma^5\} = 0 \Rightarrow \gamma^0 \gamma^5 (\gamma^\mu)^+ \gamma^0 = \begin{cases} -\gamma^5 \gamma^0, & \mu = 0 \\ -\gamma^5 \gamma^i, & \mu = i \end{cases}$

$\Rightarrow \gamma^5 \to -\gamma^5$

$$\Rightarrow \frac{1}{3}\sum_{\lambda,\sigma,\bar\sigma}|M|^2 = \frac{1}{3}\sum_{\lambda,\sigma,\bar\sigma}\bar u_\sigma(p_f)\,\gamma\cdot\varepsilon^\lambda\left[a_f(1-\gamma_5)-b_f(1+\gamma_5)\right]v_{\bar\sigma}(p_{\bar f})$$

$$\cdot\,\bar v_{\bar\sigma}(p_{\bar f})\left[a_f^*(1+\gamma_5)-b_f^*(1-\gamma_5)\right]\gamma\cdot\varepsilon^{\lambda *}u_\sigma(p_f)$$

Now, use $\sum_\sigma u_\sigma(p_f)\,\bar u_\sigma(p_f) = \gamma\cdot p_f + m_f$

$$\sum_{\bar\sigma} v_{\bar\sigma}(p_{\bar f})\,\bar v_{\bar\sigma}(p_{\bar f}) = \gamma\cdot p_{\bar f} - m_f$$

$$\Rightarrow \frac{1}{3}\sum|M|^2 = \frac{1}{3}\sum_\lambda \mathrm{Tr}\Big[(\not p_f+m_f)\,\gamma\cdot\varepsilon^\lambda\left[a_f(1-\gamma_5)-b_f\,\cdot\right.$$

$$\left.\cdot(1+\gamma_5)\right](\not p_{\bar f}-m_f)\left[a_f^*(1+\gamma_5)-b_f^*(1-\gamma_5)\right]\gamma\cdot\varepsilon^{\lambda *}\Big] =$$

$$= \frac{1}{3}\sum_\lambda \mathrm{Tr}\Big[(\not p_f+m_f)\,\gamma\cdot\varepsilon^\lambda\Big[2\left(|a_f|^2(1-\gamma_5)+|b_f|^2(1+\gamma_5)\right)\not p_{\bar f} +$$

$$+m_f\left(2b_f a_f^*(1+\gamma_5)+2a_f b_f^*(1-\gamma_5)\right)\Big]\gamma\cdot\varepsilon^{\lambda *}\Big] =$$

$$= \left(\begin{array}{l}\text{as } m_{u,d} \ll M_Z \\ \quad m_e \ll M_Z\end{array} \Rightarrow \text{neglect masses}\right) = \frac{2}{3}\sum_\lambda \mathrm{Tr}\Big[\not p_f\,\gamma\cdot\varepsilon^\lambda\,\cdot$$

$$\cdot\left(|a_f|^2(1-\gamma_5)+|b_f|^2(1+\gamma_5)\right)\not p_{\bar f}\;\gamma\cdot\varepsilon^{\lambda *}\Big]$$

In the Z-boson rest frame $\varepsilon_\mu^{(1)} = (0,1,0,0)$,

$\varepsilon_\mu^{(2)} = (0,0,1,0)$, $\varepsilon_\mu^{(3)} = (0,0,0,1)$ as $k\cdot\varepsilon = M\cdot\varepsilon_0 = 0 \Rightarrow \varepsilon_0 = 0$

$$\Rightarrow \sum_\lambda \varepsilon_\mu^\lambda\varepsilon_\nu^{\lambda *} = -g_{\mu\nu} \quad \text{for } \mu,\nu = 1,2,3 \quad (\emptyset \text{ otherwise})$$

$$\Rightarrow \frac{1}{3}\sum |M|^2 = \frac{2}{3}\ \mathrm{Tr}\Big[\gamma^i \gamma\cdot p_f \gamma^i \big(|a_f|^2(1-\gamma_5) + |b_f|^2\cdot$$

$$\cdot(1+\gamma_5)\big)\ \gamma\cdot p_{\bar f}\Big]$$

massless

$$\gamma^i\gamma\cdot p_f\gamma^i = \underbrace{\gamma^i\gamma^0\gamma^i}_{3\gamma^0} p_f - \underbrace{\gamma^i\gamma^j\gamma^i}\ p_f^j =$$

$$\gamma^i\big(-\gamma^i\gamma^j + \underbrace{\{\gamma^j,\gamma^i\}}_{-2\delta^{ij}}\big) =$$

$$= +3\gamma^j - 2\gamma^j = \gamma^j$$

$$= 3\gamma^0 p_f - \vec\gamma\cdot\vec p_f$$

$$\Rightarrow \frac{1}{3}\sum|M|^2 = \frac{2}{3}\ \mathrm{Tr}\Big[\big(3\gamma^0 p_f - \vec\gamma\cdot\vec p_f\big)\big(|a_f|^2(1-\gamma_5) + |b_f|^2(1+\gamma_5)\big)\cdot$$

$$\cdot\gamma\cdot p_{\bar f}\Big] = \left|\begin{array}{l}\text{as } \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^5)=0\\ \Rightarrow \text{drop } \gamma^5\text{'s}\\ \mathrm{tr}\,\gamma^\mu\gamma^\nu = 4g^{\mu\nu}\end{array}\right. = \frac{2}{3}\big(|a_f|^2 + |b_f|^2\big)\cdot\Big[4\cdot 3\cdot p_f\cdot p_{\bar f}$$

$$- 4\cdot\vec p_f\cdot\vec p_{\bar f}\Big] = \left|\begin{array}{l} Z \text{ CMS frame } \Rightarrow\\ \Rightarrow \vec p_{\bar f} = -\vec p_f,\ p_{\bar f} = p_f\end{array}\right. \Rightarrow = \frac{8}{3}\big(|a_f|^2 + |b_f|^2\big)\cdot$$

$$\cdot\big(3p_f^2 + \vec p_f^{\,2}\big) = \frac{32}{3}|\vec p_f|^2\big(|a_f|^2 + |b_f|^2\big) = \frac{1}{3}\sum|M|^2$$

$\Rightarrow$ the decay rate:

$$d\Gamma = \frac{1}{2M_Z}\frac{d^3p_f}{(2\pi)^3 2E_f}\frac{d^3p_{\bar f}}{(2\pi)^3 2E_{\bar f}}(2\pi)^4\delta^{(4)}(k - p_f - p_{\bar f})\ \frac{1}{3}\sum|M|^2$$

$$\delta^{(4)} = \delta(M_Z - p_f - p_{\bar f})\,\delta^3(\vec p_f + \vec p_{\bar f})$$

$$\Rightarrow \Gamma = \frac{1}{2M_Z}\int\frac{d^3p_f\, d^3p_{\bar f}}{(2\pi)^6\, 4\cancel{E_f}\cancel{E_{\bar f}}}(2\pi)^4\delta^{(4)}(k - p_f - p_{\bar f})\ \frac{32}{3}\cancel{|\vec p_f|^2}\cdot$$

$$\cdot\left(|a_f|^2+|b_f|^2\right) = \frac{1}{2M_Z}\,\frac{8}{3}\,\frac{1}{(2\pi)^2}\int d^3p_f\cdot\delta(M_Z-2p_f)\cdot$$

$$\cdot\left(|a_f|^2+|b_f|^2\right) = \frac{4}{3M_Z}\,\frac{1}{(\cancel{2}\pi)^{\cancel{2}}}\cdot\cancel{4\pi}\cdot\left(\frac{M_Z}{\cancel{2}}\right)^2\cdot\frac{1}{2}\left(|a_f|^2+|b_f|^2\right)$$

$= \dfrac{M_Z}{6\pi}\left[|a_f|^2+|b_f|^2\right]$ => finally, as a_f, b_f are real => drop $|\dots|$ =>

$$\boxed{\Gamma_{Z\to f\bar f} = \frac{M_Z}{6\pi}\left[a_f^2+b_f^2\right]}$$

a) Neutrinos: $b_\nu = 0$, $a_\nu = \dfrac{g}{4\cos\theta_W}$

$$\Rightarrow \Gamma_{Z\to\nu\bar\nu} = \frac{M_Z}{6\pi}\,\frac{g^2}{16\cos^2\theta_W} = \frac{g^2 M_Z}{96\pi\cos^2\theta_W}$$

b) Electrons: $a_e = \dfrac{g}{4\cos\theta_W}\left(2\sin^2\theta_W-1\right)$

$$b_e = \frac{-g}{4\cos\theta_W}\,2\sin^2\theta_W$$

$$\Rightarrow \Gamma_{Z\to e^+e^-} = \frac{g^2M_Z}{96\pi\cos^2\theta_W}\left[\left(2\sin^2\theta_W-1\right)^2+4\sin^4\theta_W\right]$$

$$= \frac{g^2M_Z}{96\pi\cos^2\theta_W}\,\frac{1}{2}\left[1+\left(1-4\sin^2\theta_W\right)^2\right] = \Gamma_{Z\to\nu\bar\nu}\cdot\frac{1}{2}\left[1+\left(1-4\sin^2\theta_W\right)^2\right]$$

c) u-quarks: $a_u = \dfrac{g}{4\cos\theta_W}\left(1-\frac{4}{3}\sin^2\theta_W\right)$

$$b_u = \frac{g}{4\cos\theta_W}\,\frac{4}{3}\sin^2\theta_W$$

$$\Gamma_{Z\to u\bar{u}} = \Gamma_{Z\to\nu\bar{\nu}} \times (3 \text{ colors}) \times \left[\left(1-\frac{4}{3}\sin^2\theta_W\right)^2 + \left(\frac{4}{3}\sin^2\theta_W\right)^2\right]$$

$$= \Gamma_{Z\to\nu\bar{\nu}} \cdot \frac{3}{2} \cdot \left[1 + \left(1-\frac{8}{3}\sin^2\theta_W\right)^2\right]$$

d) d-quarks $\quad a_d = -\frac{g}{4\cos\theta_W}\left(1-\frac{2}{3}\sin^2\theta_W\right)$

$$b_d = -\frac{g}{4\cos\theta_W}\,\frac{2}{3}\sin^2\theta_W$$

$$\Rightarrow \Gamma_{Z\to d\bar{d}} = \Gamma_{Z\to\nu\bar{\nu}} \cdot 3\left[\left(1-\frac{2}{3}\sin^2\theta_W\right)^2 + \left(\frac{2}{3}\sin^2\theta_W\right)^2\right]$$

$$= \Gamma_{Z\to\nu\bar{\nu}} \cdot \frac{3}{2} \cdot \left[1 + \left(1-\frac{4}{3}\sin^2\theta_W\right)^2\right].$$

The total Z-boson decay width:

$$\boxed{\Gamma_Z = \frac{g^2 M_Z}{192\pi\cos^2\theta_W}\left\{2N_\nu + \left[1+(1-4\sin^2\theta_W)^2\right]N_e + 3\left[1+\left(1-\frac{8}{3}\sin^2\theta_W\right)^2\right]N_u + 3\left[1+\left(1-\frac{4}{3}\sin^2\theta_W\right)^2\right]N_d\right\}}$$

as $M_Z \simeq 91\text{ GeV} \Rightarrow N_u = 2 \quad \left(\overset{u,c\text{ only,}}{<} m_t \simeq 170\text{ GeV}\right)$

$N_d = 3 \quad (d, s, b)$

know $N_e = 3 \quad (e, \mu, \tau)$

as $\sin^2\theta_W \simeq \frac{1}{4} \Rightarrow \Gamma_{Z\to\nu\bar{\nu}} : \Gamma_{Z\to e^+e^-} : \Gamma_{Z\to u\bar{u}} : \Gamma_{Z\to d\bar{d}} =$

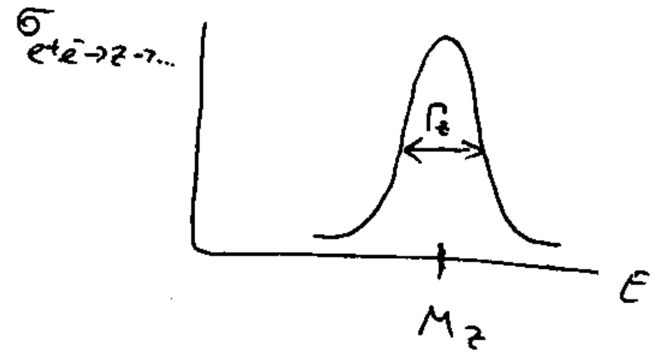
$= 2N_\nu : N_e : \frac{10}{3}N_u : \frac{13}{3}N_d$

$\Rightarrow$ measure $\Gamma_Z \simeq 2.5\text{ GeV}$, $\Gamma_{Z\to e^+e^-}$, $\Gamma_{Z\to u\bar{u}}$, $\Gamma_{Z\to d\bar{d}} \Rightarrow$

$\Rightarrow$ get $\boxed{N_\nu = 2.984 \pm 0.008}$

(look at $e^+e^- \to Z \to \ldots$ at LEP, SLAC ...)

$\Rightarrow$ there are 3 neutrino generations!



Neutrino Masses and Oscillations.

$\Rightarrow$ imagine that neutrinos have a mass (we know they do) & lepton # is not conserved ($\nu_e \to \nu_\mu, \ldots$)

$\Rightarrow$ again mass eigenstate $\neq$ weak eigenstates
(ν_1, ν_2, ν_3) (ν_e, ν_μ, ν_τ)
just like quarks

$\Rightarrow$ consider 2 generations for simplicity:

mixing angle

$$\begin{pmatrix} \nu_\mu \\ \nu_e \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Maki et al '62
Pontecorvo '58, '68
Gribov & Pontecorvo '69

$\Rightarrow$ in EW process one produces ν_e or ν_μ.

propagation: $\nu_1(t) = \nu_1(0)\, e^{-iE_1 t}$, $\nu_2(t) = \nu_2(0)\, e^{-iE_2 t}$

⇒ write (at $t=0$) that we have $\nu_\mu(0)=1$, $\nu_e(0)=0$ ~ a pure μ-neutrino:

$$\Rightarrow \begin{cases} 1 = \nu_1(0)\cos\theta + \nu_2(0)\sin\theta \\ 0 = -\nu_1(0)\sin\theta + \nu_2(0)\cos\theta \end{cases} \Rightarrow \quad \begin{aligned} \nu_1(0) &= \cos\theta \\ \nu_2(0) &= \sin\theta \end{aligned}$$

$$\Rightarrow \nu_\mu(t) = \underbrace{e^{-iE_1 t}\nu_1(0)}_{\nu_1(t)}\cos\theta + \underbrace{e^{-iE_2 t}\nu_2(0)}_{\nu_2(t)}\sin\theta =$$

$$= e^{-iE_1 t}\cos^2\theta + e^{-iE_2 t}\sin^2\theta$$

$$\Rightarrow |\nu_\mu(t)|^2 = \left| e^{-iE_1 t}\cos^2\theta + e^{-iE_2 t}\sin^2\theta \right|^2 =$$

$$= \cos^4\theta + \sin^4\theta + \cos^2\theta\cdot\sin^2\theta\cdot 2\cos\big((E_1-E_2)t\big)$$

$$= 1 + 2\sin^2\theta\cos^2\theta\left[\cos\big((E_1-E_2)t\big) - 1\right] =$$

$$= 1 - \sin^2(2\theta)\cdot\sin^2\left(\frac{E_1-E_2}{2}t\right).$$

For small masses: $E = \sqrt{m^2+p^2} \simeq p + \frac{m^2}{2p}$

$$\Rightarrow E_1 - E_2 \simeq \frac{m_1^2}{2E_1} - \frac{m_2^2}{2E_2} \simeq \frac{m_1^2 - m_2^2}{2E}$$ as $E_1 \simeq E_2$ at this order

$$\Rightarrow \boxed{P(\nu_\mu \to \nu_\mu) = 1 - \sin^2(2\theta)\cdot\sin^2\left(\frac{1{,}27\,\Delta m^2 L}{E}\right)}$$

with $\Delta m^2 \equiv m_2^2 - m_1^2$, $L = t$ (meters), E is in MeV.
$(eV)^2/c^4$

$$\boxed{P_{\nu_\mu \to \nu_e} = \sin^2(2\theta) \cdot \sin^2\left(\frac{1.27\,\Delta m^2 L}{E}\right)}$$

$\Rightarrow$ ν_μ can turn into ν_e & vice versa $\Rightarrow$

$\Rightarrow$ neutrino oscillations!

$$|\nu_\mu(0)\rangle = \cos\theta\,|\nu_1(0)\rangle + \sin\theta\,|\nu_2(0)\rangle$$

~ initially produced a true ν_μ state (EW state)

$$|\nu_e(0)\rangle = -\sin\theta\,|\nu_1(0)\rangle + \cos\theta\,|\nu_2(0)\rangle \, , \quad |\psi_i\rangle = |\nu_\mu(0)\rangle + \emptyset \cdot |\nu_e(0)\rangle$$

~ this is a true ν_e state $\perp$ ν_μ state (not there initially)

$$|\nu_\mu(t)\rangle = e^{-iE_1 t}\cos\theta\,|\nu_1(0)\rangle + e^{-iE_2 t}\sin\theta\,|\nu_2(0)\rangle$$

$$|\langle \nu_\mu(0)|\nu_\mu(t)\rangle|^2 = \left|\cos^2\theta\, e^{-iE_1 t} + \sin^2\theta\, e^{-iE_2 t}\right|^2$$

$\Rightarrow$ the rest is like above

~ solar neutrino problem : # ν_e's from the Sun was ~3 times smaller than expected

(Ray Davies '68, John Bahcall '80)

~ SNO experiment in 2003 measured ν_e and ν_μ from the sun : total # of neutrinos was just right, in agreement with solar models $\Rightarrow$ oscillations!

~ also Super-Kamiokande, KamLAND.

$\Rightarrow$ assuming 3 neutrino flavors get: (PDG):

$$\boxed{\begin{array}{ll} \sin^2\theta_{12} \simeq 0.86 & \Delta m_{12}^2 = 8\times10^{-5}\,eV^2 \\ \sin^2\theta_{23} \geq 0.92 & \Delta m_{32}^2 = 1.9 \div 3.0\times10^{-3}\,eV^2 \end{array}}$$

Note the large mixing angles

$\Rightarrow$ mass hierarchy of neutrinos has not been worked out either... experiments keep on going...

Can we "fix" Standard Model to include right-handed neutrinos? Sure we can: for instance do like for quarks: postulate right-handed neutrino singlet ν_R $\Rightarrow$ $\mathcal{L}_{R.H.\nu} = G_R\left[\underset{Y=+1}{\bar{L}}\ \underset{Y=-1}{\tilde{\phi}}\ \underset{Y=0}{\nu_R} + c.c.\right] + \cdots$ $\Rightarrow$ ok.

$\Rightarrow$ VEV of $\tilde{\phi}$ is $\langle 0|\tilde{\phi}|0\rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$

$$\Rightarrow \mathcal{L}_{RH\nu} = G_R\left[(\bar{\nu}_L\ e_L)\begin{pmatrix} v/\sqrt{2} \\ 0\end{pmatrix}\nu_R + c.c.\right] = G_R\left[\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L\right]\frac{v}{\sqrt{2}}$$

$$= \frac{G_R v}{\sqrt{2}}\bar{\nu}\nu \Rightarrow \boxed{m_\nu = \frac{G_R v}{\sqrt{2}}} \Rightarrow \text{as } m_\nu \gtrsim 0.04\,eV,\quad v \simeq 289\,GeV$$

$$\Rightarrow G_R = \frac{m_\nu\sqrt{2}}{v} \approx 2\times10^{-13}$$ ~ too much fine-tuning...

(some say "why not?")

"Naturalness" Problem:

$$\mathcal{L}_{Higgs} = \frac{1}{2}\partial_\mu \eta\, \partial^\mu \eta - \mu^2 \eta^2 - \lambda v \eta^3 - \frac{\lambda}{4}\eta^4$$

$\Rightarrow$ have diagrams like

giving
$$m_H^2 = m_{H,0}^2 + \# \cdot \lambda \cdot \underbrace{\int^{\Lambda_{UV}} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_{H,0}^2 + i\varepsilon}}_{\text{diveges like } \Lambda_{UV}^2}$$

$\uparrow$ bare mass

$$\Rightarrow \quad m_H^2 = m_{H,0}^2 + \# \cdot \lambda \cdot \Lambda_{UV}^2$$

What is Λ_{UV}? Could be large, like $M_{Planck} \simeq 10^{19}\,GeV$ ...

$\Rightarrow$ for that to work & give "finite" m_H need to fine-tune the bare mass $m_{H,0}^2$ up to 30 decimal places!

$\Rightarrow$ not natural

$\Rightarrow$ likely SM is incomplete ..