## Homework Set No. 2, Physics 8808.02 Deadline – Thursday, February 7, 2013

- 1. Construct the momentum-space Feynman rules for the following theories (draw all the propagators and vertices, and find their contributions to the diagrams):
  - a. (10 pts) Scalar electrodynamics, with the Lagrangian density

$$\mathcal{L} = (D_{\mu} \varphi)^* D^{\mu} \varphi - m^2 \varphi^* \varphi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$

where  $D_{\mu} = \partial_{\mu} + i e A_{\mu}$ . Here  $\varphi$  is a complex scalar field,  $A_{\mu}$  is a real gauge field,  $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ .

**b.** (5 pts) Complex scalar theory, with the Lagrangian density

$$\mathcal{L} = (\partial_{\mu} \varphi)^* \partial^{\mu} \varphi - \lambda^2 \varphi^* \varphi (\varphi^* \varphi - v^2)^2,$$

where  $\lambda$  and v are real positive constants and  $\varphi$  is a complex scalar field.

Note that while for the photon propagator you can simply use the expression given in class, I would like you to derive the propagator for the complex scalar field in part a.

2. (15 pts) In class when calculating the cross section for Compton scattering we encountered the following Dirac trace

$$\operatorname{Tr}\left[\left(\mathbf{p}'+m\right)\gamma^{\nu}\left(\mathbf{p}'+\mathbf{k}'+m\right)\gamma^{\mu}\left(\mathbf{p}'+m\right)\gamma_{\nu}\left(\mathbf{p}'-\mathbf{k}''+m\right)\gamma_{\mu}\right].$$

Show that this trace is equal to

$$8 \left[ 4 m^4 + m^2 (s - m^2) + m^2 (u - m^2) \right]$$

as we used in class. Here  $s = (p + k)^2 = (p' + k')^2$  and  $u = (p - k')^2 = (p' - k)^2$ , with p, p' the 4-momenta of the electron and k, k' the 4-momenta of the photon.