Homework Set No. 4, Physics 8808.02

Deadline – Thursday, March 7, 2013

1. (10 pts) Fill in the steps omitted in class to derive the one-loop electron self-energy correction in QED using dimensional regularization. That is, obtain Eq. (10.41) in Peskin and Schroeder with zero photon mass, $\mu = 0$, starting from

$$\Sigma_2(p) = -i\, e^2 \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu\,(\not k + m)\,\gamma_\mu}{[(p-k)^2 + i\,\epsilon]\,[k^2 - m^2 + i\,\epsilon]}. \tag{1}$$

Assume that $p^2 < 0$.

2. Using dimensional regularization find the one-loop beta-function

$$\beta(\lambda) \,=\, \mu^2\, \frac{d\,\lambda}{d\,\mu^2}$$

of the real scalar φ^3 theory with the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\,\partial_\mu\varphi_0\,\partial^\mu\varphi_0 - \frac{m_0^2}{2}\,\varphi_0^2 - \frac{\lambda_0}{3!}\,\varphi_0^3 \tag{2}$$

in **six** (!) space-time dimensions. Here φ_0 is the bare field, while λ_0 and m_0 are the bare coupling constant and the bare mass correspondingly.

a. (20 pts) Similar to how you did it in HW 3 for ϕ^4 theory, rewrite the Lagrangian (2) in terms of renormalized physical fields φ, coupling λ, mass m and the counterterms. By calculating the one-loop propagator and vertex corrections find the divergent ($\sim 1/\epsilon$ with $\epsilon = 6 - d$) parts of the coefficients of the counterterms δ_λ and δ_Z. (Ignore the tadpole graph.) Note that we do not need the finite parts of the counterterms to find $\beta(\lambda)$!

b. (10 pts) Using the results of part **a** find $\beta(\lambda)$ following the steps of the QED beta-function calculation performed in class.