Homework Set No. 2, Physics 8808.1

Deadline – Tuesday, September 18, 2012

1. Consider generators of some Lie group obeying Lie algebra commutation relations

$$[X_a, X_b] = i\, f_{abc}\, X_c \tag{1}$$

with anti-symmetric structure constants f_{abc}.

(a) (5 pts) Prove the Jacobi identity

$$[X_a, [X_b, X_c]] + [X_b, [X_c, X_a]] + [X_c, [X_a, X_b]] = 0$$

by expanding out the commutators.

(b) (5 pts) Use the commutation relation (1) for X_a's in the Jacobi identity to show that

$$f_{bcd}\, f_{ade} + f_{abd}\, f_{cde} + f_{cad}\, f_{bde} = 0,$$

which is also often referred to as the Jacobi identity.

2. **(a)** (5 pts) Suppose the generators of the Lie group in problem 1 are normalized such that

$$\mathrm{tr}(X_a\, X_b) = \frac{1}{2}\, \delta^{ab}. \tag{2}$$

Using this along with Eq. (1) find the structure constants f^{abc} in terms of generators X_a and prove that f^{abc} is anti-symmetric under the interchange of any pair of its indices.

(b) (5 pts) Using Gell-Mann matrices (and their commutators) find the structure constants f^{147} and f^{458} of the group $SU(3)$ employing the relation found in part (a) of this problem.

3. (10 pts) In class we defined the generators of the Lorentz group by

$$L_{\mu\nu} = i\, (x_\mu\, \partial_\nu - x_\nu\, \partial_\mu).$$

Show that these generators obey the following algebra

$$[L_{\mu\nu}, L_{\rho\sigma}] = i\, \eta_{\nu\rho}\, L_{\mu\sigma} - i\, \eta_{\mu\rho}\, L_{\nu\sigma} - i\, \eta_{\nu\sigma}\, L_{\mu\rho} + i\, \eta_{\mu\sigma}\, L_{\nu\rho}.$$