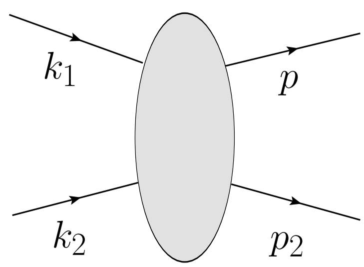
Homework Set No. 2, Physics 880.08

Deadline – Friday, February 5, 2010

1. (20 pts) Consider $2 \to 2$ scattering of identical particles of mass m. Suppose the production cross section of a particle with 4-momentum p is given by

$$\mathcal{E}_p \frac{d\sigma}{d^3p} \;=\; f(s,t,u)\,\delta(s' + t' + u' - 4\,m^2)$$

where f is some function of $s = (k_1 + k_2)^2$, $t = (k_1 - p)^2$, and $u = (k_1 - p_2)^2$, with $s' = (p + p_2)^2$, $t' = (k_2 - p_2)^2$, $u' = (k_1 - p_2)^2 = u$ (see figure below). Just like in class $k_1^\mu = (E_{k_1}, \vec{k}_1)$, $k_2^\mu = (E_{k_2}, \vec{k}_2)$, $p^\mu = (E_p, \vec{p})$, and $p_2^\mu = (E_{p_2}, \vec{k}_1 + \vec{k}_2 - \vec{p})$.



Show that

$$\frac{d\sigma}{dt} \;=\; \frac{\pi}{\sqrt{s\,(s - 4\,m^2)}}\, f(s,t,u)$$

where u is now defined by $u = 4\,m^2 - s - t$ and $s = (k_1 + k_2)^2$ still. (Hint: it is easier to prove this result in the rest frame of one of the initial state particles.)

2. In class we defined Wick contraction for two Dirac fields in the interaction representation by

$$\overbrace{\psi_\alpha(x)\,\bar{\psi}_\beta(y)} \;=\; \mathrm{T}\,\psi_\alpha(x)\,\bar{\psi}_\beta(y) - :\psi_\alpha(x)\,\bar{\psi}_\beta(y): .$$

a. (10 pts) Show that

$$\overbrace{\psi_\alpha(x)\,\bar{\psi}_\beta(y)} \;=\; \langle 0|\mathrm{T}\,\psi_\alpha(x)\,\bar{\psi}_\beta(y)|0\rangle \;=\; S_F(x-y)_{\alpha\beta}$$

where S_F is the Feynman propagator for fermions.

b. (10 pts) Prove that

$$\overbrace{\psi_\alpha(x)\,\psi_\beta(y)} \;=\; \overbrace{\bar{\psi}_\alpha(x)\,\bar{\psi}_\beta(y)} \;=\; 0.$$