Homework Set No. 3, Physics 880.08

Deadline – Monday, February 27, 2012

1. Construct the momentum-space Feynman rules for the following theories (draw all the propagators and vertices, and find their contributions to the diagrams):

a. (10 pts) Scalar electrodynamics, with the Lagrangian density

$$\mathcal{L} = (D_\mu \varphi)^* D^\mu \varphi - m^2 \varphi^* \varphi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$

where $D_\mu = \partial_\mu + i\, e\, A_\mu$. Here φ is a complex scalar field, A_μ is a real gauge field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

b. (5 pts) Complex scalar theory, with the Lagrangian density

$$\mathcal{L} = (\partial_\mu \varphi)^* \partial^\mu \varphi - \lambda^2 \varphi^* \varphi (\varphi^* \varphi - v^2)^2,$$

where λ and v are real positive constants and φ is a complex scalar field.

Note that while for the photon propagator you can simply use the expression given in class, I would like you to derive the propagator for the complex scalar field in part **a**.

2. (15 pts) In class when calculating the cross section for Compton scattering we encountered the following Dirac trace

$$\mathrm{Tr}\left[(\not{p}' + m)\, \gamma^\nu\, (\not{p} + \not{k} + m)\, \gamma^\mu\, (\not{p} + m)\, \gamma_\nu\, (\not{p} - \not{k}' + m)\, \gamma_\mu\right].$$

Show that this trace is equal to

$$8\, \left[4\, m^4 + m^2\, (s - m^2) + m^2\, (u - m^2)\right]$$

as we used in class. Here $s = (p + k)^2 = (p' + k')^2$ and $u = (p - k')^2 = (p' - k)^2$, with p, p' the 4-momenta of the electron and k, k' the 4-momenta of the photon.