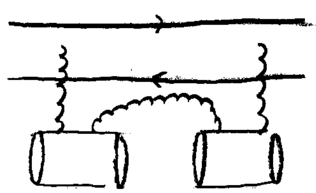
General overview of the Glauber-Gribov-Mueller model

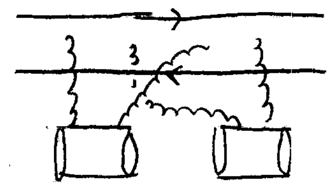
Last week we calculated the wave function in

$$\sigma_{tot}^{\gamma^* A}(x_{Bj}, Q^2) = \int \frac{d^2 x_\perp}{4\pi} \int_0^1 \frac{dz}{z(1-z)} \left|\Psi^{\gamma^* \to q\bar{q}}(\vec{x}_\perp, z)\right|^2 \sigma_{tot}^{q\bar{q}A}(\vec{x}_\perp, Y)$$

Now we need to find the cross section $\sigma^{q\bar{q}A}$. To due this we are going to use the Glauber-Gribov-Mueller model.

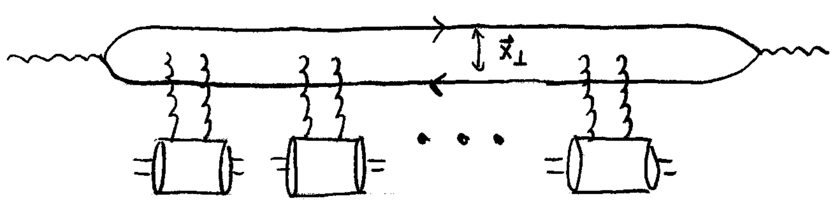
We have a very large and dilute nucleus which is made up of $A \gg 1$ nucleons. The nucleon density does not change significantly over the nucleus. Due to that any interactions which have more than one nucleon inter acting with the dipole at the same place, such as





are suppressed.

So the interaction with the nucleus consits of ordered two gluon exchanges with single neucleons.

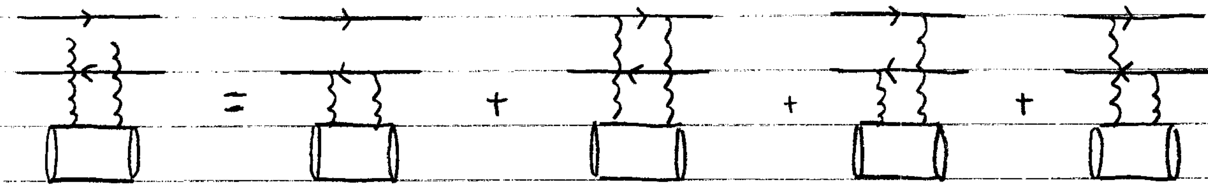


Due to the fact we are in the eikonal approximation the after each interact the quark lines are effectivily on shell. Yuri's book gives a good description of this.

So the individual interactions factorize and all one needs to due is to iterate them.

$\sigma^{q\bar{q}N}$ calculation

Modeling the nucleon as a valence quark we need to calculating the following diagrams



Using a method similar to Chun's, which was for onium - onium scattering, we find that the cross section for $q\bar{q}N$ scattering where the nucleon is a valence quark is

$$\sigma^{q\bar{q}N} = \frac{2\alpha_s^2 C_F}{N_c}\int\frac{d^2 l_\perp}{(l_\perp^2)^2}\left(2 - e^{i\vec{l}_\perp\cdot\vec{x}_\perp} - e^{-i\vec{l}_\perp\cdot\vec{x}_\perp}\right)$$

which in coordinate space is

$$\sigma^{q\bar{q}N} = \frac{2\alpha_s^2\pi C_F}{N_c}\, x_\perp^2 \ln\left(\frac{1}{|x_\perp|\Lambda}\right)$$

This calculation can be found in the notes on line. Note that there might be some factor of 2 mistakes in it.

Now the lowest two-gluon order the gluon distribution function of a single quark is

$$x_{Bj}G(x_{Bj}, Q^2) = \frac{\alpha_s C_F}{\pi}\ln\left(\frac{Q^2}{\Lambda^2}\right)$$

So the cross section is

$$\sigma^{q\bar{q}N} = \frac{\alpha_s^2\pi^2}{N_c}\, x_\perp^2\, x_{Bj}G\left(x_{Bj}, \frac{1}{x_\perp^2}\right)$$

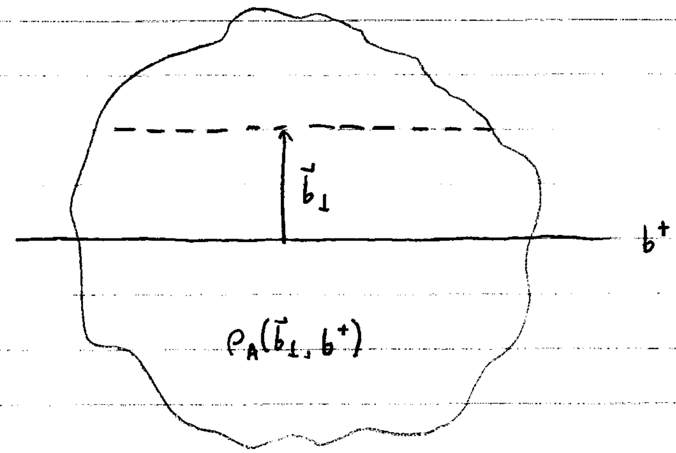
So the forward scattering amplitude is

$$N^{q\bar{q}N} = \frac{\alpha_s^2\pi^2}{2N_c}\, x_\perp^2\, x_{Bj}G\left(x_{Bj}, \frac{1}{x_\perp^2}\right)$$

which for large $x_\perp$ $N > 1$. This violates the black disk limit

Now we need to take into account multiple rescatterings, but before that we need to talk about the nuclear thickness function

First we need to talk about the nuclear thickness function.



At a given impact parameter $\vec{b}_\perp$ as one travels from one side of the nucleus to the other along the b^+ axis it will in the end see a total number of nucleons. So we define the nuclear profile function as

$$T(\vec{b}_\perp) = \int_{-\infty}^{\infty} \rho_A(\vec{b}_\perp, b^+)\, db^+$$

where ρ_A is the nucleon density of the nucleus. So $T(\vec{b}_\perp)$ is a two-dimensional number density. Notice that

$$\rho_A \sim A^{1/3}$$

So

$$T(\vec{b}_\perp) \sim A^{1/3}$$

Also in the limit we are dealing with here the density is roughly constant. So

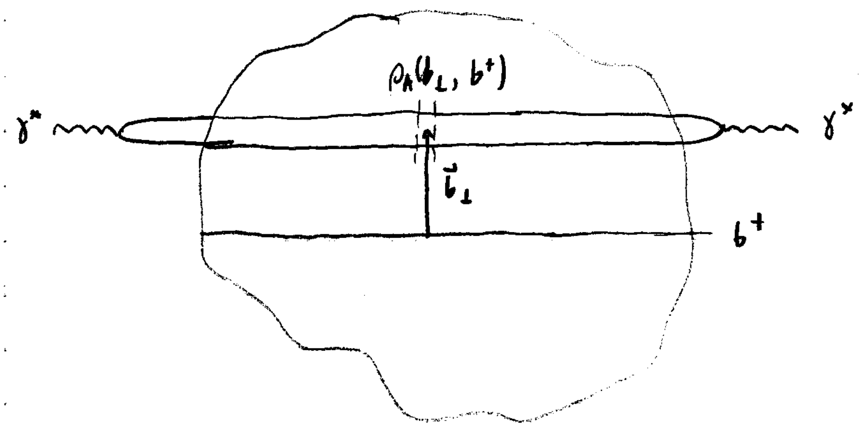
$$T(\vec{b}_\perp) \simeq L\, \rho_A(\vec{b}_\perp)$$

where L is the thickness of the nucleus.

Now we can talk about multiple rescatterings.

Multiple Rescattering

As the dipole passes through the nucleus it has a constant crossection and sees a constant nucleon density.



We are very familiar with this process (we see it in almost all fields). This is just an exponential attenuation.
Now the quantity we are interested in calculating is the S-matrix,

$$S(\vec x_\perp, \vec b_\perp, b^+, Y) = 1 - N(\vec x_\perp, \vec b_\perp, b^+, Y)$$

So with this information we can see that the differential equation this function obeys is,

$$\frac{\partial}{\partial b^+} S(\vec x_\perp, \vec b_\perp, b^+) = -\rho_A(\vec b_\perp, b^+)\, N^{q\bar q N}\, S(\vec x_\perp, \vec b_\perp, b^+)$$

So

$$\int \frac{\partial S}{S} = -N^{q\bar q N} \int \rho_A(\vec b_\perp, b^+)\, \partial b^+$$

which has the solution

$$S(\vec x_\perp, \vec b_\perp) = e^{-T(\vec b_\perp) N^{q\bar q N}}$$

So the forward scattering amplitude is

$$N(\vec x_\perp, \vec b_\perp) = 1 - e^{-T(\vec b_\perp) N^{q\bar q N}}$$

which is, for a nucleon modeled by a valence quark,

$$N(\vec x_\perp, \vec b_\perp) = 1 - e^{-\frac{\alpha_s^2 \pi C_F}{N_c} T(\vec b_\perp)\, x_\perp^2 \ln\left(\frac{1}{|x_\perp| \Lambda}\right)}$$

Now what is the physical significance of this?

It is useful to define the saturation scale,

$$\cancel{Q_s^2(\vec{b}_\perp) = \frac{\alpha_s \pi^2}{2N_c} T(\vec{b}_\perp)\, x G_N(x, Q_s^2)}$$

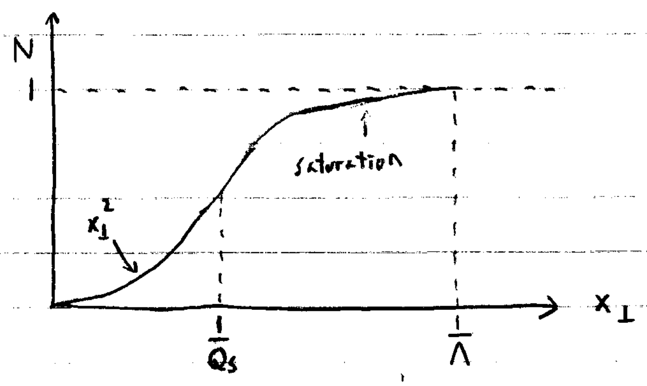
Now the quark saturation scale is defined as

$$Q_s^2(\vec{b}_\perp) = \frac{4\pi\alpha_s^2 C_F}{N_c} T(\vec{b}_\perp)$$

So our forward scattering amplitude can be written as

$$N(\vec{x}_\perp, \vec{b}_\perp) = 1 - e^{-\frac{Q_s^2(\vec{b}_\perp)}{4} x_\perp^2 \ln\left(\frac{1}{|x_\perp|\Lambda}\right)}$$

Lets plot this



Notice for small $x_\perp$

$$N \sim x_\perp^2$$

So for zero dipole size the target becomes transparent. Think of it as the colors of the quarks canceling each other out. This is known as "color transparency". Now when the dipole is above the saturation scale $x_\perp \gtrsim 1/Q_s$ the amplitude saturates

$$N = 1$$

This corresponds to the black disk limit.

Also since

$$Q_s^2 \sim A^{1/3}$$

The saturation scale increases with atomic number A.