

# **New Qualitative Phenomena in Classical (Intramolecular) Dynamics and their Quantum Analogs.**

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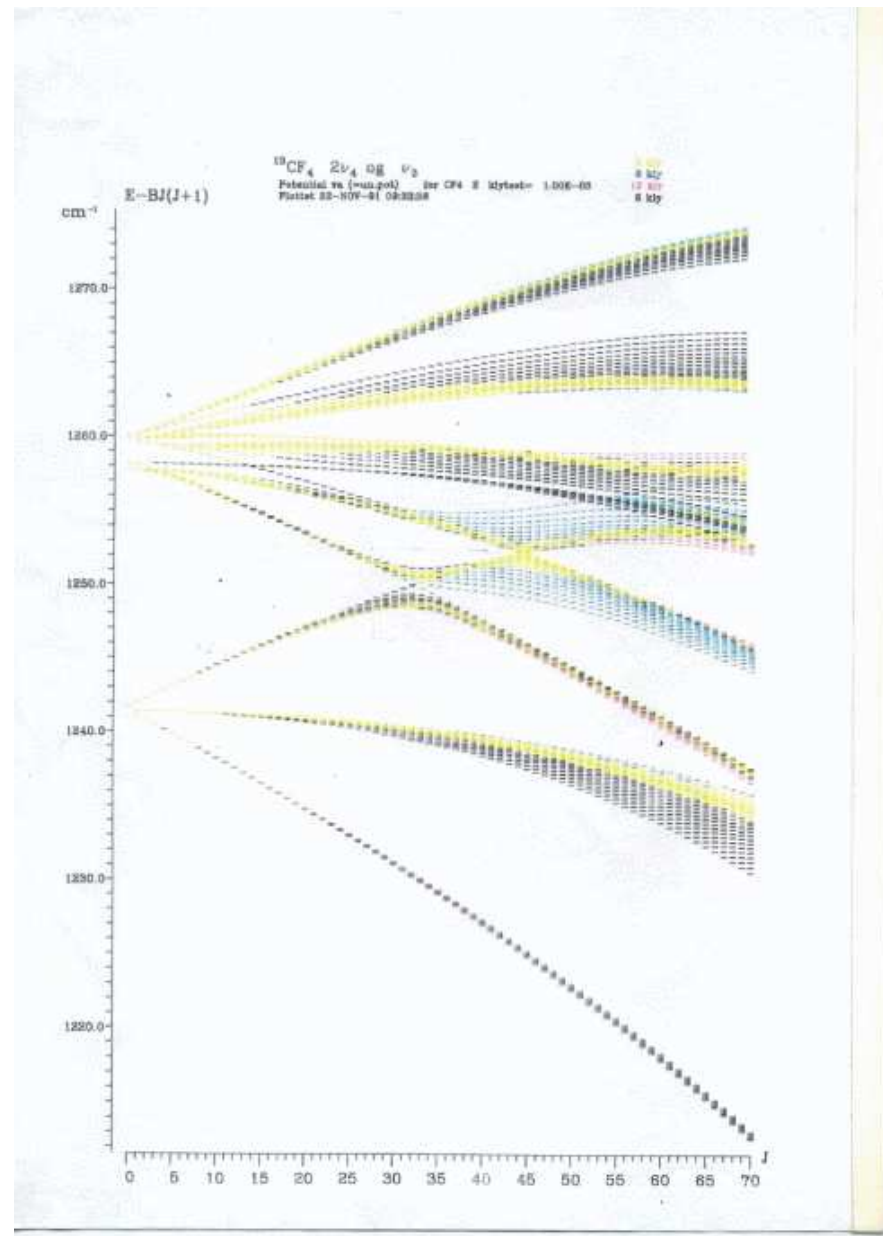
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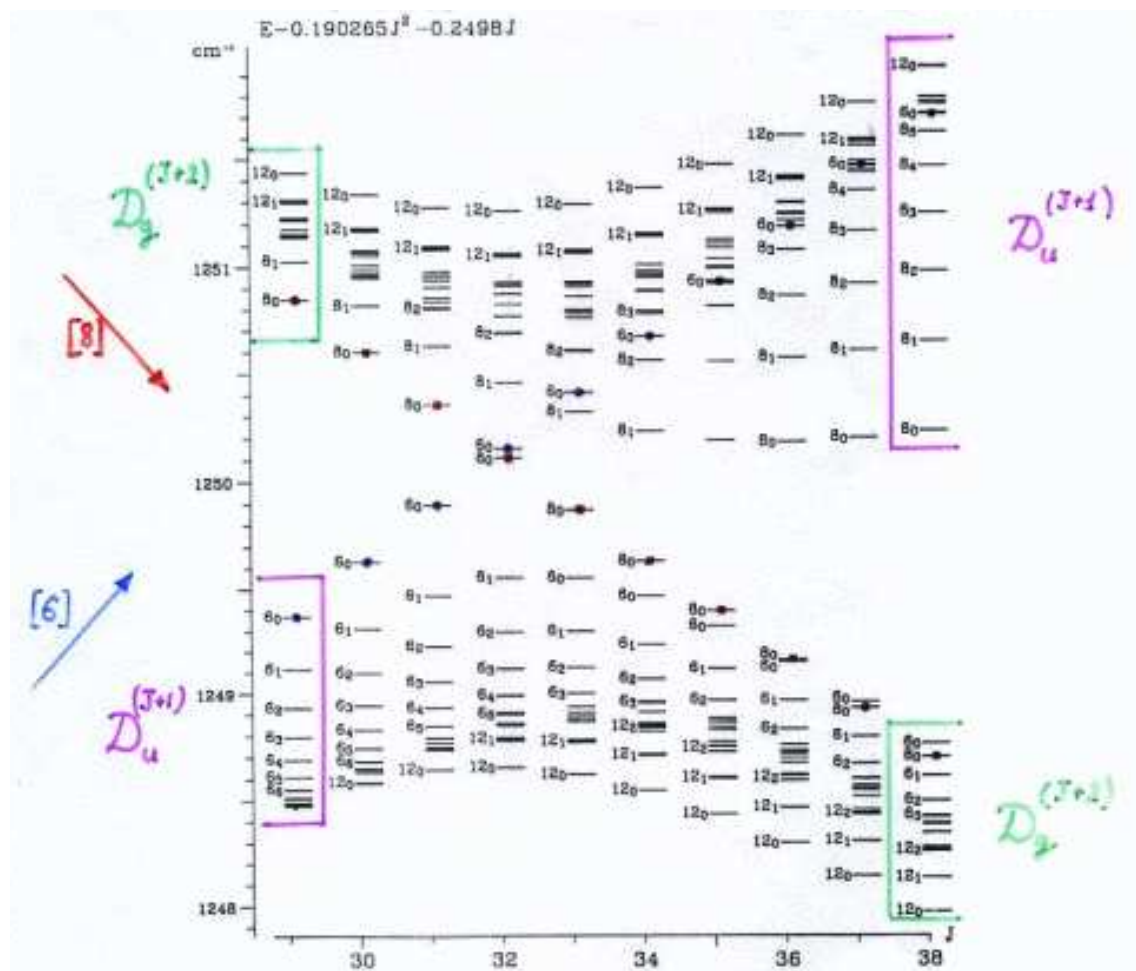
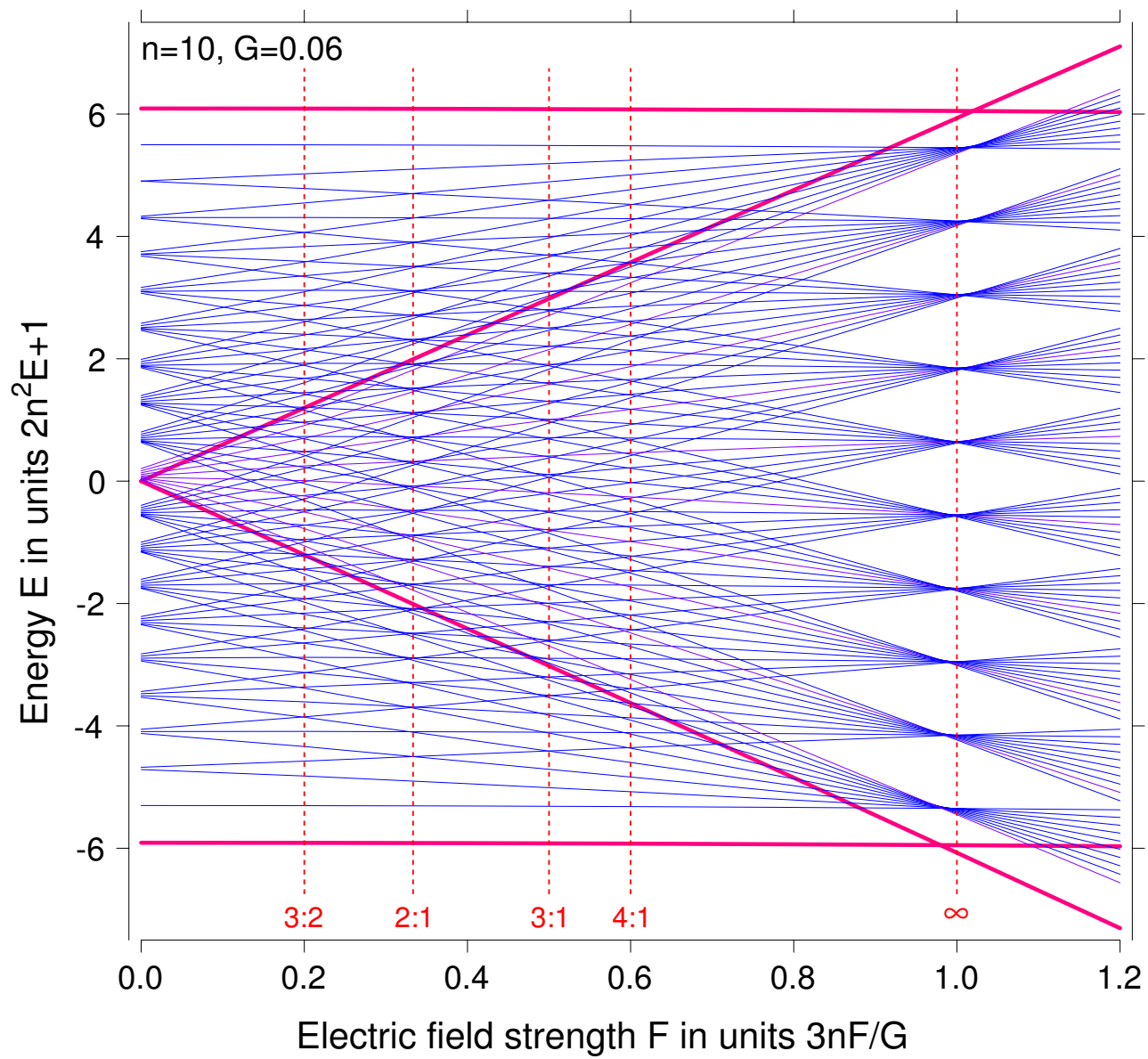
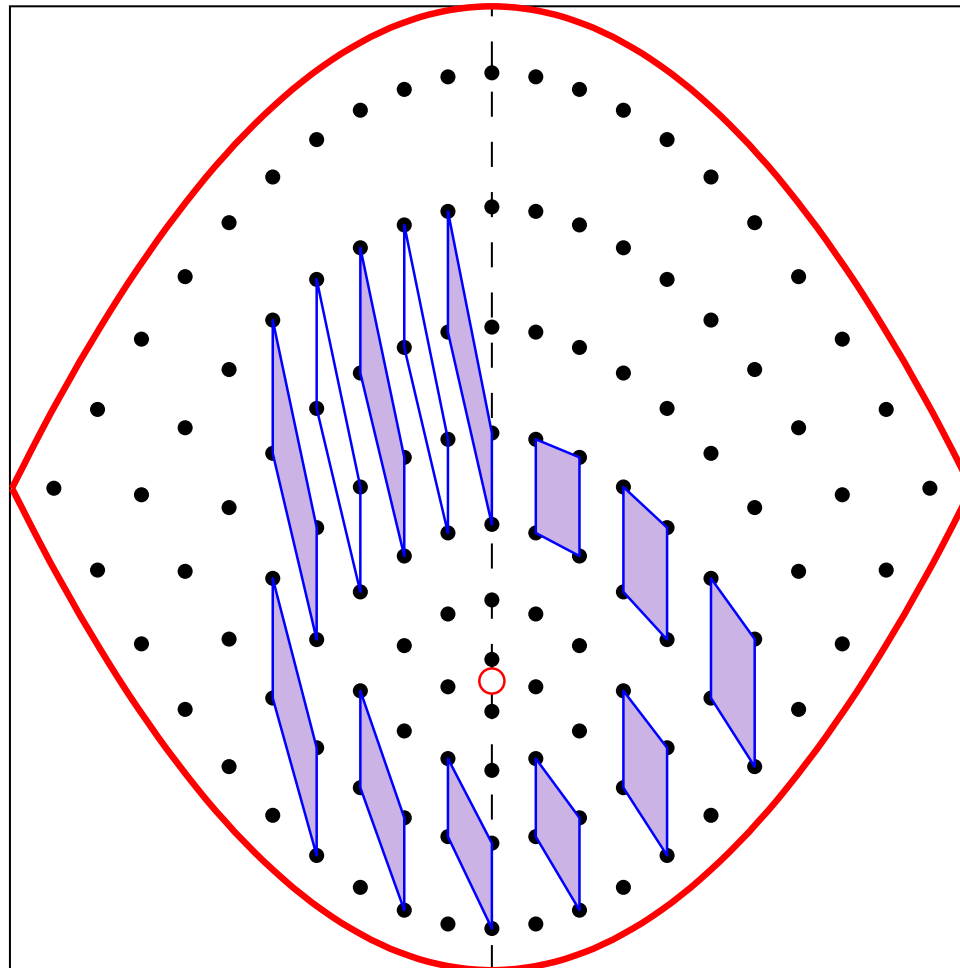
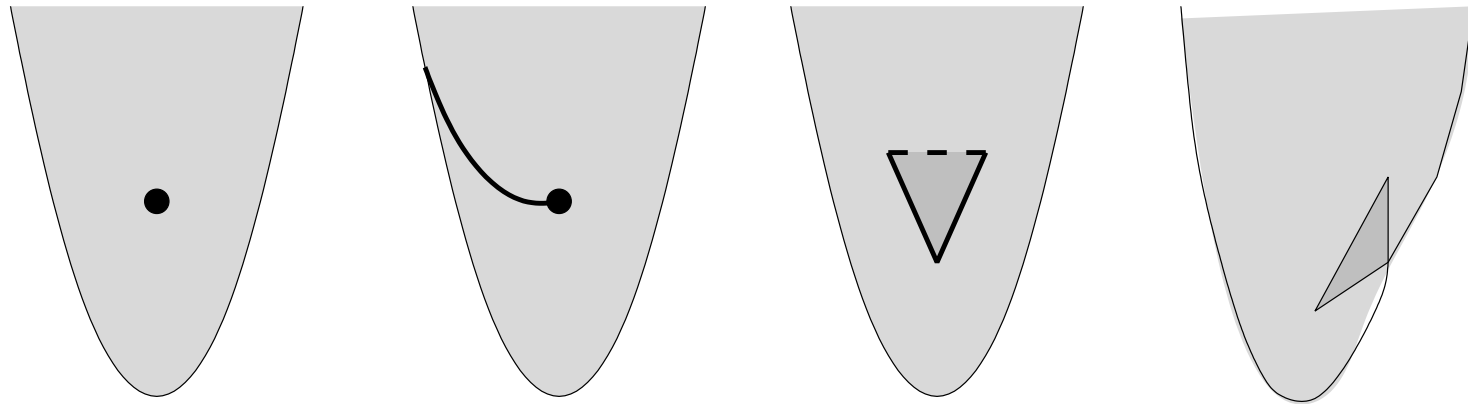


FIG. 2. A reduced energy level diagram showing the crossing of the  $D_2^{(J+1)}$  component of the  $\nu_1$  vibrational state (lower component at low  $J$ ; upper component at high  $J$ ), and the  $D_4^{(J+1)}$  component of the  $2\nu_4$  vibrational state (upper component at low  $J$ ; lower component at high  $J$ ) in  $^{13}\text{CF}_4$ . The series of transferring 8-fold clusters is marked by squares. The series of transferring 6-fold clusters is marked by circles.

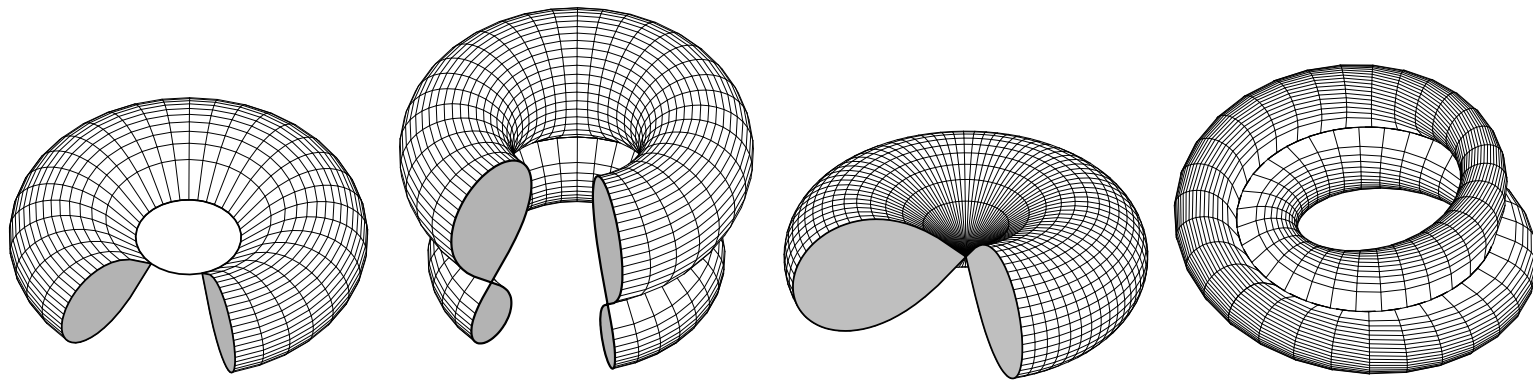




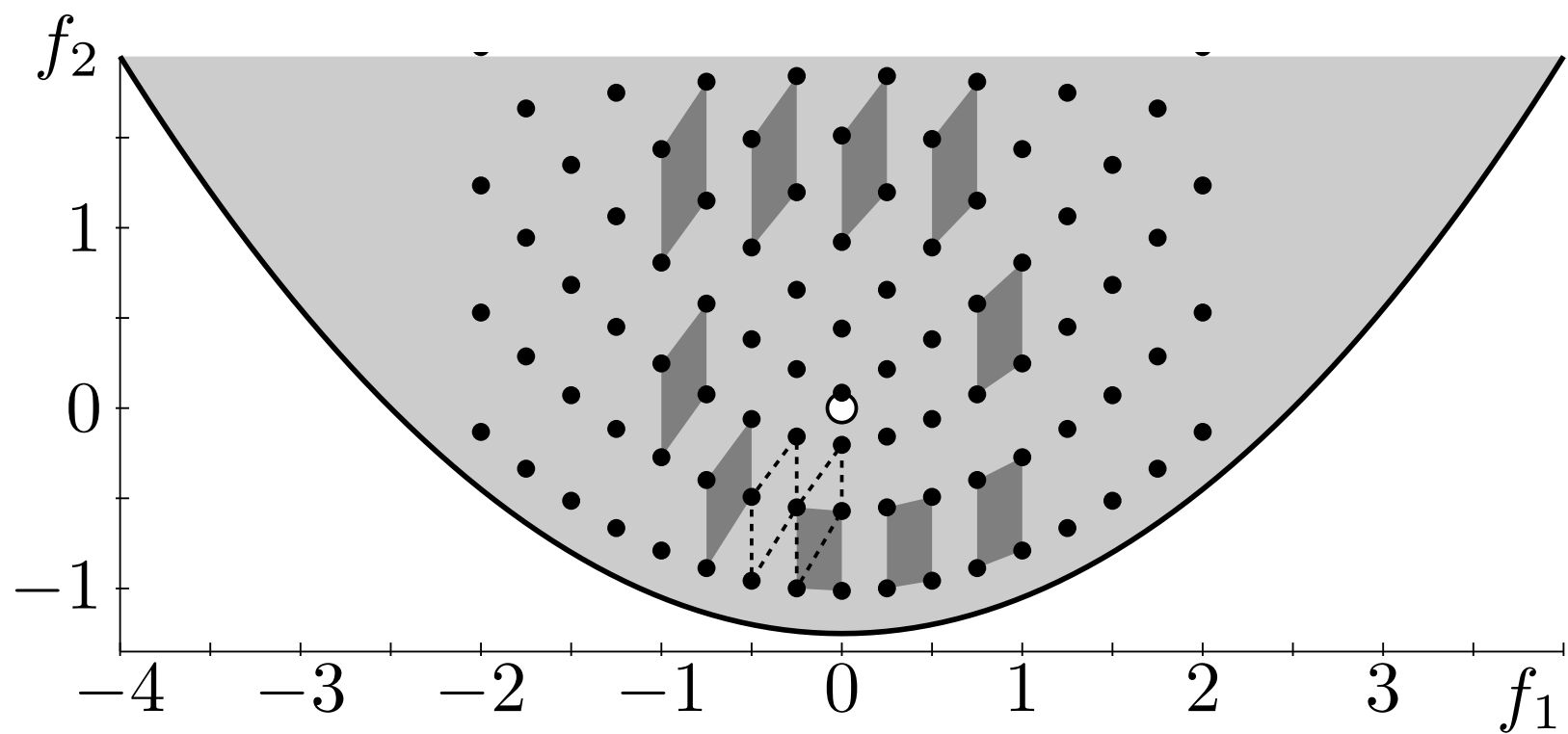
$n$ -shell splitting of hydrogen atom in orthogonal electric and magnetic fields.



Typical images of the energy momentum map for completely integrable Hamiltonian systems with two degree of freedom in the case of integer monodromy, fractional monodromy, nonlocal monodromy, and bidromy. Values in light shaded area lift to single 2-tori; values in dark shaded area lift to two 2-tori.

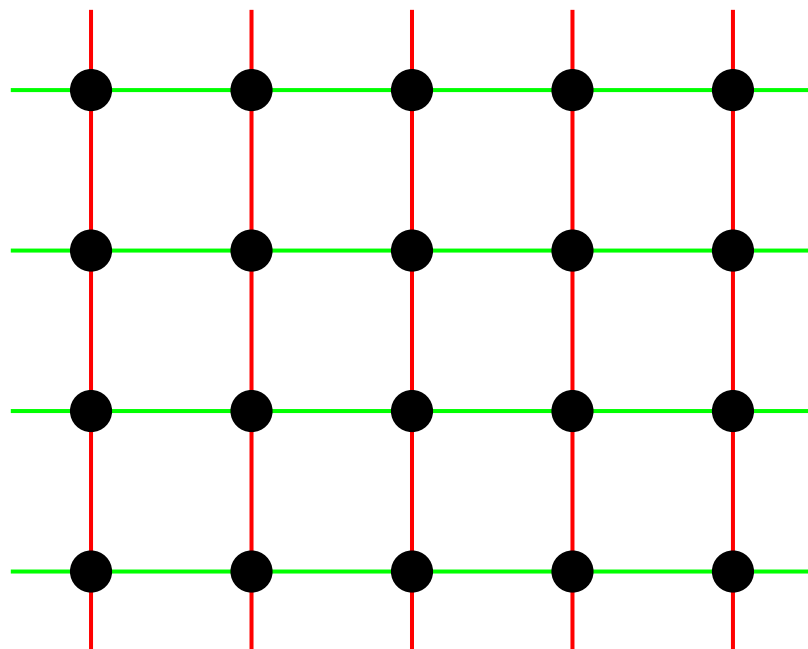


Two dimensional singular fibers in the case of integrable Hamiltonian systems with two degrees of freedom (left to right): singular torus, bitorus, pinched and curled tori.

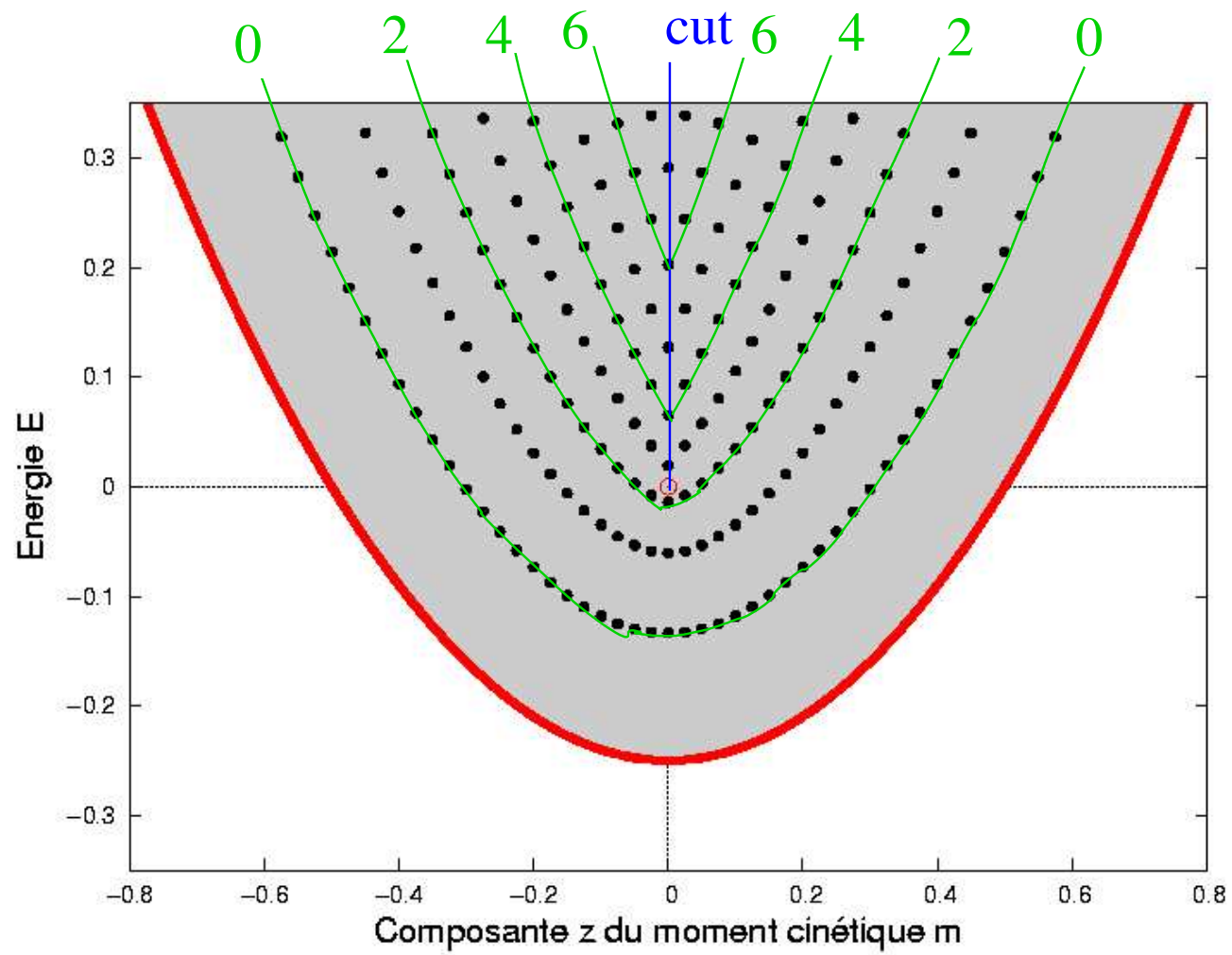


Quantum monodromy for  $1 : (-1)$  resonant oscillator system.

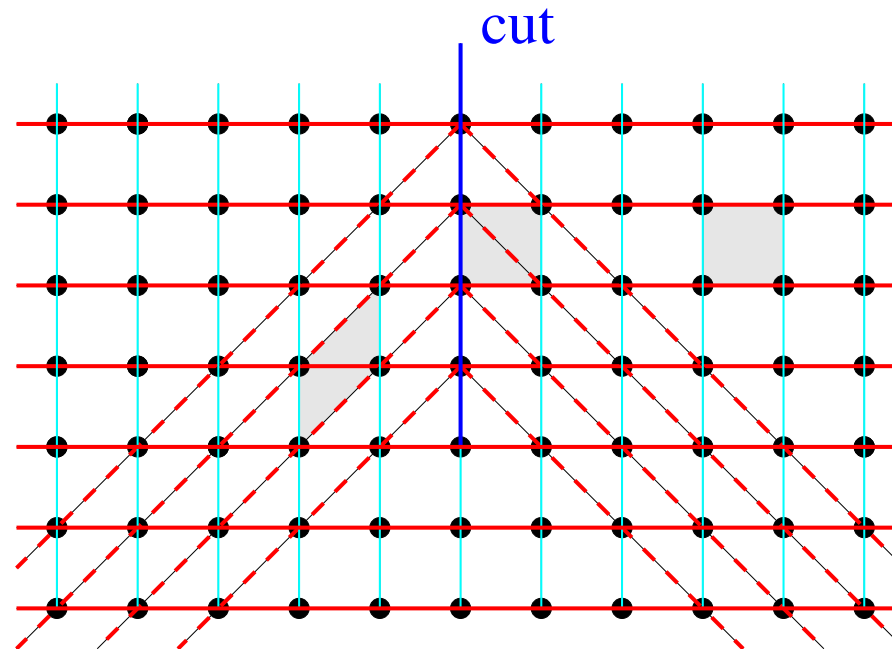




Local representation of the regular part of the joint spectrum.

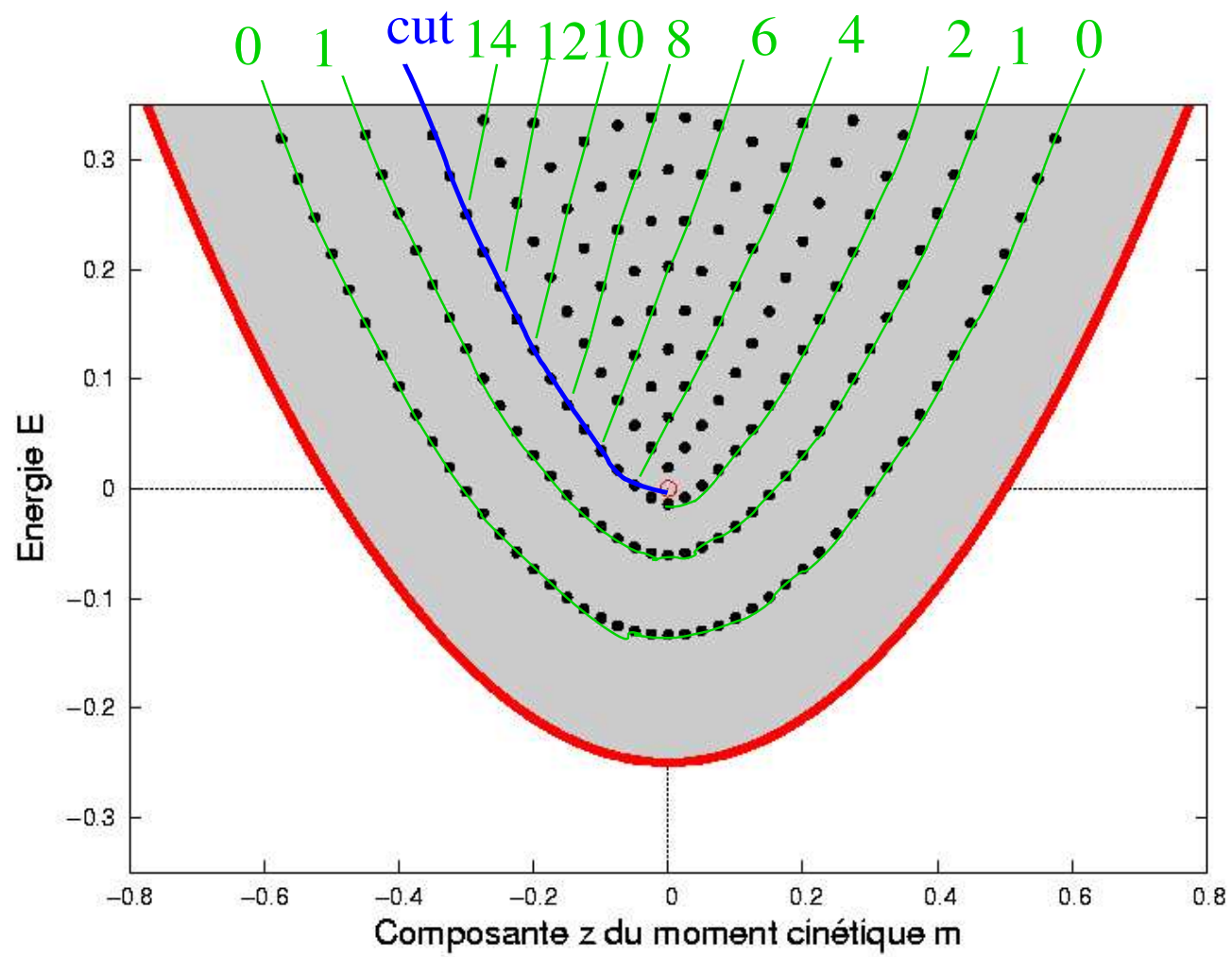


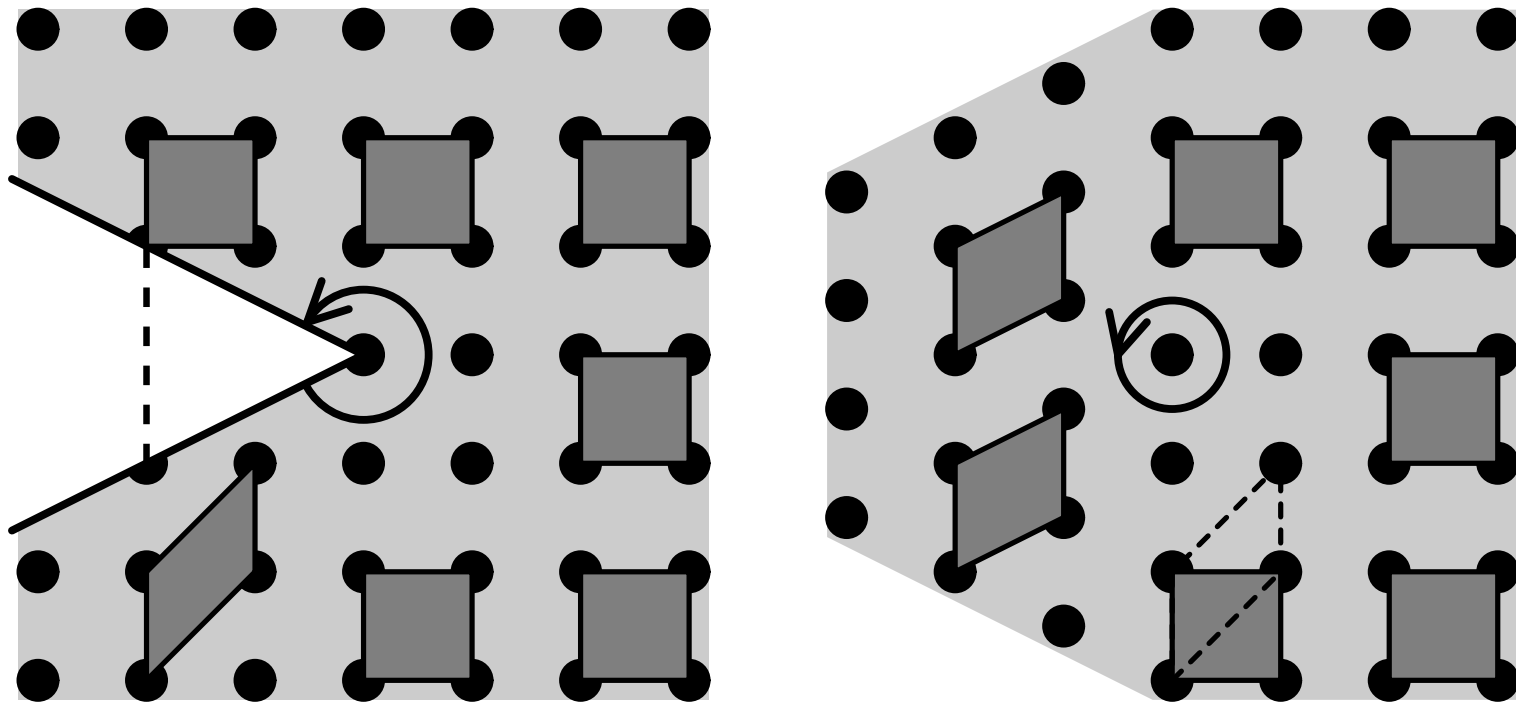
1 :  $(-1)$  resonant oscillator system with a cut along an *eigenray* represented in action variables.



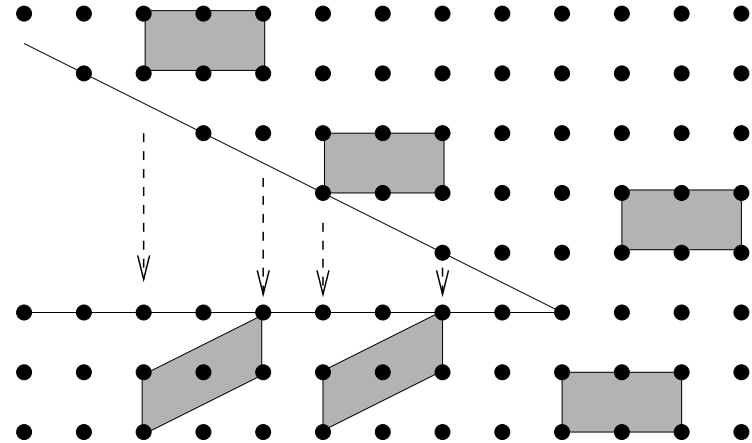
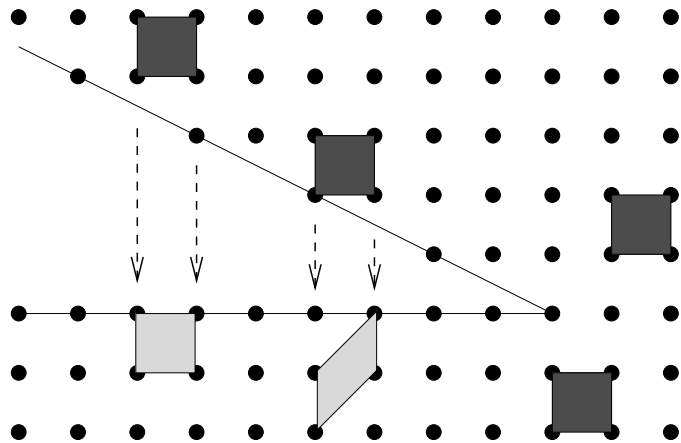
“Eigenray“ is introduced by M.Symington [math.SG/0210033].

Line of “kink“ according to Child.





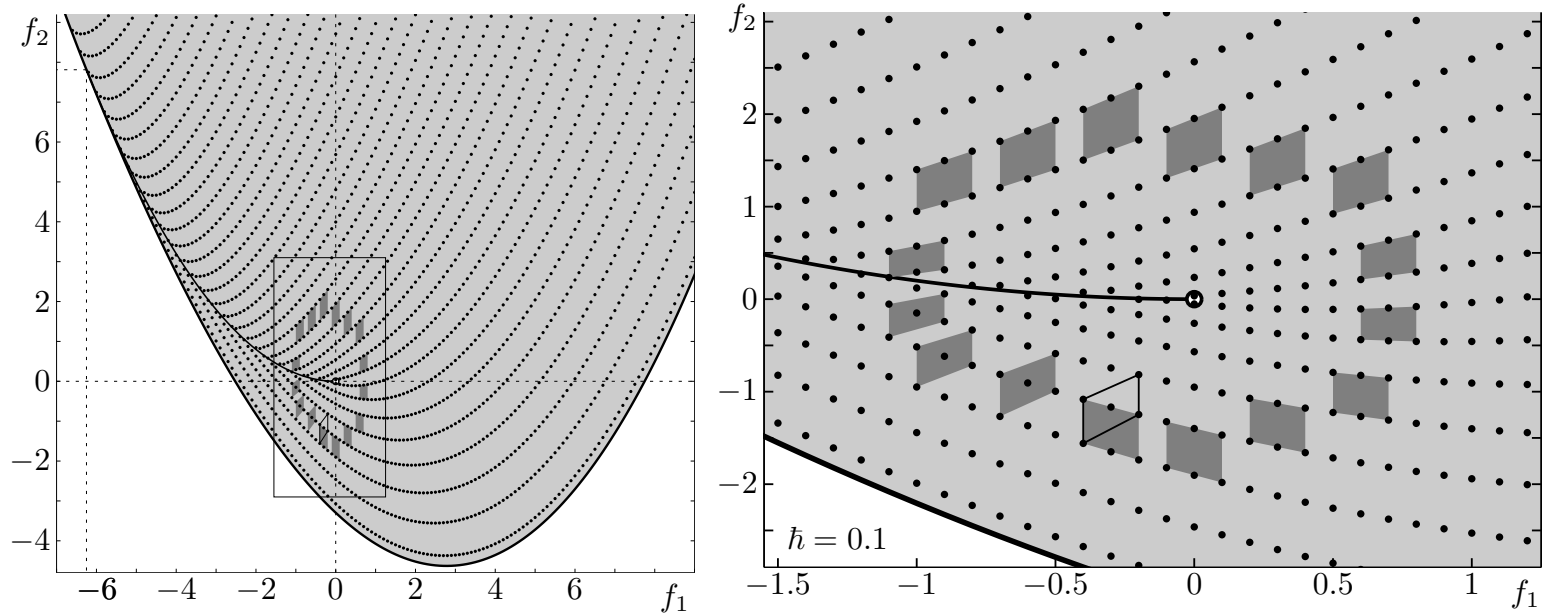
Construction of the  $1:(-1)$  lattice defect starting from the regular  $Z^2$  lattice. Dark grey quadrangles show the evolution of an elementary lattice cell along a closed path around the defect point.



Construction of 1 : 2 rational lattice defect.

Left: Elementary cell does not pass unambiguously.

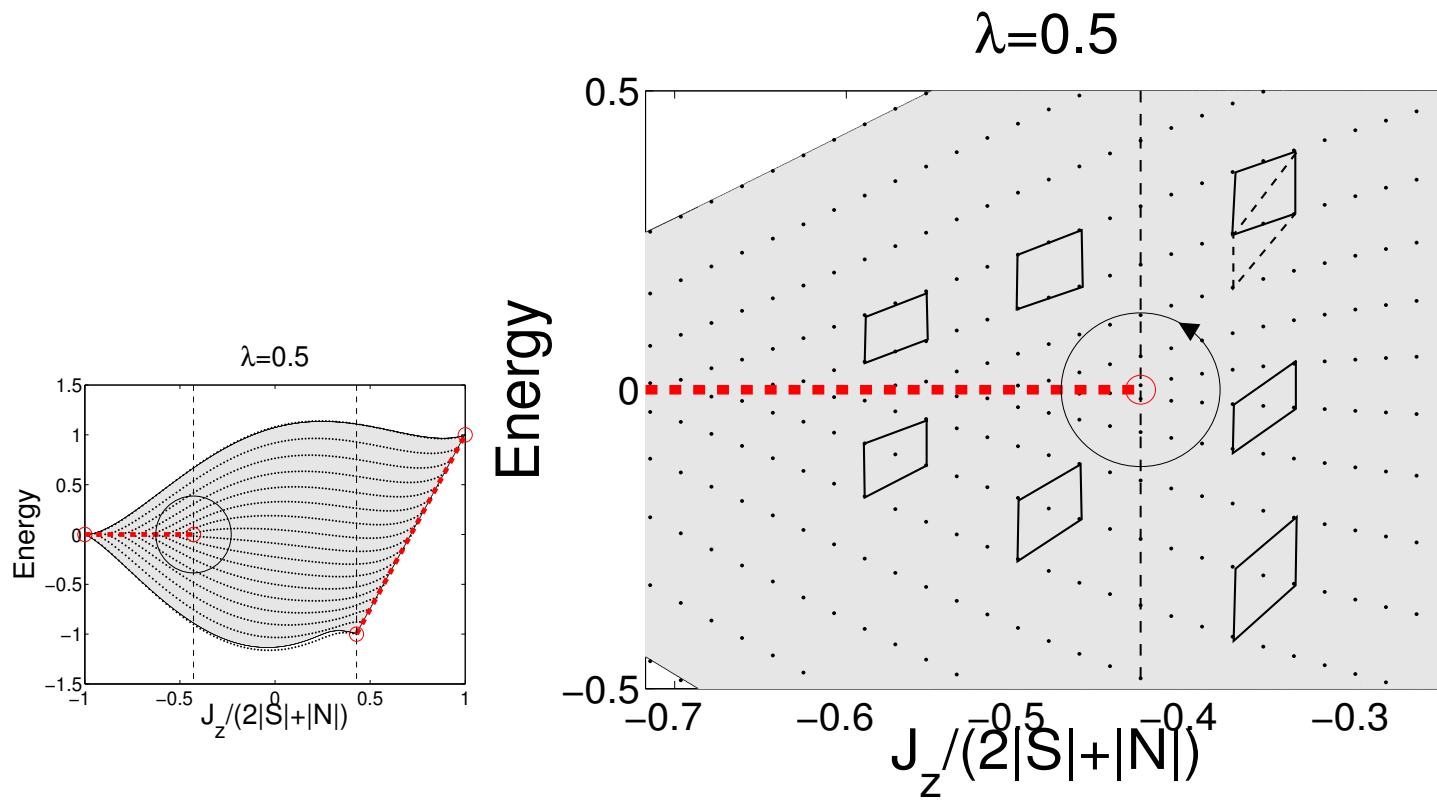
Right: Double cell passes.



Quantum fractional monodromy for 1 :  $(-2)$  resonant oscillator system.

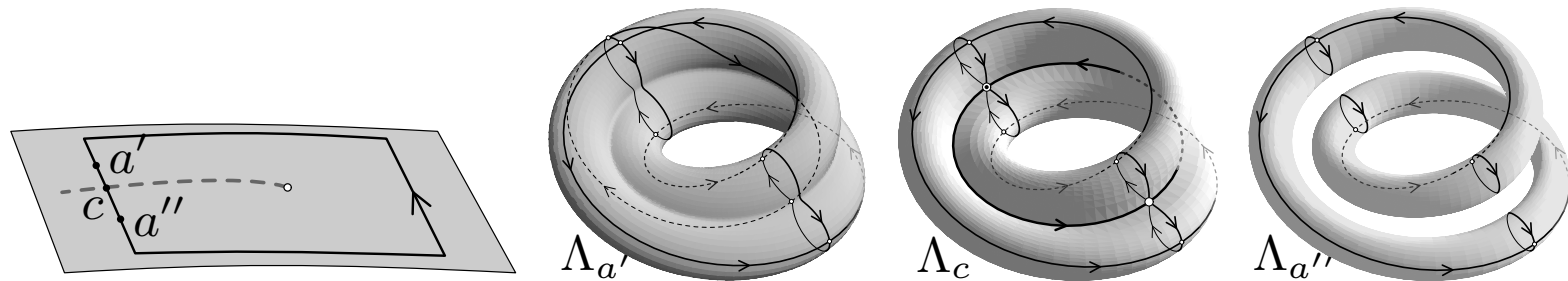
$$F_1 = \frac{\omega}{2}(p_1^2 + q_1^2) - \frac{2\omega}{2}(p_2^2 + q_2^2) + R_1(q, p), \quad (1)$$

$$F_2 = \text{Im}[(q_1 + ip_1)^2(q_2 + ip_2)] + R_2(q, p). \quad (2)$$

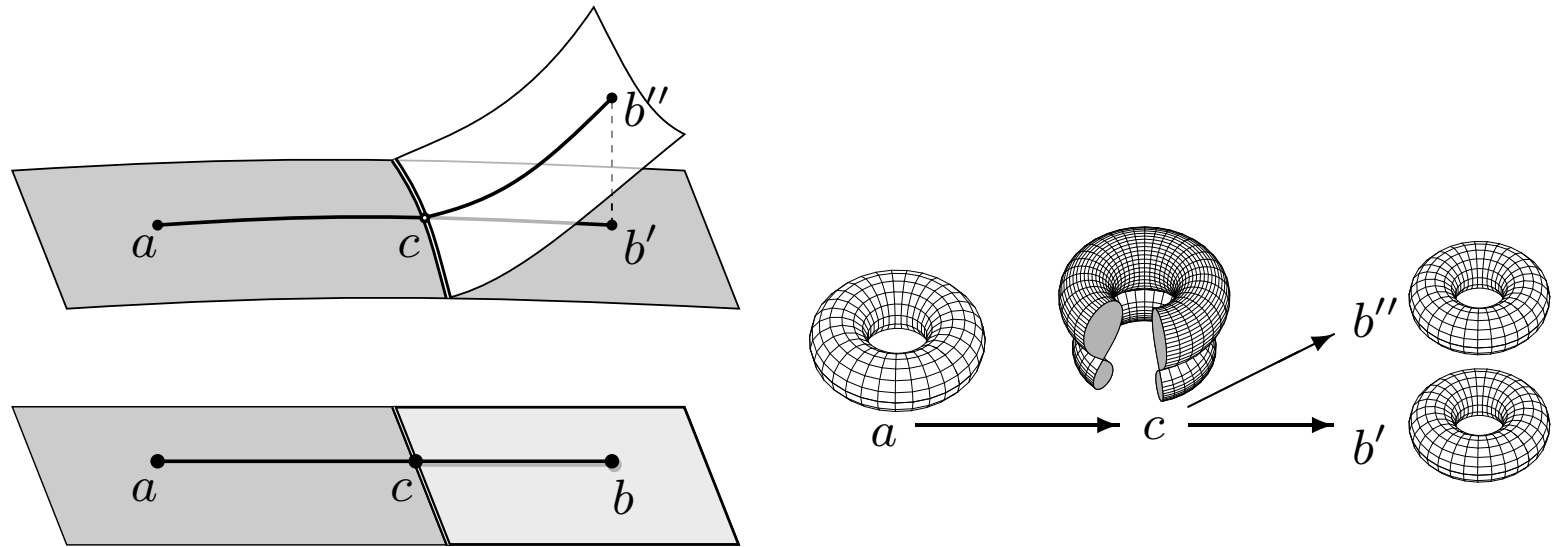


$$H_\lambda = \frac{1-\lambda}{|S|} S_z + \lambda \left( \frac{1}{|S||N|} S_z N_z + \frac{1}{2|S||N|^2} (N_-^2 S_+ + N_+^2 S_-) \right)$$

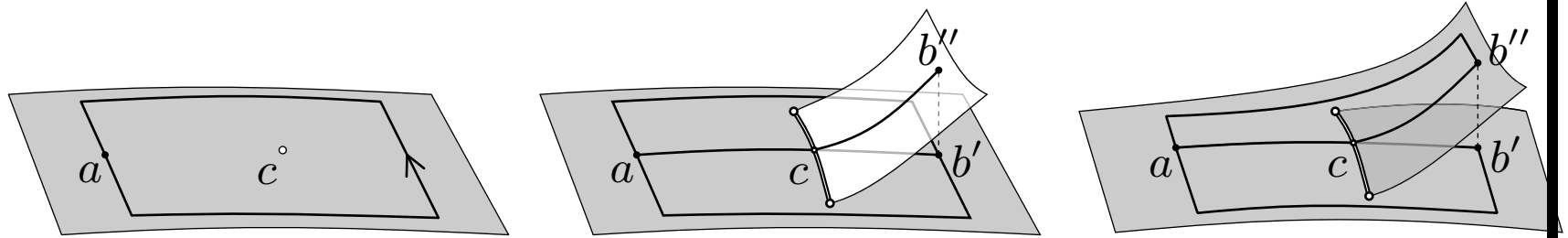




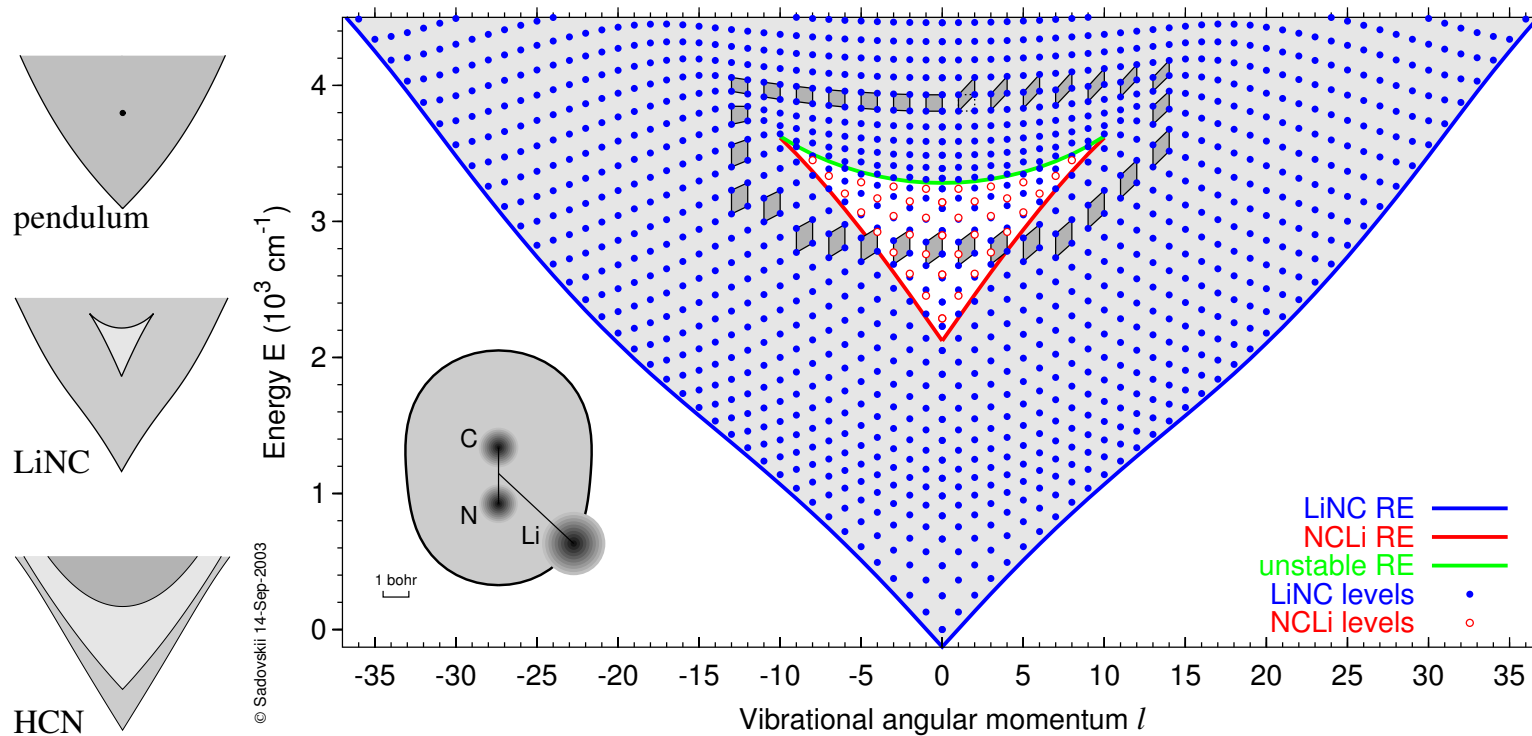
Bifurcation diagram of a system with fractional monodromy with a line of weakly critical values  $c$  (dashes) that lift to curled tori  $\Lambda_c$ . Fractional monodromy corresponds to the closed directed path (solid bold line) which goes around the strongly critical value (open circle) and crosses the critical line at  $c$ . Further plots illustrate the deformation of the regular fibres  $\Lambda_{a'}$  and  $\Lambda_{a''}$  and of the cycles on them as we go from  $a'$  to  $a''$ .



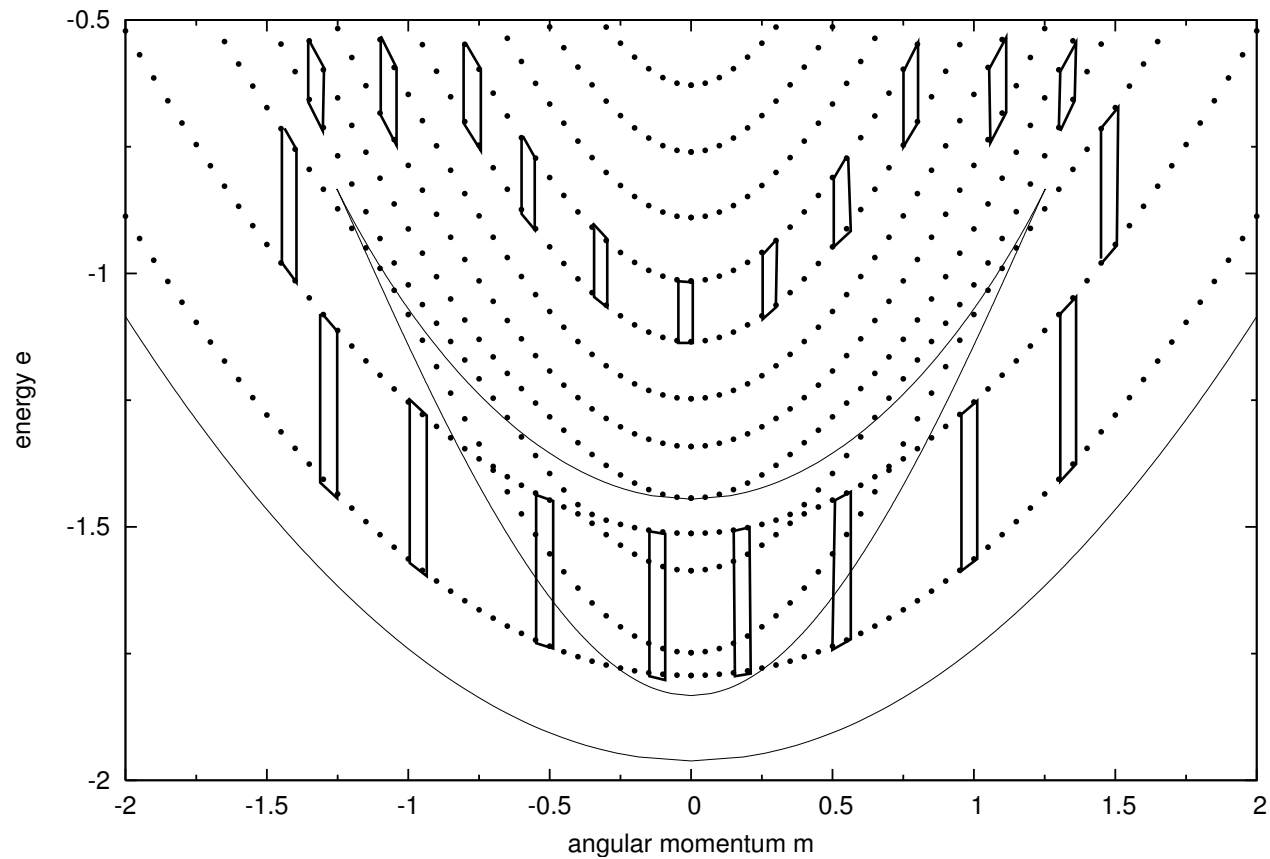
Example of overlapping lower cells in the 2D-image of an energy-momentum map (bottom left) and its two-sheet cell unfolding surface (top left). Points  $a$ ,  $b'$ ,  $b''$ , and  $c$  lift each to a connected component (right);  $b'$  and  $b''$  correspond to the same value  $b$ . Double line marks branching boundary; bold solid line marks a path connecting  $a$ ,  $c$  and  $b$ .



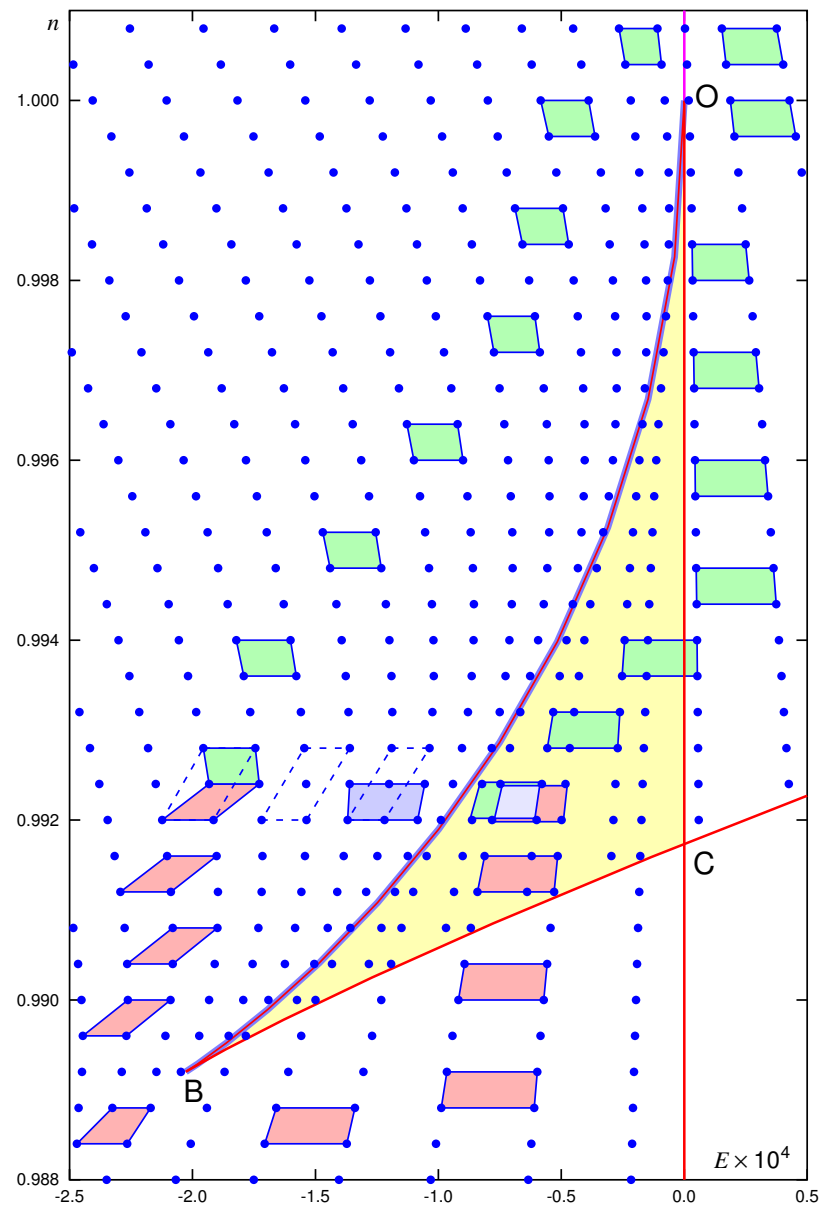
A simple bifurcation diagram of a system with monodromy (left), and cell unfolding surfaces with two overlapping lower cells of a system with nonlocal monodromy (centre), and with a single self-overlapping cell of a system with bidromy (right).



Nonlocal monodromy for LiCN type molecule or quadratic spherical pendulum.



Integrable model with island formed by second component and with *NO* monodromy associated with closed path surrounding island.



## References

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