Notes from Kovchegov - Levin.

- $V^+ = V^0 + V^3$: becomes the time coordinate
- $V^- = V^0 - V^3$: the space like coordinate

$$V^2 = V^+ V^- - V_\perp^2 \quad , \quad \vec{V}_\perp = (V^1, V^2)$$

$$u \cdot v = \frac{1}{2} u^+ v^- + \frac{1}{2} u^- v^+ - \vec{u}_\perp \cdot \vec{v}_\perp$$

$$g_{\alpha\beta} = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad g^{\alpha\beta} = \begin{pmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$V_- = g_{-+} V^+ = \frac{1}{2} V^+$$

$$V_+ = g_{+-} V^- = \frac{1}{2} V^-$$

$$\partial_+ = \frac{1}{2} \partial^- \quad , \quad \partial_- = \frac{1}{2} \partial^+$$

light cone gauge

$$A^+ = 0$$

defined by

$$\eta \cdot A^a = \eta^\mu A^a{}_\mu = 0$$

η^μ : constant 4-vector that is light like

$\Rightarrow \eta^2 = 0$

$$\eta = (0, 2, \vec{0}_\perp)$$

$$\eta \cdot A = \eta^+ A^- + \eta^- A^+ + \vec{0}_\perp \cdot \vec{A}_\perp$$

$$= \eta^- A^+ = 0, \Rightarrow A^+ = 0$$

A^- can be expressed in terms of $A^i_\perp$ by QCD Lagrangian and the equations of motion

$$\mathcal{L}_{QCD} = \sum_f \bar{q}_i^f [i \gamma^\mu D_\mu - m_f]_{ij} q_j^f - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

two spinor fields for the quarks

$$q_\pm = \Lambda_\pm q$$

$$A_\pm = \frac{1}{2}\gamma^0\gamma^\pm$$

$$\gamma^\pm = \gamma^0 \pm \gamma^3$$

$$\Lambda_+\Lambda_- = 0 \qquad \Lambda_\pm{}^2 = \Lambda_\pm$$

$$\Lambda_+\Lambda_- = \frac{1}{4}\gamma^0(\gamma^0+\gamma^3)\gamma^0(\gamma^0-\gamma^3)$$
$$= \frac{1}{4}\gamma^0(\gamma^0+\gamma^3)(\mathbb{1}-\gamma^0\gamma^3)$$
$$= \frac{1}{4}\gamma^0(\gamma^0+\gamma^3-\gamma^3-\gamma^3\gamma^0\gamma^3)$$
$$= \frac{1}{4}\gamma^0(\gamma^0-\gamma^3\gamma^0\gamma^3)$$
$$= \frac{1}{4}\gamma^0(\gamma^0-\gamma^3\{\gamma^3\gamma^0\}) = 0$$

$$\Lambda_\pm\Lambda_\pm = \frac{1}{4}\gamma^0(\gamma^0\pm\gamma^3)\gamma^0(\gamma^0\pm\gamma^3)$$
$$= \frac{1}{2}\gamma^0(\gamma^0\pm\gamma^3)$$

$$K^2 = \frac{1}{2}K^+K^- + \frac{1}{2}K^-K^+ - K_\perp^2 = m^2$$

$$K^- = \frac{K_\perp^2 + m^2}{K^+}$$

$$K^\mu = \left(K^+, \frac{K_\perp^2 + m^2}{K^+}, \vec{K}_\perp \right)$$

Rules

1) . Draw all diagrams at desired order in coupling constant,

- Include all orderings of interaction vertices in light cone time x^+

- Assign $4K^\mu$ to each line, so that $k^2 = m^2$ "on schell"

- Each vertex conserves K^+ & $K_\perp$ componets of the 4-momentum only *

Each line has

$$K^\mu = \left(K^+, \frac{K_\perp^2 + m^2}{K^+}, K_\perp \right) \quad (1.49)$$

2) for quarks

$$u_\sigma(p) = \frac{1}{\sqrt{p^+}}\left(p^+ + m\gamma^0 + \gamma^0\vec{\gamma}_\perp\cdot\vec{p}_\perp\right)\chi(\sigma)$$

$$v_\sigma(p) = \frac{1}{\sqrt{p^+}}\left(p^+ - m\gamma^0 + \gamma^0\vec{\gamma}_\perp\cdot\vec{p}_\perp\right)\chi(-\sigma)$$

$$\chi(+1) = \begin{pmatrix}1\\0\\1\\0\end{pmatrix}\frac{1}{\sqrt{2}} \quad \chi(-1) = \begin{pmatrix}0\\1\\0\\-1\end{pmatrix}\frac{1}{\sqrt{2}}$$

for Gluons, polarization vectors in $A^+=0$ gauge

$$\varepsilon^\mu_\lambda(k) = \left(0,\ \frac{2\,\vec{\varepsilon}_\perp^{\,\lambda}\cdot\vec{k}_\perp}{k^+},\ \vec{\varepsilon}_\perp^{\,\lambda}\right)$$

$\Rightarrow$ from $\varepsilon^+_\lambda = 0$

& $\varepsilon_\lambda(k)\cdot k = 0$

3 For each intermediate state a factor of

$$\frac{1}{\sum_{inc} k^- - \sum_{interm} k^- + i\varepsilon} \qquad (*)$$

For each ~~incom~~ particle

$$k^- = \frac{(k_\perp^2 + m^2)}{k^+}$$

k^- is not conserved
the intermediate states are not on "energy shell" &
the denominator of $(*)$ is not zero.

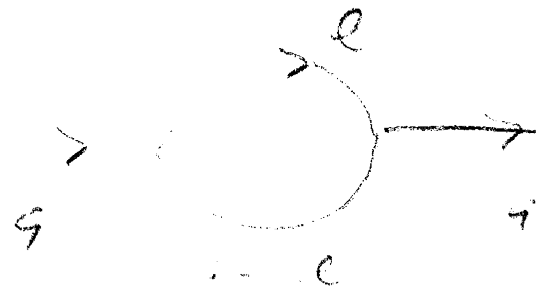
The light cone energy is ~~not~~ conserved for the whole scattering process

$$\sum_{inc} k^- = \underbrace{\sum_{out}}_{} k^-$$

Sum over all outgoing particles

Now: ϕ^3 non-simple

$$\mathcal{L} = \frac{1}{2}[\partial\phi]^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{3!}\phi^3$$



$$iA = \frac{(-i\lambda)^2}{2!}\int\frac{d^4 l}{(2\pi)^4}\,\frac{i}{l^2-m^2+i\varepsilon}\,\frac{i}{(q-l)^2-m^2+i\varepsilon}$$

$$q^\mu = (q^+, q^-, q_\perp)\,,\quad l^\mu = (l^+, l^-, l_\perp)$$

$$(q-l)^2 = (q^+ - l^+)(q^- - l^-) - (\vec{q}_\perp - \vec{l}_\perp)^2$$

$$iA = \frac{\lambda^2}{4} \int \frac{dl^+ dl^- d^2 l_\perp}{(2\pi)^4}$$

$$\times \frac{1}{l^+ l^- - l_\perp^2 - m^2 + i\varepsilon}$$

$$\times \frac{1}{(q^+ - l^+)(q^- - l^-) - (\vec{q}_\perp - \vec{l}_\perp)^2 - m^2 + i\varepsilon}$$

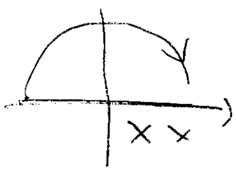
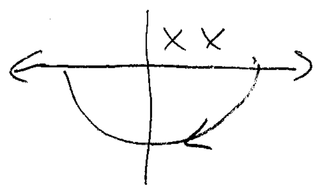
Integrate over l^-. In the l^- complex plane there are two poles

$$l_1^- = \frac{l_\perp^2 - m^2 + i\varepsilon}{l^+}$$

&

$$l_2^- = q^- - \left[\frac{(\vec{q}_\perp - \vec{l}_\perp)^2 + m^2 - i\varepsilon}{q^+ - l^+}\right]$$

If l_1 & l_2 lie in either the upper or lower plane we get zero

 , $\int = 0$  , $\int = 0$

II $\quad l^+ > 0$ & $q^+ - l^+ > 0$

l_1 is in lower plane

l_2 is in upper plane

III $\quad l^+ < 0 \;, \quad q^+ - l^+ < 0$

l_1 is in upper plane

l_2 is in lower plane.

Only the case II is possible

Close the contour in the lower half plane.

pole at

$$l_1 = \frac{l_\perp^2 + m^2 - i\varepsilon}{l^+} = l^- - \frac{i\varepsilon}{l^+}$$

$$= l^- - i\varepsilon$$

$$A = -\frac{2\pi\lambda^2}{4}\int \frac{dl^+ \, d^2 l_\perp}{(2\pi)(2\pi)^3} \frac{\theta(l^+)\,\theta(q^+ - l^+)}{l^+}$$

$$\times \cancel{\frac{1}{l^+\left\{\frac{l_\perp^2 + m^2 - i\varepsilon}{l^+}\right\} - l_\perp^2 - m^2 + i\varepsilon}}$$

$$\times \frac{1}{(q^+ - l^+)\left(q^- - \left[\frac{l_\perp^2 + m^2 - i\varepsilon}{l^+}\right]\right) - (\vec{q}_\perp - \vec{l}_\perp)^2 - m^2 + i\varepsilon}$$

$$= -\frac{\lambda^2}{4}\int \frac{dl^+ \, d^2 l_\perp}{(2\pi)^3} \frac{\theta(l^+)\,\theta(q^+ - l^+)}{l^+}$$

$$\times \frac{1}{(q^+ - l^+)} \times \frac{1}{\left(q^- - \left[\frac{l_\perp^2 + m^2 - i\varepsilon}{l^+}\right]\right) - \frac{(\vec{q}_\perp - \vec{l}_\perp)^2 - m^2 + i\varepsilon}{(q^+ + l^+)}}$$

$$= -\frac{\lambda^2}{2!}\int \frac{dl^+ d^2 l_\perp}{2(2\pi)^3}\,\frac{\Theta(l^+)\,\Theta(q^+-l^+)}{l^+(q^+-l^+)}$$

$$\cdot\ \frac{1}{q^- - \underbrace{\frac{l_\perp^2 - m^2}{l^+}}_{l^-} - \underbrace{\frac{(\vec{q}_\perp - \vec{l}_\perp)^2 + m^2}{(q^+-l^+)}}_{q^- + l^-} + i\varepsilon}$$

- Same result as you get by LCPT Rules.

$$\frac{1}{\sum_{inc} K^- - \sum_{interm} K^- + i\varepsilon} = \frac{1}{q^- - \frac{l_\perp^2 + m^2}{l^+} - \frac{(\vec{q}_\perp - \vec{l}_\perp)^2 + m^2}{q^+ - l^+} + i\varepsilon}$$

rule #3

rule #4

assign $\frac{\Theta(l^+)}{l^+}$ & $\frac{\Theta(q^+ - l^+)}{q^+ - l^+}$

for each internal line.

$\int \frac{dl^+ d^2 l_\perp}{2(2\pi)^3}$ rule #6