$\boxed{\begin{array}{l}\text{High Energy QCD Journal Club} \\ \text{Lecture II: The Parton Model}\end{array}}$

9/28
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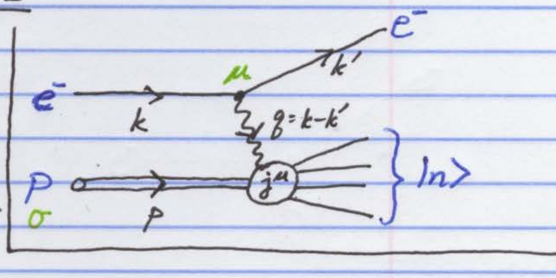
I. Review: Kinematics of DIS

- Deep Inelastic Scattering: lepton-hadron ($e^- p$) scattering at high virtuality. The hadronic final state $|n\rangle$ is summed over.



- Spacelike virtual photon γ^* with momentum q couples to EM currents j^μ at scales inside the proton.

- Characterized by 2 independent kinematic invariants:
  1) $Q^2 \equiv -q^2$ (Photon virtuality)
  2) $x_B \equiv \frac{Q^2}{2p\cdot q}$ (Bjorken x ~ electron recoil)

- "Deep inelastic" criterion: $\boxed{Q^2 \gg m_p^2}$

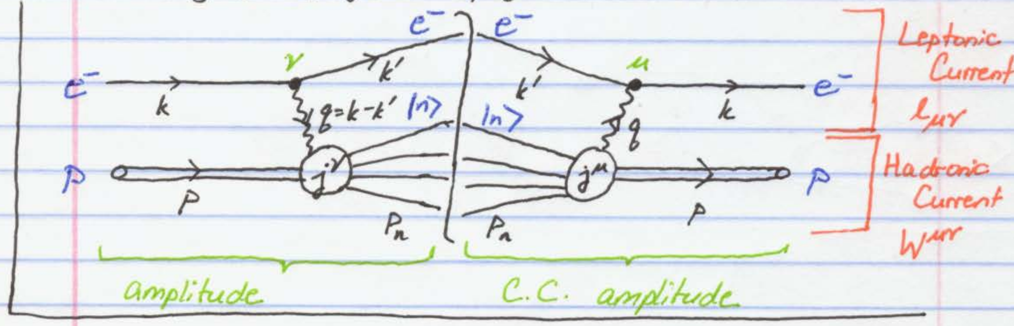
II. Review: General Formalism

- Electron cross-section decomposes into a product of currents:

$$\boxed{E' \frac{d\sigma}{d^3k'} = \frac{2\alpha_{EM}^2}{Q^4}\frac{M_P}{E}\, l_{\mu\nu} W^{\mu\nu}}$$

- Leptonic current: $$l_{\mu\nu} \equiv \frac{1}{2}\sum_{spins}\left(\bar{u}_{k'}\gamma_\mu u_k\right)^*\left(u_{k'}\gamma_\nu u_k\right)$$
$$= 2\left(k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu}\, k\cdot k'\right)$$
  ↳ explicitly symmetric ($\mu \leftrightarrow \nu$)

- Hadronic current: $W^{\mu\nu} = \frac{1}{8\pi m_p} \sum_{\sigma, |n\rangle} \langle n | j^\mu(0) | p\sigma \rangle^* \langle n | j^\nu(0) | p\sigma \rangle (2\pi)^4 \delta^4(p+q-p_n)$
$$= \frac{1}{4\pi m_p} \int d^4y \; e^{iq\cdot y} \langle p | j^\mu(y) j^\nu(0) | p \rangle$$
  ↪ only $(\mu \leftrightarrow \nu)$ symmetric projection contributes.



- Without knowing anything about the internal structure of the proton, we can constrain its Lorentz structure:
  1) $W_{\mu\nu}(P, q)$ is built from 2 vectors: 6 possible tensors
     $P_\mu P_\nu$, $q_\mu q_\nu$, $P_\mu q_\nu$, $q_\mu P_\nu$, $g_{\mu\nu}$, $\varepsilon_{\mu\nu\rho\sigma} P^\rho q^\sigma$
  2) Current conservation: $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$
  3) Parity invariance: eliminates $\varepsilon_{\mu\nu\rho\sigma}$
  4) Linear independence of p_μ, q_μ

↪ only 2 independent structure functions W_1, W_2

$$\boxed{\begin{aligned} W_{\mu\nu}(P,q) &= \left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right] W_1(x_B, Q^2) + \frac{1}{m_p^2}\left[P^\mu - q^\mu \frac{P\cdot q}{q^2}\right]\left[P^\nu - q^\nu \frac{P\cdot q}{q^2}\right] W_2(x_B, Q^2) \\ &= \frac{1}{m_p}\left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right] F_1(x_B, Q^2) + \frac{1}{P\cdot q\, m_p}\left[P^\mu - q^\mu \frac{P\cdot q}{q^2}\right]\left[P^\nu - q^\nu \frac{P\cdot q}{q^2}\right] F_2(x_B, Q^2) \end{aligned}}$$

- Structure functions are Lorentz scalar and depend only on Lorentz scalar kinematic variables.

- F_1 & F_2 defined to be dimensionless

- Deep inelastic neutrino scattering breaks parity invariance due to axial current j_5^μ, permitting $\varepsilon_{\mu\nu\rho\sigma} P^\rho q^\sigma$ and $W_3(x_B, Q^2)$

Today: The Parton Model — DIS as a Microscope

III. Frozen Partons and the Bjorken Frame

- If the proton is a composite state of sub-nucleon constituents ("partons"), their interactions occur on a scale set by m_p: (ie, Λ_{QCD})

  Rest frame: $\tau_p \sim \frac{1}{m_p}$ $\longrightarrow$ Boosted frame: $\tau_p \sim \frac{1}{\gamma} \cdot \frac{1}{m_p}$

- The EM interaction occurs over a time scale set by the photon energy:

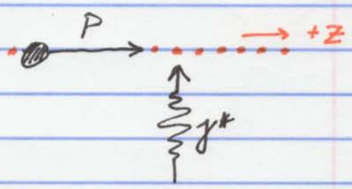
  $$\tau_{EM} \sim \frac{1}{q^0}$$

- Seen from a frame in which $E_p \gg m_p$, the time-scales separate: $\tau_{EM} \ll \tau_p$. The parton distribution of the proton wave function appears "frozen", and DIS acts as an "instantaneous" probe of the wave function.

  > "The idea of 'quanta' is a frame-dependent concept." — Al Mueller

- Bjorken Frame (an "infinite momentum frame")
  - $\mapsto$ start in a frame collinear to proton (ie, $e^- p$ CMS frame)
  - $\mapsto$ boost until $q_z = 0$ (no longitudinal γ^* momentum)

$$p^\mu = (E_p, \underline{0}, p)$$
$$q^\mu = (q_0, \underline{q}, 0)$$



DIS criterion: $Q^2 \equiv -q^2 \gg m_p^2$

$$q_T^2 - q_0^2 \gg m_p^2$$
$$q_T \gg \sqrt{q_0^2 + m_p^2} > q_0 \Rightarrow \boxed{q_T \gg q_0}$$

↳ Since q^2 is large and spacelike and $q_z = 0$ in the Bjorken frame, this results in q^μ being predominantly transverse:

$$q^\mu \approx (0, \underline{q}, 0) \quad ; \quad Q^2 \approx q_T^2$$

· Further, the $\gamma^* p$ cms energy is

$$\hat{s} \equiv (p+q)^2 \geq m_p^2 \qquad\qquad p\cdot q = q_0 E_p$$

$$\hookrightarrow \quad \cancel{m_p^2} - Q^2 + 2p\cdot q \geq \cancel{m_p^2}$$

$$\hookrightarrow \quad p\cdot q \geq Q^2/2$$

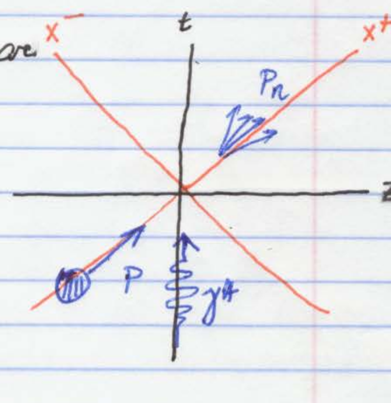
$$\hookrightarrow \quad E_p \geq \frac{Q^2}{2q_0} \gg \frac{Q^2}{2Q} \gg Q \gg m_p$$

$$\hookrightarrow \quad \boxed{E_p \gg m_p} \qquad \Rightarrow \quad E_p = p + \frac{m_p^2}{2p} + \mathcal{O}\left(\frac{1}{p^2}\right)$$

$\Longrightarrow$ Thus DIS in the Bjorken frame automatically yields an ultra-relativistic proton. The Bjorken frame is an "infinite momentum frame" at high Q^2.

Thus the Bjorken frame kinematics are

$$\boxed{\begin{cases} p^\mu = \left(p + \frac{m_p^2}{2p}, \underline{0}, p\right) \\ q^\mu = (q_0, \underline{q}, 0) \\ \text{with } q_0 \ll q_T \end{cases}}$$



· Since the proton is traveling along the light cone in this frame, it makes sense to introduce light-cone coordinates:

$$\boxed{\begin{aligned} v^\pm &\equiv \tfrac{1}{\sqrt{2}}(v^0 \pm v^3) \\ u\cdot v &= u^+ v^- + u^- v^+ - \underline{u}\cdot\underline{v} \end{aligned}} \longrightarrow \begin{cases} p^+ \approx \sqrt{2}\, p \\ p^- = \frac{m_p^2}{2\sqrt{2}p} = \frac{m_p^2}{2p^+} \\ q^+ = q^- = q_0/\sqrt{2} \ll Q \end{cases}$$

· Separation of scales: $\boxed{p^+ \gg m_p \gg p^-}$

IV. DIS as a Forward Compton Amplitude

• Recall $W^{\mu\nu} \equiv \frac{1}{4\pi m_p} \sum_{|n\rangle} \langle n | j^\mu(0) | p \rangle^* \langle n | j^\nu(0) | p \rangle (2\pi)^4 \delta^4(p+q-p_n)$

• If the current matrix elements are expressed in terms of (truncated) interacting Green functions $G_{p\to n}$, they differ by factors of i from the QED interaction:

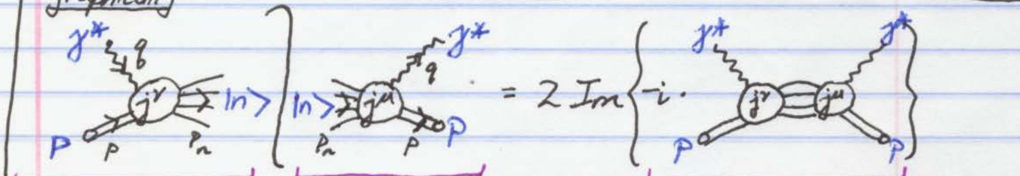
$$\langle n | j^\mu(0) | p \rangle = -i\, G^\mu_{p\to n} = \mathcal{M}^\mu_{p\to n}$$

$\hookrightarrow$ $W^{\mu\nu} = \frac{1}{4\pi m_p} \sum_{|n\rangle} \mathcal{M}^{\mu *}_{p\to n} \mathcal{M}^\nu_{p\to n}$

• The optical theorem relates this square to a forward amplitude:

$$\sum_{|n\rangle} \mathcal{M}^{\mu *}_{p\to n} \mathcal{M}^\nu_{p\to n} = 2\,\mathrm{Im}(\mathcal{M}_{p\to p}) = 2\,\mathrm{Im}(-i\,G_{p\to p})$$

graphically:



amplitude: $\gamma^* p \to n$ C.C.: $\gamma^* p \to n$ Forward Compton Amplitude: $\gamma^* p \to \gamma^* p$

• Thus DIS can be written as a forward Compton scattering process with a virtual photon:

$$W^{\mu\nu} = \frac{1}{2\pi m_p} \mathrm{Im}\left[-i \right.$$

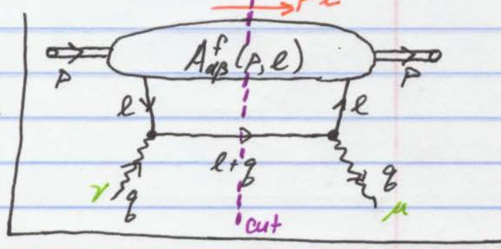
$$\left.\right] = \frac{1}{2\pi m_p} \mathrm{Im}\left\{ +i \int d^4y\, e^{iq\cdot y} \langle p | T\, j^\mu(y)\, j^\nu(0) | p \rangle \right\}$$

$$W^{\mu\nu} \equiv 2\,\mathrm{Im}(i\,T^{\mu\nu})$$

(sign changes due to i's missing from j^μ)

V. Calculation of $W^{\mu\nu}$ in the Parton Model

- Suppose the electrically charged partons are fermions (ie, quarks). Let $A^f_{\alpha\beta}(p,\ell)$ represent the part of the diagram that selects a parton of flavor f, momentum ℓ.



$$W^{\mu\nu} = \frac{1}{2\pi m_p} \mathrm{Im}\left\{ \sum_f \int d^4\ell \,(-i)\, A^f_{\alpha\beta}(p,\ell) \cdot \left[(ie_f \gamma^\mu) \frac{i(\not\ell + \not q)}{(\ell+q)^2 + i\varepsilon} (ie_f \gamma^\nu) \right]_{\beta\alpha} \right\}$$

$$= \frac{-1}{2\pi m_p} \sum_f e_f^2 \int d^4\ell \cdot \mathrm{Im}\left\{ A^f_{\alpha\beta}(p,\ell) \frac{[\gamma^\mu (\not\ell + \not q)\gamma^\nu]_{\beta\alpha}}{(\ell+q)^2 + i\varepsilon} \right\}$$

- The Cutkosky rules tell us that the imaginary part of the diagram comes from cutting the diagram in all possible places. The only cut not forbidden by $1 \to 2$ decay or spontaneous decay of stable particles is the one shown.

- The imaginary part comes from the fermion propagator:

$$\mathrm{Im}\left(\frac{1}{(\ell+q)^2 + i\varepsilon}\right) = -\pi\, \delta((\ell+q)^2)$$

so $$W^{\mu\nu} = \frac{1}{2m_p} \sum_f e_f^2 \int d^4\ell \, A^f_{\alpha\beta}(p,\ell) \, [\gamma^\mu (\not\ell + \not q)\gamma^\nu]_{\beta\alpha} \, \delta((\ell+q)^2)$$

- Putting the cut fermion on shell gives:

$$\begin{aligned}
\delta[(\ell+q)^2] &= \delta[2(\ell^+ + q^+)(\cancel{\ell^-} + q^-) - (\cancel{\ell} + q)^2] \\
&\approx \delta[2\ell^+ q^- + 2q^+ q^- - q_T^2] \\
&= \delta[2\ell^+ q^- - Q^2] \\
&= \delta[2p^+ q^- (\ell^+/p^+ - Q^2/2p^+q^-)] \\
&\approx \frac{1}{2p^+ q^-} \delta(\ell^+/p^+ - Q^2/2p^+q^-) \\
&\approx x_B/Q^2 \, \delta(\ell^+/p^+ - x_B)
\end{aligned}$$

$(p-\ell)^2 = \text{finite}$
$\to p^+ \ell^- = \text{finite}$
$\to \ell^- \sim \mathcal{O}(1/p^+) \ll q^-$

Expect $\underline{\ell}$ cut off by an intrinsic scale (ie, m_p) $\ll Q$

$$W^{\mu\nu} = \frac{x_B}{2m_pQ^2} \sum_f e_f^2 \int d^4\ell \, A^f_{\alpha\beta}(p,\ell) \left[\gamma^\mu (\not{\ell}+\not{q})\gamma^\nu\right]_{\beta\alpha} \delta\left(\frac{\ell^+}{p^+} - x_B\right)$$

- The longitudinal momentum fraction $x_F \equiv \ell^+/p^+$ of the parton is a property of the proton wave function. x_B is a property of the electron recoil. The $\delta(x_F - x_B)$ shows that DIS probes the parton distribution in the Bjorken frame.

- We can further simplify the Dirac structure:

$$(\not{\ell}+\not{q}) = \gamma^+(\ell^- + q^-) + \gamma^-(\ell^+ + q^+) - \underline{\gamma}\cdot(\underline{\ell}+\underline{q})$$

- After integrating $d^4\ell$, the Lorentz indices of γ^μ can be converted to p^μ by $A^f_{\alpha\beta}(p,\ell)$; it is the only external 4-vector originating in that part of the diagram.

Hence

$$(\not{\ell}+\not{q}) \approx \underbrace{\gamma^+q^-}_{\ll p^+} + \underbrace{\gamma^-(\ell^+ + q^+)}_{\ll p^- \sim \frac{M_p^2}{p^+}} - \underbrace{\underline{\gamma}\cdot(\underline{\ell}+\underline{q})}_{\ll \underline{p}=0} \approx \gamma^+ q^-$$

So $\left[\gamma^\mu(\not{\ell}+\not{q})\gamma^\nu\right]_{\beta\alpha} \to q^-\left[\gamma^\mu\gamma^+\gamma^\nu\right]_{\beta\alpha}$

- Further simplification:
  1) $(\gamma^+)^2 = 0$, so $\mu,\nu \neq +$
  2) $\gamma^-\gamma^+\underline{\gamma}_\perp \to (p^+p^-)\underline{p} \to M_p^2 \cdot \underline{0} = 0$
  3) $\gamma^-\gamma^+\gamma^- \to \mathcal{O}(p^-)$ is suppressed
  4) $\gamma_\perp^i\gamma^+\gamma_\perp^j$ contains a $\sim p^+ g_{ij}$ contribution (dominant)

Thus $(\mu,\nu) \to (i,j)$ dominates $W^{\mu\nu}$.

$$\left[\gamma^\mu(\not{\ell}+\not{q})\gamma^\nu\right]_{\beta\alpha} \to q^-(\gamma^i\gamma^+\gamma^j)_{\beta\alpha} = -q^-(\gamma^+\gamma^i\gamma^j)_{\beta\alpha}.$$

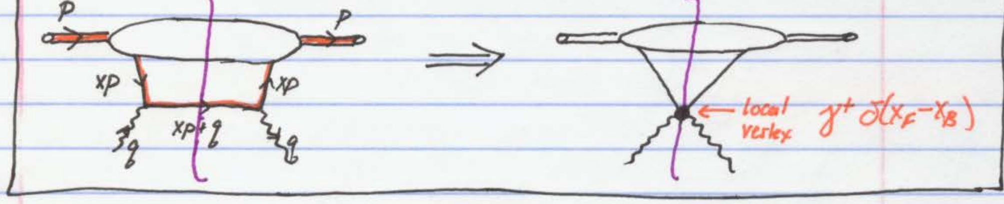
- Since $L_{\mu\nu}$ is symmetric, we can project out only the symmetric part of $W^{\mu\nu}$:

$$W^{\mu\nu} \to \frac{1}{2}(W^{\mu\nu} + W^{\nu\mu})$$

$$W^{ij} = \frac{x_B}{4m_pQ^2} \sum_f e_f^2 \int d^4l \, A^f_{\alpha\beta}(p,l)(-q^-) \left[\gamma^+\{\gamma^i,\gamma^j\}\right]_{\beta\alpha} \delta(l^+/p^+ - x_B)$$

$$= -\frac{x_B}{2m_pQ^2} q^- g^{ij} \sum_f e_f^2 \int d^4l \, A^f_{\alpha\beta}(p,l) \underbrace{\gamma^+_{\beta\alpha} \, \delta(l^+/p^+ - x_B)}_{\text{Effective Vertex}}$$

• Since p^+ is very large in this frame, the current probed is effectively a <u>local Mueller vertex</u>.



• Furthermore, the tensor structure is $W^{ij} \propto g^{ij}$. This means

$$W_{ij} = -g_{ij} W_1 + \underbrace{\frac{q_iq_j}{q^2}W_1 + \frac{1}{M_p^2}\left[\cancel{p_i}^{\,0} - q_i\frac{p\cdot q}{q^2}\right]\left[\cancel{p_j}^{\,0} - q_j\frac{p\cdot q}{q^2}\right]W_2}_{\text{Must }=\ 0}$$

so $$\frac{q_iq_j}{\cancel{q^2}} W_1 + \frac{1}{M_p^2}\frac{q_iq_j}{\cancel{q^2}}\frac{(p\cdot q)^2}{q^2} W_2 = 0$$

$$\hookrightarrow \quad \boxed{\frac{W_1}{W_2} = \frac{(p\cdot q)^2}{M_p^2Q^2} \quad \text{or} \quad \frac{F_1}{F_2} = \frac{m_pW_1}{\frac{p\cdot q}{m_p}W_2} = \frac{p\cdot q}{Q^2} = \frac{1}{2x_B}} \quad \text{Callan-Gross Relation}$$

• This relationship between the structure functions proved that the electrically charged partons were fermions (quarks).

Then
$$W_1 = \frac{x_B}{2m_pQ^2} q^- \sum_f e_f^2 \int d^4l \, A^f_{\alpha\beta}(p,l)\gamma^+_{\beta\alpha} \, \delta\left(\frac{l^+}{p^+} - x_B\right)$$

$$q^- \approx \frac{p\cdot q}{p^+} = \frac{Q^2}{2x_Bp^+}$$

$$W_1 = \frac{1}{4m_p p^+} \sum_f e_f^2 \int d^4\ell\, A^f_{\alpha\beta}(p,\ell)\, \gamma^+_{\beta\alpha}\, \delta(\ell^+/p^+ - x_B)$$

Then

$$F_2(x_B, Q^2) = 2x_B F_1(x_B, Q^2) = 2m_p x_B\, W_1(x_B, Q^2)$$
$$= \frac{x_B}{2p^+} \sum_f e_f^2 \int d^4\ell\, A^f_{\alpha\beta}(p,\ell)\, \gamma^+_{\beta\alpha}\, \delta(\ell^+/p^+ - x_B)$$

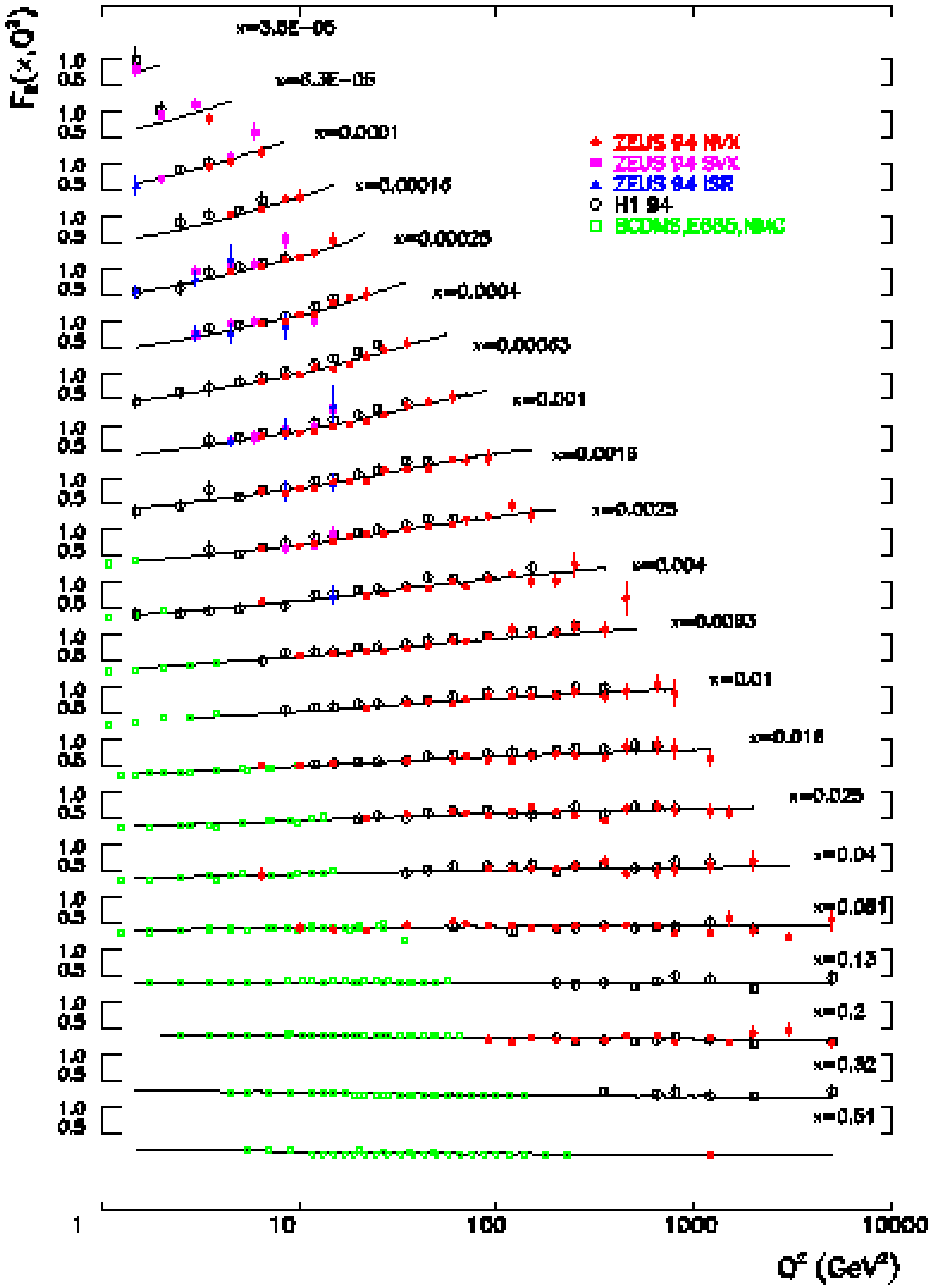
$$= \sum_f e_f^2 \cdot x_B \left[\underbrace{\frac{1}{2p^+} \int d^4\ell\, A^f_{\alpha\beta}(p,\ell)\, \gamma^+_{\beta\alpha}\, \delta\left(\frac{\ell^+}{p^+} - x_B\right)}_{\equiv P_f(x_B) \;\to\; \text{no } Q^2 \text{ dependence}} \right]$$

$$\boxed{F_2(x_B, Q^2) = \sum_f e_f^2 \cdot x_B P_f(x_B)}$$

- The structure function F_2 appears as a simple superposition of individual parton contributions. The function $x_B P_f(x_B)$ measures the probability to encounter a parton with flavor f and momentum $\ell^+ = x_B p^+$.

- The parton model predicts F_2 should be a function of x_B alone (Bjorken scaling). This is a consequence of the fact that p^+ is large in the Bjorken frame and the $A^f_{\alpha\beta}$ portion of the diagram is independent of Q^2.

- One loophole in Bjorken scaling is the possibility that Q^2 enters as a necessary transverse momentum cutoff in the $d^2\ell$ integral. This leads to violations of Bjorken scaling at very high Q^2 due to the quantum corrections (evolution) of $x_B P_f(x_B, Q^2)$ called Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution. [Next lecture topic.]



$F_2(x,Q^2)$
x=3.5E-05
x=6.3E-05
x=0.0001
x=0.00016
x=0.00025
x=0.0004
x=0.00063
x=0.001
x=0.0016
x=0.0025
x=0.004
x=0.0063
x=0.01
x=0.016
x=0.025
x=0.04
x=0.061
x=0.13
x=0.2
x=0.32
x=0.51
ZEUS 94 NVX
ZEUS 94 SVX
ZEUS 94 ISR
H1 94
BCDMS,E665,NMC
1.0
0.5
1
10
100
1000
10000
Q^2 (GeV^2)