Homework Set No. 4, Physics 880.08

Deadline – Wednesday, March 17, 2010

1. (15 pts) In class when calculating the cross section for Compton scattering we encountered the following Dirac trace

$$\mathrm{Tr}\left[(\not p' + m)\,\gamma^\nu\,(\not p + \not k + m)\,\gamma^\mu\,(\not p + m)\,\gamma_\nu\,(\not p - \not k' + m)\,\gamma_\mu\right].$$

Show that this trace is equal to

$$8\,\left[4\,m^4 + m^2\,(s - m^2) + m^2\,(u - m^2)\right]$$

as we used in class. Here $s = (p+k)^2 = (p'+k')^2$ and $u = (p-k')^2 = (p'-k)^2$, with p, p' the 4-momenta of the electron and k, k' the 4-momenta of the photon.

2. Consider ϕ^4 scalar theory with the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4!}\,\varphi^4.$$

Below use Pauli-Villars regularization with two new particles, such that the propagators of the internal lines are replaced by

$$\frac{i}{k^2 - m^2 + i\,\epsilon} \to \frac{i}{k^2 - m^2 + i\,\epsilon} + \sum_{i=1}^{2} \frac{i\,C_i}{k^2 - M_i^2 + i\,\epsilon}$$

with $C_1 = 1$, $C_2 = -2$, and $M_1^2 = m^2 + 2\,M^2$, $M_2^2 = m^2 + M^2$. M is some UV cutoff.

a. (30 pts) Calculate regularized one-loop corrections to the propagator and to the connected four-point Green function, keeping only divergent and finite terms at large M.

b. (15 pts) Set up the renormalization program for the ϕ^4 theory at one loop. Find all counterterms and coefficients in front of them using "on-shell" renormalization conditions. In particular show that the renormalized truncated 4-point Green function $\Gamma^4(s,t,u)$ is given by

$$\Gamma^4(s,t,u) \;=\; -i\,\left\{\lambda + \frac{\lambda^2}{32\,\pi^2}\int\limits_0^1 dx\,\left[\ln\left(\frac{m^2 - s\,x\,(1-x)}{m^2 - 4\,m^2 x\,(1-x)}\right) + (s\leftrightarrow t) + (s\leftrightarrow u)\right] + o(\lambda^3)\right\}.$$

Choose the counter-terms such that $\Gamma^4(s,t,u) = -i\,\lambda$ when $s = t = u = 4\,m^2$. For a 4-point function with two external lines having incoming 4-momenta p_1^μ and p_2^μ and two other external lines having outgoing 4-momenta p_3^μ and p_4^μ we define as usual $s = (p_1+p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_1 - p_4)^2$. (You may find the discussion in Chapter 10.2 of Peskin and Schroeder useful, though they use dimensional regularization.)