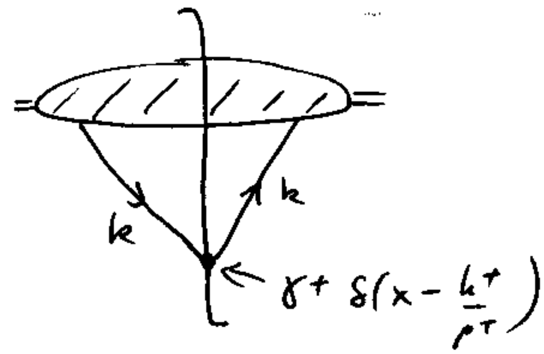
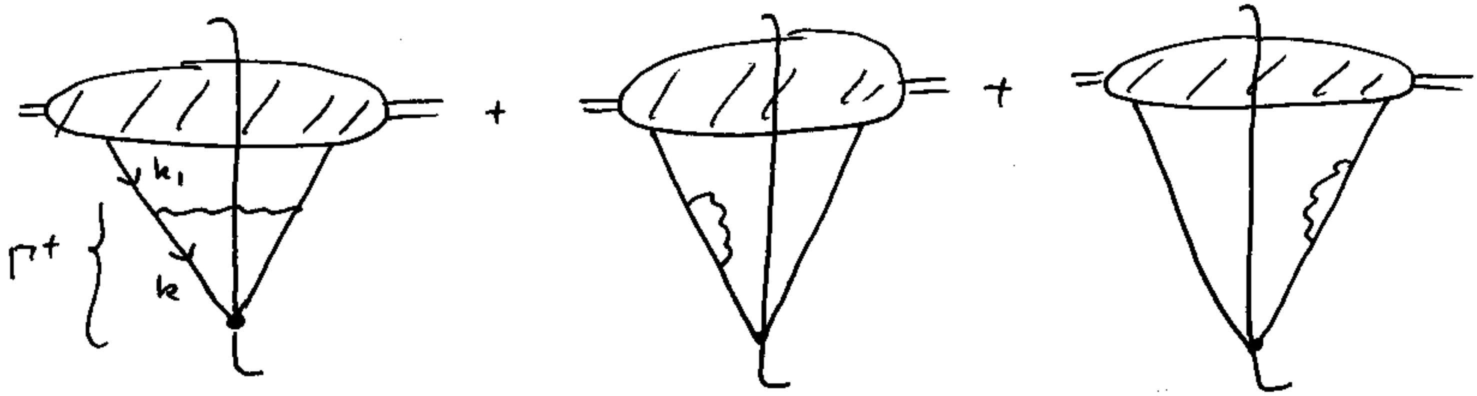
Last time:

QCD-Improved Parton Model (DGLAP Equation)
(cont'd)

$q^f(x) = \frac{\varepsilon_p}{p^+}$ 

"naive" parton model

QCD $O(\alpha_s)$ corrections to this picture:



A B C

We calculated diagram A assuming that

$\underline{k}^2 \gg \underline{k}_1^2, \, k^2 \ldots$ We got

$$\boxed{\Gamma^+ = \gamma^+ \frac{1}{x_1} \frac{\alpha_s C_F}{2\pi} \int \frac{dk_\perp^2}{k_\perp^2} \frac{1+z^2}{1-z}}$$

with $x_1 = \frac{k_1^+}{p^+}$, $z = \frac{k^+}{k_1^+}$ $\Rightarrow$ as $z \leq 1 \Rightarrow x = z x_1 \leq x_1$

$$\Gamma^+ \sim \alpha_s \int_{\Lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \sim \alpha_s \ln \frac{Q^2}{\Lambda^2} \Rightarrow \left.\begin{array}{l} \alpha_s \ll 1 \\ \ln \frac{Q^2}{\Lambda^2} \gg 1 \end{array}\right\} \alpha_s \ln \frac{Q^2}{\Lambda^2} \sim 1$$

leading log approximation (LLA)

$\Rightarrow$ one can resum powers of α_s enhanced by $\ln \frac{Q^2}{\Lambda^2}$ $\Rightarrow$ resum powers of $\left(\alpha_s \ln Q^2/\Lambda^2\right)^{\cdots}$.

We had:

$$q_0^f(x) = \frac{\varepsilon_p}{p^+} \int d^4k \; A_{ab}^f(p,k) \left(\gamma^+\right)_{ba} \delta\left(x - \frac{k^+}{p^+}\right)$$

We now have:

$$\delta q^f = \frac{\varepsilon_p}{p^+} \int d^4k_1 \; A_{ab}^f(p,k_1) \left(\Gamma^+\right)_{ba} = \frac{\varepsilon_p}{p^+} \int d^4k_1 \; A_{ab}^f(p,k_1) \cdot$$

inserting $\int dx_1 \, \delta(x_1 - \frac{k_1^+}{p^+}) = 1$

$$\cdot \left(\gamma^+\right)_{ba} \cdot \frac{1}{x_1} \frac{\alpha_s C_F}{2\pi} \int_{\underline{k}_1^2}^{Q^2} \frac{d^2k_\perp}{k_\perp^2} \frac{1+z^2}{1-z} = \frac{\varepsilon_p}{p^+} \int_x^1 \frac{dx_1}{x_1} \cdot \underbrace{\int d^4k_1 \cdot A_{ab}^f(p,k_1) \left(\gamma^+\right)_{ba} \delta\left(x_1 - \frac{k_1^+}{p^+}\right)}_{\sim q_0^f(x_1)} \frac{\alpha_s C_F}{2\pi} \int_{\underline{k}_1^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \frac{1+z^2}{1-z}$$

$$\Rightarrow \delta q^f = \int_x^1 \frac{dx_1}{x_1} \int_{\underline{k}_1^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \frac{\alpha C_F}{2\pi} \underbrace{\frac{1+\left(\frac{x}{x_1}\right)^2}{1-\frac{x}{x_1}}}_{\text{as } z = \frac{x}{x_1}} \cdot q^f(x_1, Q^2)$$

put Q^2-dep. back.

$$\Rightarrow \boxed{\frac{\partial q^f(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha C_F}{2\pi} \int_x^1 \frac{dx_1}{x_1} \frac{1+\left(\frac{x}{x_1}\right)^2}{1-\frac{x}{x_1}} q^f(x_1, Q^2)}$$

$\Rightarrow$ but this can not be the full story, as we are forgetting graphs B & C.

$$\Gamma_+ = \gamma_+ \frac{1}{x_1} \frac{\alpha_s C_F}{2\pi} \int_{\underline{k}_1^2}^{Q^2} \frac{d\underline{k}^2}{\underline{k}^2} \frac{1+z^2}{1-z}$$

$$\Gamma_+ \sim \alpha_s \cdot \ln\left(Q^2/\underline{k}_1^2\right) \sim \alpha_s \ln Q^2$$

$\alpha_s << 1$ (perturbation theory, small coupling)

$\ln Q^2 >> 1$ (DIS with large Q^2)

$\alpha \ln Q^2 \sim 1$ our resummation parameter!

"Leading Logarithmic Approximation"

Remember: we neglected terms suppressed by $\frac{\underline{k}_1^2}{\underline{k}^2}, \frac{\underline{k}_1^4}{\underline{k}^4}, \ldots \Rightarrow$ they give

$$\int_{\underline{k}_1^2}^{Q^2} \frac{dk^2}{k^4} \underline{k}_1^2 \sim \left(\frac{1}{\underline{k}_1^2} - \frac{1}{Q^2}\right) \underline{k}_1^2 \sim \underbrace{1}_{\text{no log}} - \underbrace{\frac{\underline{k}_1^2}{Q^2}}_{\text{higher twist}}$$

Old (LO) Parton Model vertex (Mueller vertex) was $\gamma_+ \delta\left(x - \frac{k_+}{p_+}\right)$ ~ same γ_+ matrix as Γ_+

$$\Rightarrow \boxed{Q^2 \frac{\partial}{\partial Q^2} q^f(x, Q^2) = \frac{\alpha}{2\pi} \int_x^1 \frac{dx_1}{x_1} \gamma_{qq}\left(\frac{x}{x_1}\right) q^f(x_1, Q^2)}$$

where $x = \frac{k_+}{p_+}$, $x_1 = \frac{k_{1+}}{p_+} \Rightarrow z = \frac{k_+}{k_{1+}} = \frac{x}{x_1}$

as $z < 1 \Rightarrow x_1 > x$ in the integral.

Including the virtual terms (B and C)

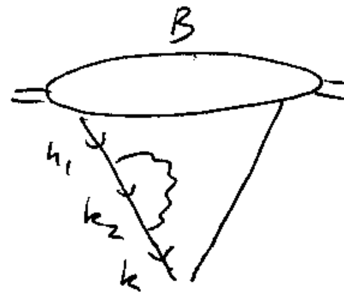
gives

$$\gamma_{qq}(z) = C_F \left(\frac{1+z^2}{1-z}\right)_+$$

where

$$\int_0^1 dz \,[h(z)]_+ \, f(z) = \int_0^1 dz \, h(z) \,[f(z) - f(1)]$$

Easy to understand:



$\propto \delta(k - k_1)$

$\Downarrow$

$x = x_1$

$\Rightarrow q_f(x_1, Q^2) = q_f(x, Q^2)$

as $x = x_1$

"real" part, diagram A

$$Q^2 \frac{\partial}{\partial Q^2} q^f(x, Q^2) = \frac{\alpha C_F}{2\pi} \int_x^1 \frac{dx_1}{x_1} \frac{1 + (x/x_1)^2}{1 - x/x_1} \cdot \left[q^f(x_1, Q^2) - q_f(x, Q^2) \right]$$

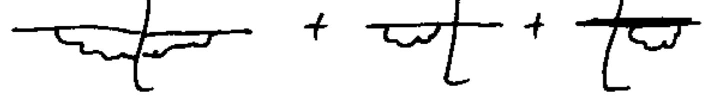
Virtual corrections, graphs B & C

bare quark state $|\psi_0\rangle =$ —— $\Rightarrow \langle\psi_0|\psi_0\rangle = 1$
(normalization)

dressed quark state $|\psi\rangle =$ 

$|\psi_0\rangle \qquad |\psi_1\rangle \qquad V|\psi_0\rangle$

normalization:

$\langle\psi|\psi\rangle = 1 = \langle\psi_0|\psi_0\rangle +$ 

$= 1 + \langle\psi_1|\psi_1\rangle + 2V\langle\psi_0|\psi_0\rangle = 1 + \langle\psi_1|\psi_1\rangle + 2V$

$\Rightarrow V = -\frac{1}{2}\langle\psi_1|\psi_1\rangle$

$\Rightarrow$ graphs $B, C = -\frac{1}{2}A \Rightarrow B + C = -A$.

~ simply imposed probability conservation!

Defining flavor singlet distribution

$$\Sigma(x, Q^2) \equiv \sum_f \left[q^f(x, Q^2) + q^{\bar{f}}(x, Q^2) \right]$$

and flaver non-singlet

$$\Delta_{ff'}(x, Q^2) \equiv q^f(x, Q^2) - q^{f'}(x, Q^2)$$

we write

$$Q^2 \frac{\partial}{\partial Q^2} \Delta_{ff'}(x, Q^2) = \frac{\alpha(Q^2)}{2\pi} \int_x^1 \frac{dx_1}{x_1} \gamma_{qq}\left(\frac{x}{x_1}\right) \cdot \Delta^{ff'}(x_1, Q^2)$$

and

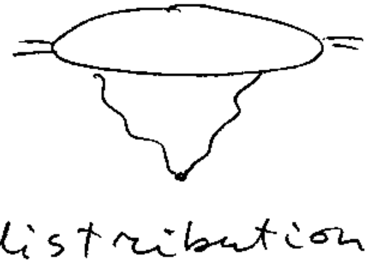
$$Q^2 \frac{\partial}{\partial Q^2} \begin{pmatrix} \Sigma(x, Q^2) \\ G(x, Q^2) \end{pmatrix} = \frac{\alpha(Q^2)}{2\pi} \int_x^1 \frac{dx_1}{x_1} \begin{pmatrix} \gamma_{qq}\left(\frac{x}{x_1}\right) & \gamma_{qG}\left(\frac{x}{x_1}\right) \\ \gamma_{Gq}\left(\frac{x}{x_1}\right) & \gamma_{GG}\left(\frac{x}{x_1}\right) \end{pmatrix} \cdot \begin{pmatrix} \Sigma(x_1, Q^2) \\ G(x_1, Q^2) \end{pmatrix}$$

Dokshitzer, Gribov, Lipatov, Altarelli, Parisi

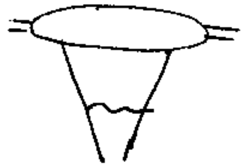
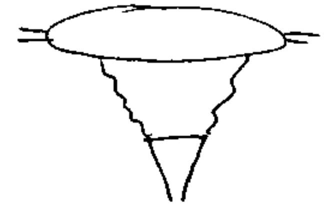
(DGLAP) Equations

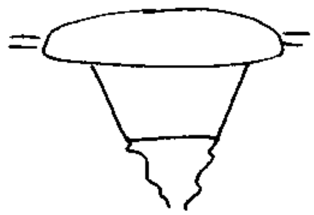
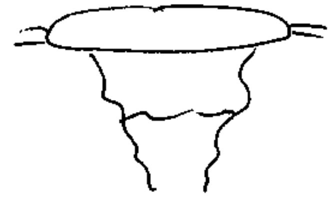
GL ~ QED case ~ '72

D, A & P ~ QCD case, '77

$G(x, Q^2) =$  $\sim \langle A_i A_i \rangle$ in $A_+ = 0$ gauge

gluon distribution function

$\gamma_{qq} \sim$  , $\gamma_{qG} =$ 

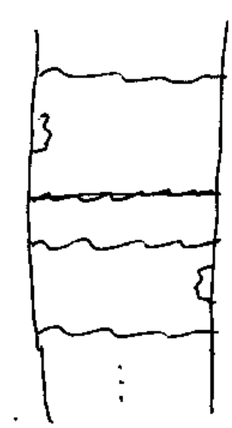
$\gamma_{Gq} \sim$  , $\gamma_{GG} \sim$ 

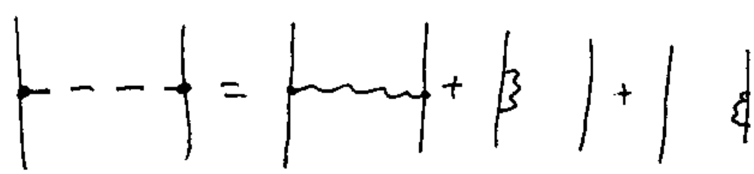
After explicit calculations one gets:

$$
\begin{cases}
\gamma_{qq}(z) = C_F \left(\dfrac{1+z^2}{1-z} \right)_+ \\[2ex]
\gamma_{Gq}(z) = C_F \, \dfrac{1+(1-z)^2}{z} \\[2ex]
\gamma_{qG}(z) = N_f \left[z^2 + (1-z)^2 \right] \\[2ex]
\gamma_{GG}(z) = 2N_c \left[\dfrac{z}{(1-z)_+} + \dfrac{1-z}{z} + z(1-z) \right] + \dfrac{11N_c - 2N_f}{6} \delta(z-1)
\end{cases}
$$

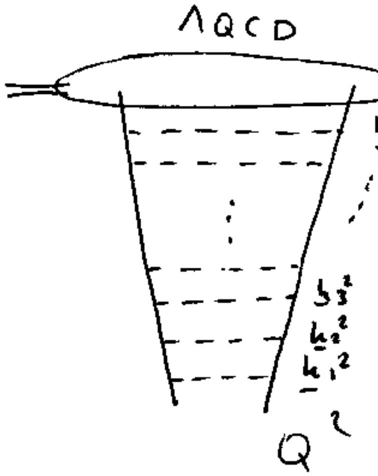
Note that $\gamma_{Gq}(z)$ can be obtained from $\gamma_{qq}(z)$ by substituting $z \to 1-z$ and dropping virtual corrections.

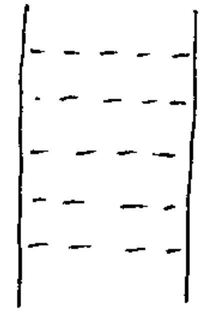
Iterate the evolution for $\rho_f(x, Q^2)$:



Defining 

We get a <u>ladder</u> diagram:



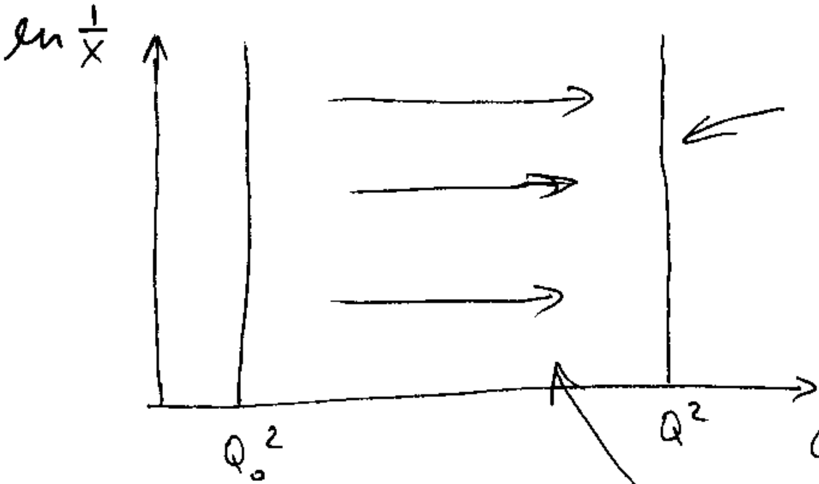


$Q^2 \gg k_1^2 \gg k_2^2 \gg \ldots \gg k_N^2 \gg \Lambda_{QCD}^2$

DGLAP resums ladder graphs with the ladder connecting scales Q^2 and Λ_{QCD}^2, $Q^2 \gg \Lambda_{QCD}^2$

such that $\ln Q^2/\Lambda_{QCD}^2 \gg 1$ and $\alpha_s \ln \frac{Q^2}{\Lambda_{QCD}^2} \sim 1$ is the resummation parameter.

<u>How does DGLAP work?</u>



obtain distribution at whatever Q^2 you wanted.

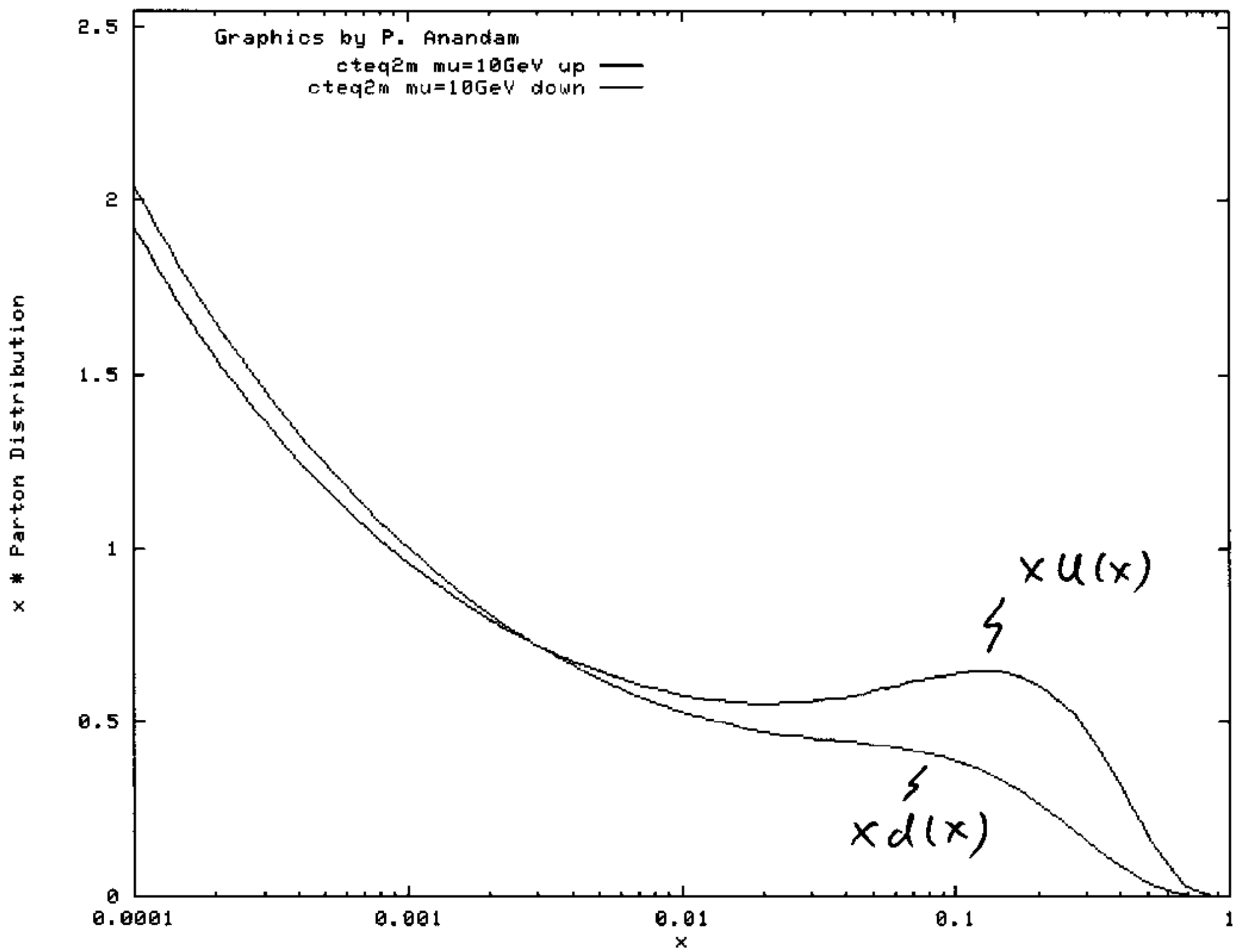
start with some initial condition = function $p^f(x, Q_0^2)$

Solve the DGLAP equations ("evolve" the distribution function)

$\Rightarrow$ people calculate PDF's (Parton Distribution Functions) & fit the data. See attachements for PDF examples.

Parton Distribution Graph

(Number of graphs plotted since 21 November 2000: 658)



$Q = 10$ GeV $\Rightarrow Q^2 = 100$ GeV^2

at large-x valence quarks dominate

$\Rightarrow \quad x u_v(x) = 2\, x d_v(x)$

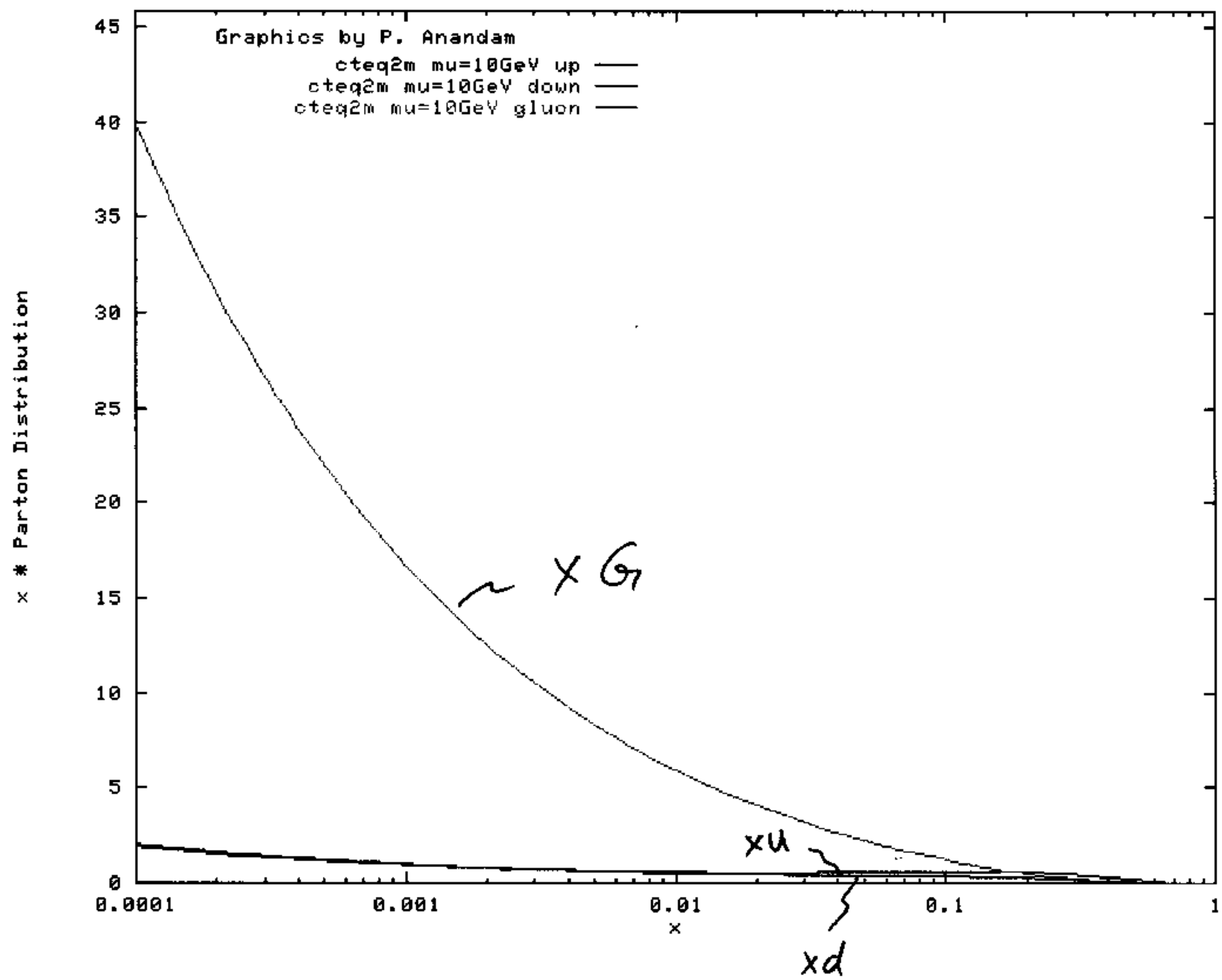
$\Rightarrow$ not so at small-x

go to http://zebu.uoregon.edu/~parton/partongraph.html

to plot more.

Parton Distribution Graph

(Number of graphs plotted since 21 November 2000: 659)



the same plot with xG (gluon distribution) plotted as well ....

now, who's ya daddy?

=> at small-x gluons dominate by far...