Homework Set No. 4, Physics 880.08

Deadline – Monday, June 7, 2010

1. (20 pts) Consider a non-Abelian gauge theory in the light cone gauge $\eta \cdot A^a = A^{a\,+} = 0$ ($\eta^- = 1, \eta^+ = 0, \eta^1 = \eta^2 = 0$). By inverting the operator in the quadratic in A_μ^a term in the exponent of the path integral, show that the gauge field (gluon) propagator in this gauge is given by

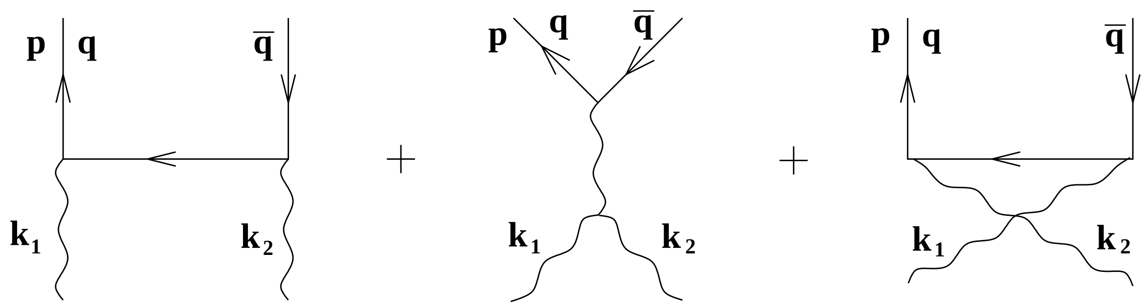
$$D_{\mu\nu}^{ab}(k) \;=\; \frac{-i\,\delta^{ab}}{k^2 + i\,\epsilon} \left(g_{\mu\nu} - \frac{\eta_\mu k_\nu + \eta_\nu k_\mu}{\eta \cdot k} \right).$$

(Hint: do not forget to take the gauge-fixing parameter ξ to zero, $\xi \to 0$, at the end of the calculation.)

2. (Unrelated to problem 1.)
a. (30 pts) Calculate the cross section for

$$\text{gluon} + \text{gluon} \to \text{quark} + \text{antiquark}$$

at the Born level shown in the figure below. The figure is for the *amplitude*, which needs to be squared and multiplied by appropriate factors to get the cross section.



You should find

$$\frac{d\sigma_{gg\to q\bar{q}}}{dt} \;=\; \frac{3\,\pi\,\alpha_s^2}{8\,s^2}\,(t^2+u^2)\left(\frac{4}{9\,t\,u} - \frac{1}{s^2}\right)$$

with the Mandelstam variables $s = (k_1+k_2)^2$, $t = (k_1-p)^2$, $u = (k_2-p)^2$. (q and $\bar{q}$ in the figure denote the quark and the antiquark. Time flows upward.) Assume that quarks are massless.

b. (5 pts) Using the result of part **a** find the cross section $d\sigma_{q\bar{q}\to gg}/dt$ for the inverse process

$$\text{quark} + \text{antiquark} \to \text{gluon} + \text{gluon}.$$

Hints:

One quick and dirty way of arriving at the solution is to use the fact that incoming gluons are physical,

$$k^\mu \, \epsilon_\mu^\lambda(k) \;=\; 0, \tag{1}$$

in the amplitude to eliminate some of the terms. Then after squaring the amplitude you may replace the polarization sums by

$$\sum_{\lambda=\pm 1} \epsilon_\mu^{*\,\lambda}(k) \, \epsilon_\nu^\lambda(k) \;\to\; -g_{\mu\nu}. \tag{2}$$

Alternatively you may use Eq. (2) without Eq. (1). However, if you follow this path you would have to subtract ghost loop contributions (amplitudes with ghost-antighost pair in initial state instead of gluons, see Sterman pp. 233-237). This strategy is a more systematic way of arriving at the right answer.

Other ways of solving the problem include the use of explicit parameterizations for polarization vectors in calculating amplitudes. One may also use polarization sum

$$\sum_{\lambda=\pm 1} \epsilon_\mu^{*\,\lambda}(k) \, \epsilon_\nu^\lambda(k) \;=\; -g_{\mu\nu} + \frac{\bar{k}_\mu \, k_\nu + k_\mu \, \bar{k}_\nu}{k \cdot \bar{k}},$$

where for $k^\mu = (k^0, \vec{k})$ we defined $\bar{k}^\mu = (k^0, -\vec{k})$.

The following γ-matrix formulas may be useful:

$$\gamma^\mu \, \gamma^\nu \, \gamma_\mu \;=\; -2 \, \gamma^\nu$$

$$\gamma^\mu \, \gamma^\nu \, \gamma^\rho \, \gamma_\mu \;=\; 4 \, g^{\nu\,\rho}$$

$$\gamma^\mu \, \gamma^\nu \, \gamma^\rho \, \gamma^\sigma \, \gamma_\mu \;=\; -2 \, \gamma^\sigma \, \gamma^\rho \, \gamma^\nu$$

$$\mathrm{tr}[\gamma^\mu \, \gamma^\nu] \;=\; 4 \, g^{\mu\nu}$$

$$\mathrm{tr}[\gamma^\mu \, \gamma^\nu \, \gamma^\rho \, \gamma^\sigma] \;=\; 4 \, (g^{\mu\nu} \, g^{\rho\sigma} + g^{\mu\sigma} \, g^{\nu\rho} - g^{\mu\rho} \, g^{\nu\sigma}).$$

For color traces the following expressions may come in handy:

$$T^a \, T^a \;=\; C_F \, \mathbf{1}$$

with

$$C_F \;=\; \frac{N_c^2 - 1}{2 \, N_c},$$

$$\mathrm{tr}[T^a T^b T^a T^b] \;=\; -\frac{C_F}{2}$$

$$f^{abc} \, f^{abd} \;=\; N_c \, \delta^{cd}.$$