ROTATIONAL AND ROVIBRATIONAL CONSTANTS FOR TRIATOMIC MOLECULES FROM FOURTH-ORDER PERTURBATION THEORY

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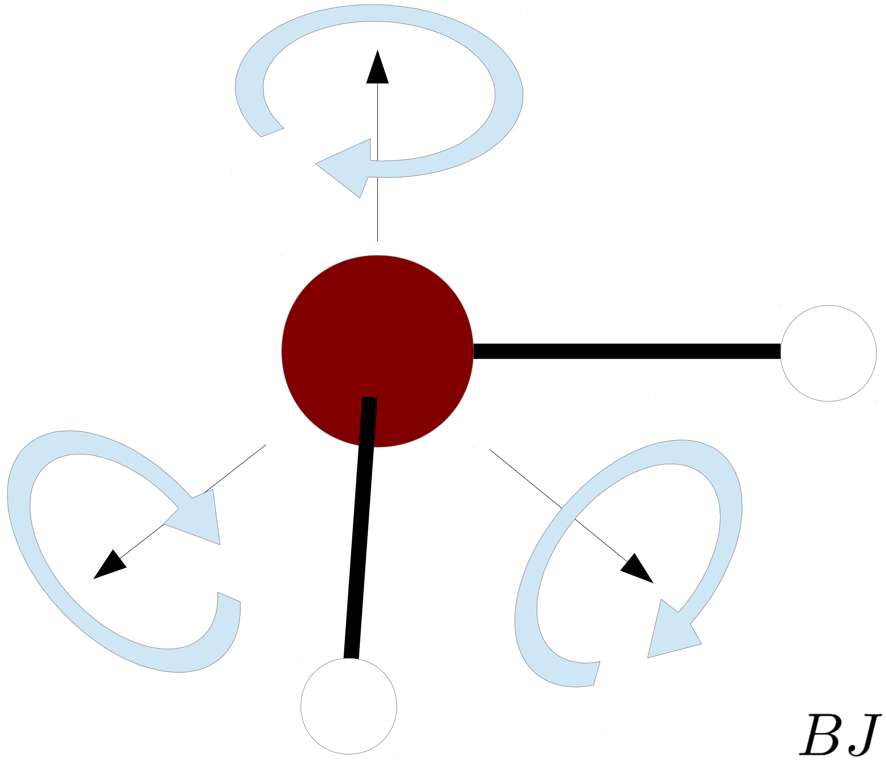


Rigid-rotor Approximation:

Inverse moments of inertia about principal axes

$$E_{RR} = E_v + \sum_\alpha B_e^\alpha \langle P_\alpha^2 \rangle$$

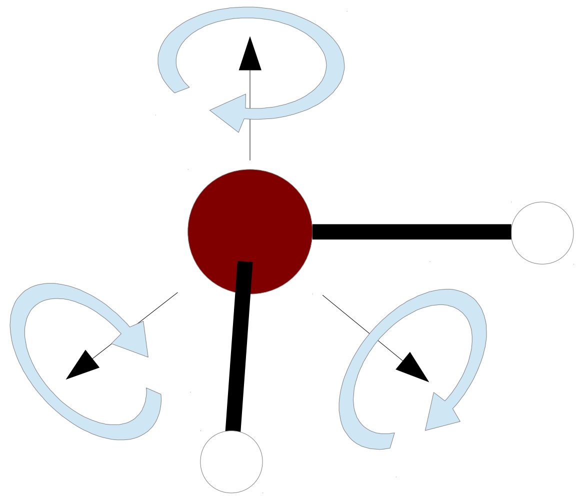
$$BJ(J+1)$$

For linear and spherical top molecules

$$BJ(J+1) + (A-B)K^2$$

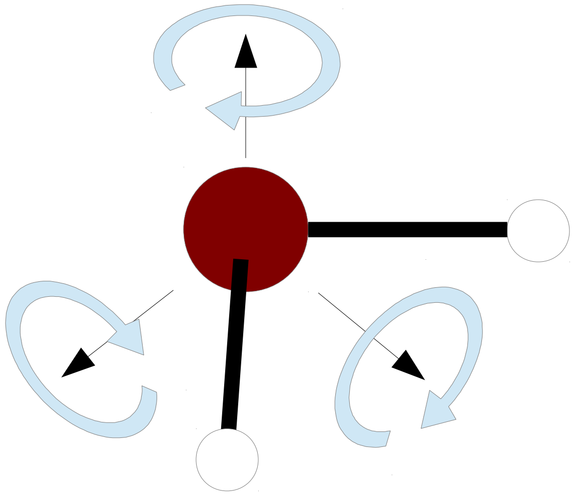
For symmetric top molecules

No simple form for asymmetric top molecules



For general (non-linear) molecules:

$$
\begin{aligned}
E \;=\; & E_{\mathrm{RR}} + \frac{1}{2}\sum_{\alpha\beta}\langle P_\alpha P_\beta\rangle \left(\sum_i \sigma_{\alpha\beta}(i)(n_i+\frac{1}{2}) + \frac{1}{2}\tilde{\sigma}_{\alpha\beta} + \frac{1}{2}\sum_{i\le j}\sigma_{\alpha\beta}(i,j)(n_i+\frac{1}{2})(n_j+\frac{1}{2}) + \ldots\right) \\
& + \frac{1}{4}\sum_{\alpha\beta\gamma\delta}\langle P_\alpha P_\beta P_\gamma P_\delta\rangle \left(\tau_{\alpha\beta\gamma\delta} + \sum_i \tau_{\alpha\beta\gamma\delta}(i)(n_i+\frac{1}{2}) + \ldots\right) \\
& + \sum_{\alpha\beta\gamma\delta\epsilon\eta}\langle P_\alpha P_\beta P_\gamma P_\delta P_\epsilon P_\eta\rangle \left(\xi_{\alpha\beta\gamma\delta\epsilon\eta} + \ldots\right) \\
& + \ldots
\end{aligned}
$$



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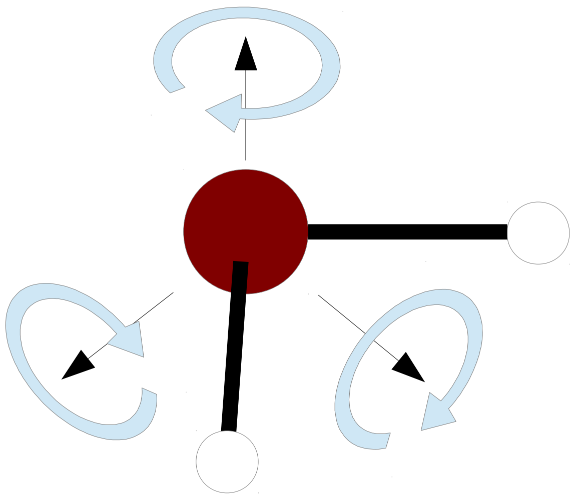
$$
\begin{aligned}
E \;=\;& E_{\mathrm{RR}} + \frac{1}{2}\sum_{\alpha\beta}\langle P_\alpha P_\beta\rangle \left(\sum_i \sigma_{\alpha\beta}(i)(n_i+\frac{1}{2}) + \frac{1}{2}\tilde{\sigma}_{\alpha\beta} + \frac{1}{2}\sum_{i\le j}\sigma_{\alpha\beta}(i,j)(n_i+\frac{1}{2})(n_j+\frac{1}{2}) + \ldots\right) \\
&+ \frac{1}{4}\sum_{\alpha\beta\gamma\delta}\langle P_\alpha P_\beta P_\gamma P_\delta\rangle \left(\tau_{\alpha\beta\gamma\delta} + \sum_i \tau_{\alpha\beta\gamma\delta}(i)(n_i+\frac{1}{2}) + \ldots\right) \\
&+ \sum_{\alpha\beta\gamma\delta\epsilon\eta}\langle P_\alpha P_\beta P_\gamma P_\delta P_\epsilon P_\eta\rangle \left(\xi_{\alpha\beta\gamma\delta\epsilon\eta} + \ldots\right) \\
&+ \ldots
\end{aligned}
$$

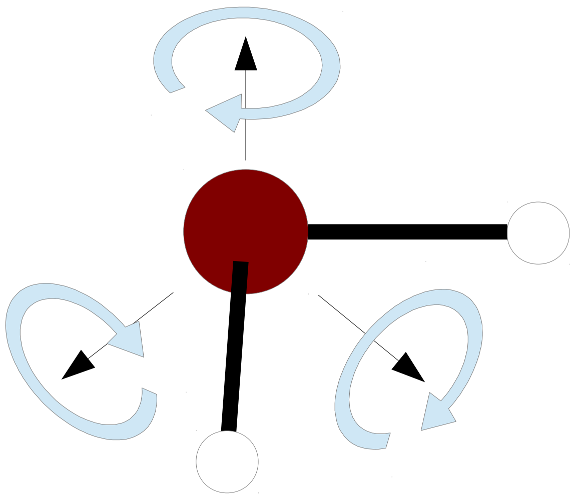
"Vibration-rotation interaction"



For general (non-linear) molecules:

$$
\begin{aligned}
E \;=\; & E_{\mathrm{RR}} + \frac{1}{2}\sum_{\alpha\beta}\langle P_\alpha P_\beta\rangle \left(\sum_i \sigma_{\alpha\beta}(i)(n_i+\frac{1}{2}) + \frac{1}{2}\tilde{\sigma}_{\alpha\beta} + \frac{1}{2}\sum_{i\leq j}\sigma_{\alpha\beta}(i,j)(n_i+\frac{1}{2})(n_j+\frac{1}{2}) + \ldots\right) \\
& + \frac{1}{4}\sum_{\alpha\beta\gamma\delta}\langle P_\alpha P_\beta P_\gamma P_\delta\rangle \left(\tau_{\alpha\beta\gamma\delta} + \sum_i \tau_{\alpha\beta\gamma\delta}(i)(n_i+\frac{1}{2}) + \ldots\right) \\
& + \sum_{\alpha\beta\gamma\delta\epsilon\eta}\langle P_\alpha P_\beta P_\gamma P_\delta P_\epsilon P_\eta\rangle \left(\xi_{\alpha\beta\gamma\delta\epsilon\eta} + \ldots\right) \\
& + \ldots
\end{aligned}
$$

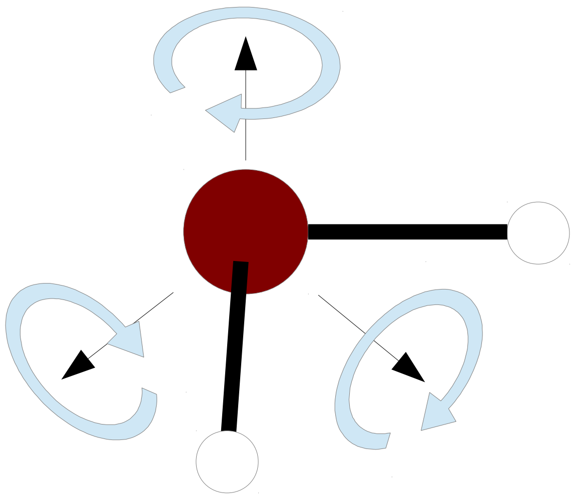
“Quartic centrifugal distortion”



For general (non-linear) molecules:

$$
\begin{aligned}
E \;=\; & E_{\mathrm{RR}} + \frac{1}{2}\sum_{\alpha\beta}\langle P_\alpha P_\beta\rangle \left(\sum_i \sigma_{\alpha\beta}(i)(n_i+\frac{1}{2}) + \frac{1}{2}\tilde{\sigma}_{\alpha\beta} + \frac{1}{2}\sum_{i\le j}\sigma_{\alpha\beta}(i,j)(n_i+\frac{1}{2})(n_j+\frac{1}{2}) + \ldots\right) \\
& + \frac{1}{4}\sum_{\alpha\beta\gamma\delta}\langle P_\alpha P_\beta P_\gamma P_\delta\rangle \left(\tau_{\alpha\beta\gamma\delta} + \sum_i \tau_{\alpha\beta\gamma\delta}(i)(n_i+\frac{1}{2}) + \ldots\right) \\
& + \sum_{\alpha\beta\gamma\delta\epsilon\eta}\langle P_\alpha P_\beta P_\gamma P_\delta P_\epsilon P_\eta\rangle \left(\xi_{\alpha\beta\gamma\delta\epsilon\eta} + \ldots\right) \\
& + \ldots
\end{aligned}
$$

"Sextic centrifugal distortion"



For general (non-linear) molecules:

Rigid-rotor

VPT2

VPT4

$$
\begin{aligned}
E \;=\;& E_{\mathrm{RR}} + \frac{1}{2}\sum_{\alpha\beta}\langle P_\alpha P_\beta\rangle \left(\sum_i \sigma_{\alpha\beta}(i)(n_i+\frac{1}{2}) + \frac{1}{2}\tilde{\sigma}_{\alpha\beta} + \frac{1}{2}\sum_{i\le j}\sigma_{\alpha\beta}(i,j)(n_i+\frac{1}{2})(n_j+\frac{1}{2}) + \ldots\right)\\
&+\frac{1}{4}\sum_{\alpha\beta\gamma\delta}\langle P_\alpha P_\beta P_\gamma P_\delta\rangle \left(\tau_{\alpha\beta\gamma\delta} + \sum_i \tau_{\alpha\beta\gamma\delta}(i)(n_i+\frac{1}{2}) + \ldots\right)\\
&+\sum_{\alpha\beta\gamma\delta\epsilon\eta}\langle P_\alpha P_\beta P_\gamma P_\delta P_\epsilon P_\eta\rangle\left(\xi_{\alpha\beta\gamma\delta\epsilon\eta} + \ldots\right)\\
&+\ldots
\end{aligned}
$$

The Watson Rovibrational Hamiltonian

Rotational operators. Neglecting all vibration gives RR approximation.

Vibrational "angular momentum". Vibrations combine and couple to rotation.

$$\hat{H} = \frac{1}{2}\sum_{\alpha\beta} \mu_{\alpha\beta}(P_\alpha - \pi_\alpha)(P_\beta - \pi_\beta) + \hat{T} + \hat{V} + \hat{U}$$

Instantaneous inverse moment-of-inertia tensor. Vibrations change the mass distribution and hence the moments of inertia (and even the principal axes).

$$\frac{1}{2}\mu_{\alpha\beta} = B_e^\alpha \delta_{\alpha\beta} - \sum_i B_e^\alpha B_e^\beta a_i^{\alpha\beta} q_i + \frac{3}{4}\sum_{\gamma ij} B_e^\alpha B_e^\beta B_e^\gamma a_i^{\alpha\gamma} a_j^{\gamma\beta} q_i q_j + \ldots$$

Hamiltonian expanded in perturbation series, order = # of P's, p's and q's – 2:

$$
\begin{aligned}
\hat{H} &= \hat{H}_v + \sum_{\alpha} \hat{H}_\alpha + \sum_{\alpha\beta} \hat{H}_{\alpha\beta} = \left[\hat{H}_v^{[0]} + \sum_{\alpha} B_e^\alpha (P^\alpha)^2 \right] \\
&+ \lambda \left[\hat{H}_v^{[1]} - \sum_{\alpha i j} 2B_e^\alpha \sqrt{\frac{\omega_j}{\omega_i}} \zeta_{ij}^\alpha P^\alpha q_i p_j - \sum_{\alpha\beta i} B_e^\alpha B_e^\beta a_i^{\alpha\beta} P^\alpha P^\beta q_i \right] \\
&+ \lambda^2 \left[\hat{H}_v^{[2]} + \sum_{\alpha\beta ijk} 2B_e^\alpha B_e^\beta a_k^{\alpha\beta} \sqrt{\frac{\omega_j}{\omega_i}} \zeta_{ij}^\alpha P^\alpha q_k q_i p_j + \sum_{\alpha\beta\gamma ij} \frac{3}{4} B_e^\alpha B_e^\beta B_e^\gamma a_i^{\alpha\gamma} a_j^{\gamma\beta} P^\alpha P^\beta q_i q_j \right] \\
&+ \lambda^3 \left[\hat{H}_v^{[3]} - \sum_{\alpha\beta\gamma ijkl} \frac{3}{2} B_e^\alpha B_e^\beta B_e^\gamma a_k^{\alpha\gamma} a_l^{\gamma\beta} \sqrt{\frac{\omega_j}{\omega_i}} \zeta_{ij}^\alpha P^\alpha q_k q_l q_i p_j - \sum_{\alpha\beta\gamma\delta ijk} \frac{1}{2} B_e^\alpha B_e^\beta B_e^\gamma B_e^\delta a_i^{\alpha\gamma} a_j^{\gamma\delta} a_k^{\delta\beta} P^\alpha P^\beta q_i q_j q_k \right] \\
&+ \lambda^4 \left[\hat{H}_v^{[4]} + \sum_{\alpha\beta\gamma\delta ijklm} \frac{1}{2} B_e^\alpha B_e^\beta B_e^\gamma B_e^\delta a_k^{\alpha\gamma} a_l^{\gamma\delta} a_m^{\delta\beta} \sqrt{\frac{\omega_j}{\omega_i}} \zeta_{ij}^\alpha P^\alpha q_k q_l q_m q_i p_j \right. \\
&\left. + \sum_{\alpha\beta\gamma\delta\epsilon ijkl} \frac{5}{16} B_e^\alpha B_e^\beta B_e^\gamma B_e^\delta B_e^\epsilon a_i^{\alpha\gamma} a_j^{\gamma\delta} a_k^{\delta\epsilon} a_l^{\epsilon\beta} P^\alpha P^\beta q_i q_j q_k q_l \right]
\end{aligned}
$$

Rigid-rotor+Harmonic Oscillator

Rotation+expansion of $\mu_{\alpha\beta}$

Vibration

Rotation+π_α+expansion of $\mu_{\alpha\beta}$

$$
\begin{aligned}
\langle \sigma_{\alpha\beta} \rangle_i^{[4]} \quad = \quad & 2 {\sum_{klm}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_{\alpha\beta}^{[1]} | \Psi_k^{[0]} \rangle \langle \Psi_k^{[0]} | \hat{H}_v^{[1]} | \Psi_l^{[0]} \rangle \langle \Psi_l^{[0]} | \hat{H}_v^{[1]} | \Psi_m^{[0]} \rangle \langle \Psi_m^{[0]} | \hat{H}_v^{[1]} | \Psi_i^{[0]} \rangle}{(E_i^{[0]} - E_k^{[0]})(E_i^{[0]} - E_l^{[0]})(E_i^{[0]} - E_m^{[0]})} + 2 {\sum_{klm}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_{\alpha}^{[1]} | \Psi_k^{[0]} \rangle \langle \Psi_k^{[0]} | \hat{H}_\beta^{[1]} | \Psi_l^{[0]} \rangle \langle \Psi_l^{[0]} | \hat{H}_v^{[1]} | \Psi_m^{[0]} \rangle \langle \Psi_m^{[0]} | \hat{H}_v^{[1]} | \Psi_i^{[0]} \rangle}{(E_i^{[0]} - E_k^{[0]})(E_i^{[0]} - E_l^{[0]})(E_i^{[0]} - E_m^{[0]})} \\
+ \quad & 2 {\sum_{klm}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_{\alpha}^{[1]} | \Psi_k^{[0]} \rangle \langle \Psi_k^{[0]} | \hat{H}_v^{[1]} | \Psi_l^{[0]} \rangle \langle \Psi_l^{[0]} | \hat{H}_\beta^{[1]} | \Psi_m^{[0]} \rangle \langle \Psi_m^{[0]} | \hat{H}_v^{[1]} | \Psi_i^{[0]} \rangle}{(E_i^{[0]} - E_k^{[0]})(E_i^{[0]} - E_l^{[0]})(E_i^{[0]} - E_m^{[0]})} + {\sum_{klm}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_{\alpha}^{[1]} | \Psi_k^{[0]} \rangle \langle \Psi_k^{[0]} | \hat{H}_v^{[1]} | \Psi_l^{[0]} \rangle \langle \Psi_l^{[0]} | \hat{H}_v^{[1]} | \Psi_m^{[0]} \rangle \langle \Psi_m^{[0]} | \hat{H}_\beta^{[1]} | \Psi_i^{[0]} \rangle}{(E_i^{[0]} - E_k^{[0]})(E_i^{[0]} - E_l^{[0]})(E_i^{[0]} - E_m^{[0]})} \\
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+ \quad & 2 {\sum_{kl}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_{\alpha\beta}^{[2]} | \Psi_k^{[0]} \rangle \langle \Psi_k^{[0]} | \hat{H}_v^{[1]} | \Psi_l^{[0]} \rangle \langle \Psi_l^{[0]} | \hat{H}_v^{[1]} | \Psi_i^{[0]} \rangle}{(E_i^{[0]} - E_k^{[0]})(E_i^{[0]} - E_l^{[0]})} + 2 {\sum_{kl}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_\alpha^{[2]} | \Psi_k^{[0]} \rangle \langle \Psi_k^{[0]} | \hat{H}_\beta^{[1]} | \Psi_l^{[0]} \rangle \langle \Psi_l^{[0]} | \hat{H}_v^{[1]} | \Psi_i^{[0]} \rangle}{(E_i^{[0]} - E_k^{[0]})(E_i^{[0]} - E_l^{[0]})} \\
+ \quad & 2 {\sum_{kl}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_\alpha^{[2]} | \Psi_k^{[0]} \rangle \langle \Psi_k^{[0]} | \hat{H}_v^{[1]} | \Psi_l^{[0]} \rangle \langle \Psi_l^{[0]} | \hat{H}_\beta^{[1]} | \Psi_i^{[0]} \rangle}{(E_i^{[0]} - E_k^{[0]})(E_i^{[0]} - E_l^{[0]})} + 2 {\sum_{kl}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_v^{[2]} | \Psi_k^{[0]} \rangle \langle \Psi_k^{[0]} | \hat{H}_{\alpha\beta}^{[1]} | \Psi_l^{[0]} \rangle \langle \Psi_l^{[0]} | \hat{H}_v^{[1]} | \Psi_i^{[0]} \rangle}{(E_i^{[0]} - E_k^{[0]})(E_i^{[0]} - E_l^{[0]})} \\
+ \quad & 2 {\sum_{kl}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_v^{[2]} | \Psi_k^{[0]} \rangle \langle \Psi_k^{[0]} | \hat{H}_\alpha^{[1]} | \Psi_l^{[0]} \rangle \langle \Psi_l^{[0]} | \hat{H}_\beta^{[1]} | \Psi_i^{[0]} \rangle}{(E_i^{[0]} - E_k^{[0]})(E_i^{[0]} - E_l^{[0]})} + 2 {\sum_{kl}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_v^{[2]} | \Psi_k^{[0]} \rangle \langle \Psi_k^{[0]} | \hat{H}_v^{[1]} | \Psi_l^{[0]} \rangle \langle \Psi_l^{[0]} | \hat{H}_{\alpha\beta}^{[1]} | \Psi_i^{[0]} \rangle}{(E_i^{[0]} - E_k^{[0]})(E_i^{[0]} - E_l^{[0]})} \\
+ \quad & 2 {\sum_{kl}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_{\alpha\beta}^{[1]} | \Psi_k^{[0]} \rangle \langle \Psi_k^{[0]} | \hat{H}_v^{[2]} | \Psi_l^{[0]} \rangle \langle \Psi_l^{[0]} | \hat{H}_v^{[1]} | \Psi_i^{[0]} \rangle}{(E_i^{[0]} - E_k^{[0]})(E_i^{[0]} - E_l^{[0]})} + 2 {\sum_{kl}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_\alpha^{[1]} | \Psi_k^{[0]} \rangle \langle \Psi_k^{[0]} | \hat{H}_\beta^{[2]} | \Psi_l^{[0]} \rangle \langle \Psi_l^{[0]} | \hat{H}_v^{[1]} | \Psi_i^{[0]} \rangle}{(E_i^{[0]} - E_k^{[0]})(E_i^{[0]} - E_l^{[0]})} \\
+ \quad & {\sum_{kl}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_\alpha^{[1]} | \Psi_k^{[0]} \rangle \langle \Psi_k^{[0]} | \hat{H}_v^{[2]} | \Psi_l^{[0]} \rangle \langle \Psi_l^{[0]} | \hat{H}_\beta^{[1]} | \Psi_i^{[0]} \rangle}{(E_i^{[0]} - E_k^{[0]})(E_i^{[0]} - E_l^{[0]})} + {\sum_{kl}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_v^{[1]} | \Psi_k^{[0]} \rangle \langle \Psi_k^{[0]} | \hat{H}_{\alpha\beta}^{[2]} | \Psi_l^{[0]} \rangle \langle \Psi_l^{[0]} | \hat{H}_v^{[1]} | \Psi_i^{[0]} \rangle}{(E_i^{[0]} - E_k^{[0]})(E_i^{[0]} - E_l^{[0]})} \\
+ \quad & 2 {\sum_{k}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_{\alpha\beta}^{[3]} | \Psi_k^{[0]} \rangle \langle \Psi_k^{[0]} | \hat{H}_v^{[1]} | \Psi_i^{[0]} \rangle}{E_i^{[0]} - E_k^{[0]}} + 2 {\sum_{k}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_\alpha^{[3]} | \Psi_k^{[0]} \rangle \langle \Psi_k^{[0]} | \hat{H}_\beta^{[1]} | \Psi_i^{[0]} \rangle}{E_i^{[0]} - E_k^{[0]}} + 2 {\sum_{k}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_v^{[3]} | \Psi_k^{[0]} \rangle \langle \Psi_k^{[0]} | \hat{H}_{\alpha\beta}^{[1]} | \Psi_i^{[0]} \rangle}{E_i^{[0]} - E_k^{[0]}} \\
+ \quad & 2 {\sum_{k}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_{\alpha\beta}^{[2]} | \Psi_k^{[0]} \rangle \langle \Psi_k^{[0]} | \hat{H}_v^{[2]} | \Psi_i^{[0]} \rangle}{E_i^{[0]} - E_k^{[0]}} + {\sum_{k}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_\alpha^{[2]} | \Psi_k^{[0]} \rangle \langle \Psi_k^{[0]} | \hat{H}_\beta^{[2]} | \Psi_i^{[0]} \rangle}{E_i^{[0]} - E_k^{[0]}} - {\sum_{k}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_v^{[1]} | \Psi_k^{[0]} \rangle \sigma_{\alpha\beta}^{[2]}(a) \langle \Psi_k^{[0]} | \hat{H}_v^{[1]} | \Psi_i^{[0]} \rangle}{(E_i^{[0]} - E_k^{[0]})^2} \\
- \quad & {\sum_{k}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_\alpha^{[1]} | \Psi_k^{[0]} \rangle E_i^{[2]} \langle \Psi_k^{[0]} | \hat{H}_\beta^{[1]} | \Psi_i^{[0]} \rangle}{(E_i^{[0]} - E_k^{[0]})^2} - 2 {\sum_{k}}' \frac{\langle \Psi_i^{[0]} | \hat{H}_{\alpha\beta}^{[1]} | \Psi_k^{[0]} \rangle E_i^{[2]} \langle \Psi_k^{[0]} | \hat{H}_v^{[1]} | \Psi_i^{[0]} \rangle}{(E_i^{[0]} - E_k^{[0]})^2} + \langle \Psi_i^{[0]} | \hat{H}_{\alpha\beta}^{[4]} | \Psi_i^{[0]} \rangle
\end{aligned}
$$

Placement of rotational operators gives many terms.

Both $H_{\alpha\beta}$- and $H_\alpha H_\beta$-type terms contribute.

$$
\begin{aligned}
\langle\sigma_{\alpha\beta}\rangle_i^{[4]} \;=\;& 2\sum_{klm}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_{\alpha\beta}^{[1]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_v^{[1]}|\Psi_l^{[0]}\rangle\langle\Psi_l^{[0]}|\hat{H}_v^{[1]}|\Psi_m^{[0]}\rangle\langle\Psi_m^{[0]}|\hat{H}_v^{[1]}|\Psi_i^{[0]}\rangle}{(E_i^{[0]}-E_k^{[0]})(E_i^{[0]}-E_l^{[0]})(E_i^{[0]}-E_m^{[0]})} + 2\sum_{klm}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_{\alpha}^{[1]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_\beta^{[1]}|\Psi_l^{[0]}\rangle\langle\Psi_l^{[0]}|\hat{H}_v^{[1]}|\Psi_m^{[0]}\rangle\langle\Psi_m^{[0]}|\hat{H}_v^{[1]}|\Psi_i^{[0]}\rangle}{(E_i^{[0]}-E_k^{[0]})(E_i^{[0]}-E_l^{[0]})(E_i^{[0]}-E_m^{[0]})}\\
+\;& 2\sum_{klm}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_{\alpha}^{[1]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_v^{[1]}|\Psi_l^{[0]}\rangle\langle\Psi_l^{[0]}|\hat{H}_\beta^{[1]}|\Psi_m^{[0]}\rangle\langle\Psi_m^{[0]}|\hat{H}_v^{[1]}|\Psi_i^{[0]}\rangle}{(E_i^{[0]}-E_k^{[0]})(E_i^{[0]}-E_l^{[0]})(E_i^{[0]}-E_m^{[0]})} + \sum_{klm}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_{\alpha}^{[1]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_v^{[1]}|\Psi_l^{[0]}\rangle\langle\Psi_l^{[0]}|\hat{H}_v^{[1]}|\Psi_m^{[0]}\rangle\langle\Psi_m^{[0]}|\hat{H}_\beta^{[1]}|\Psi_i^{[0]}\rangle}{(E_i^{[0]}-E_k^{[0]})(E_i^{[0]}-E_l^{[0]})(E_i^{[0]}-E_m^{[0]})}\\
+\;& 2\sum_{klm}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_v^{[1]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_{\alpha\beta}^{[1]}|\Psi_l^{[0]}\rangle\langle\Psi_l^{[0]}|\hat{H}_v^{[1]}|\Psi_m^{[0]}\rangle\langle\Psi_m^{[0]}|\hat{H}_v^{[1]}|\Psi_i^{[0]}\rangle}{(E_i^{[0]}-E_k^{[0]})(E_i^{[0]}-E_l^{[0]})(E_i^{[0]}-E_m^{[0]})} + \sum_{klm}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_v^{[1]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_\alpha^{[1]}|\Psi_l^{[0]}\rangle\langle\Psi_l^{[0]}|\hat{H}_\beta^{[1]}|\Psi_m^{[0]}\rangle\langle\Psi_m^{[0]}|\hat{H}_v^{[1]}|\Psi_i^{[0]}\rangle}{(E_i^{[0]}-E_k^{[0]})(E_i^{[0]}-E_l^{[0]})(E_i^{[0]}-E_m^{[0]})}\\
+\;& 2\sum_{kl}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_{\alpha\beta}^{[2]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_v^{[1]}|\Psi_l^{[0]}\rangle\langle\Psi_l^{[0]}|\hat{H}_v^{[1]}|\Psi_i^{[0]}\rangle}{(E_i^{[0]}-E_k^{[0]})(E_i^{[0]}-E_l^{[0]})} + 2\sum_{kl}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_\alpha^{[2]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_\beta^{[1]}|\Psi_l^{[0]}\rangle\langle\Psi_l^{[0]}|\hat{H}_v^{[1]}|\Psi_i^{[0]}\rangle}{(E_i^{[0]}-E_k^{[0]})(E_i^{[0]}-E_l^{[0]})}\\
+\;& 2\sum_{kl}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_\alpha^{[2]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_v^{[1]}|\Psi_l^{[0]}\rangle\langle\Psi_l^{[0]}|\hat{H}_\beta^{[1]}|\Psi_i^{[0]}\rangle}{(E_i^{[0]}-E_k^{[0]})(E_i^{[0]}-E_l^{[0]})} + 2\sum_{kl}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_v^{[2]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_{\alpha\beta}^{[1]}|\Psi_l^{[0]}\rangle\langle\Psi_l^{[0]}|\hat{H}_v^{[1]}|\Psi_i^{[0]}\rangle}{(E_i^{[0]}-E_k^{[0]})(E_i^{[0]}-E_l^{[0]})}\\
+\;& 2\sum_{kl}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_v^{[2]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_\alpha^{[1]}|\Psi_l^{[0]}\rangle\langle\Psi_l^{[0]}|\hat{H}_\beta^{[1]}|\Psi_i^{[0]}\rangle}{(E_i^{[0]}-E_k^{[0]})(E_i^{[0]}-E_l^{[0]})} + 2\sum_{kl}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_v^{[2]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_v^{[1]}|\Psi_l^{[0]}\rangle\langle\Psi_l^{[0]}|\hat{H}_{\alpha\beta}^{[1]}|\Psi_i^{[0]}\rangle}{(E_i^{[0]}-E_k^{[0]})(E_i^{[0]}-E_l^{[0]})}\\
+\;& 2\sum_{kl}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_{\alpha\beta}^{[1]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_v^{[2]}|\Psi_l^{[0]}\rangle\langle\Psi_l^{[0]}|\hat{H}_v^{[1]}|\Psi_i^{[0]}\rangle}{(E_i^{[0]}-E_k^{[0]})(E_i^{[0]}-E_l^{[0]})} + 2\sum_{kl}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_\alpha^{[1]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_\beta^{[2]}|\Psi_l^{[0]}\rangle\langle\Psi_l^{[0]}|\hat{H}_v^{[1]}|\Psi_i^{[0]}\rangle}{(E_i^{[0]}-E_k^{[0]})(E_i^{[0]}-E_l^{[0]})}\\
+\;& \sum_{kl}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_\alpha^{[1]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_v^{[2]}|\Psi_l^{[0]}\rangle\langle\Psi_l^{[0]}|\hat{H}_\beta^{[1]}|\Psi_i^{[0]}\rangle}{(E_i^{[0]}-E_k^{[0]})(E_i^{[0]}-E_l^{[0]})} + \sum_{kl}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_v^{[1]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_{\alpha\beta}^{[2]}|\Psi_l^{[0]}\rangle\langle\Psi_l^{[0]}|\hat{H}_v^{[1]}|\Psi_i^{[0]}\rangle}{(E_i^{[0]}-E_k^{[0]})(E_i^{[0]}-E_l^{[0]})}\\
+\;& 2\sum_{k}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_{\alpha\beta}^{[3]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_v^{[1]}|\Psi_i^{[0]}\rangle}{E_i^{[0]}-E_k^{[0]}} + 2\sum_{k}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_\alpha^{[3]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_\beta^{[1]}|\Psi_i^{[0]}\rangle}{E_i^{[0]}-E_k^{[0]}} + 2\sum_{k}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_v^{[3]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_{\alpha\beta}^{[1]}|\Psi_i^{[0]}\rangle}{E_i^{[0]}-E_k^{[0]}}\\
+\;& 2\sum_{k}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_{\alpha\beta}^{[2]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_v^{[2]}|\Psi_i^{[0]}\rangle}{E_i^{[0]}-E_k^{[0]}} + \sum_{k}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_\alpha^{[2]}|\Psi_k^{[0]}\rangle\langle\Psi_k^{[0]}|\hat{H}_\beta^{[2]}|\Psi_i^{[0]}\rangle}{E_i^{[0]}-E_k^{[0]}} - \sum_{k}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_v^{[1]}|\Psi_k^{[0]}\rangle\sigma_{\alpha\beta}^{[2]}(a)\langle\Psi_k^{[0]}|\hat{H}_v^{[1]}|\Psi_i^{[0]}\rangle}{(E_i^{[0]}-E_k^{[0]})^2}\\
-\;& \sum_{k}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_\alpha^{[1]}|\Psi_k^{[0]}\rangle E_i^{[2]}\langle\Psi_k^{[0]}|\hat{H}_\beta^{[1]}|\Psi_i^{[0]}\rangle}{(E_i^{[0]}-E_k^{[0]})^2} - 2\sum_{k}^{\prime}\frac{\langle\Psi_i^{[0]}|\hat{H}_{\alpha\beta}^{[1]}|\Psi_k^{[0]}\rangle E_i^{[2]}\langle\Psi_k^{[0]}|\hat{H}_v^{[1]}|\Psi_i^{[0]}\rangle}{(E_i^{[0]}-E_k^{[0]})^2} + \langle\Psi_i^{[0]}|\hat{H}_{\alpha\beta}^{[4]}|\Psi_i^{[0]}\rangle
\end{aligned}
$$

Order of rotational operators in renormalization terms is important.

$\sigma_{\alpha\beta}(i,i)$

$$+\frac{15B^a_e a^{ac}_i B^b_e a^{bc}_i B^c_e a^{cd}_i B^d_e a^{de}_i B^e_e}{16}+\frac{5B^a_e a^{ac}_i B^b_e a^{bd}_i B^c_e a^{cd}_i B^d_e \phi_{iii}}{4\omega_i}+\frac{B^a_e a^{ac}_i B^b_e a^{bd}_j B^c_e a^{cd}_i B^d_e \phi_{iij}(3\omega_j^2-8\omega_i^2)}{4\omega_j(\omega_j^2-4\omega_i^2)}+\frac{B^a_e a^{ac}_i B^b_e a^{bd}_j B^c_e a^{cd}_j B^d_e \phi_{iij}(3\omega_j^2-8\omega_i^2)}{4\omega_j(\omega_j^2-4\omega_i^2)}+\frac{B^a_e a^{ac}_j B^b_e a^{bd}_i B^c_e a^{cd}_i B^d_e \phi_{iij}(3\omega_j^2-8\omega_i^2)}{4\omega_j(\omega_j^2-4\omega_i^2)}-\frac{3B^a_e a^{ad}_i B^b_e \zeta^b_{ij} B^c_e a^{cd}_i \zeta^c_{ij} B^d_e(3\omega_j^2+5\omega_i^2)}{2\omega_i(\omega_j^2-\omega_i^2)}-\frac{3B^a_e \zeta^a_{ij} B^b_e a^{bd}_i B^c_e a^{cd}_i \zeta^c_{ij} B^d_e(3\omega_j^2+5\omega_i^2)}{2\omega_i(\omega_j^2-\omega_i^2)}+\frac{B^a_e a^{ab}_i B^b_e \phi_{iiii}}{8\omega_i}+\frac{B^a_e a^{ab}_j B^b_e \phi_{iijj}}{8\omega_j}-\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_i B^c_e \phi_{iiii}}{16\omega_i}-\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_j B^c_e \phi_{iijj}\omega_j}{8(\omega_j^2-\omega_i^2)}$$

$$-\frac{3B^a_e a^{ac}_j B^b_e a^{bc}_i B^c_e \phi_{iijj}\omega_j}{8(\omega_j^2-\omega_i^2)}-\frac{6B^a_e a^{ac}_i B^b_e a^{bd}_i B^c_e \zeta^c_{ij} B^d_e \zeta^d_{ij}(\omega_j^2-\omega_i^2)}{\omega_i(\omega_j^2-4\omega_i^2)}$$

$$+\frac{5B^a_e a^{ac}_i B^b_e a^{bc}_i B^c_e \phi^2_{iii}}{8\omega_i^2}+\frac{3B^a_e a^{ac}_j B^b_e a^{bc}_j B^c_e \phi^2_{iij}(3\omega_j^4-16\omega_i^2\omega_j^2+32\omega_i^4)}{16\omega_j^2(\omega_j^2-4\omega_i^2)^2}+\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_i B^c_e \phi^2_{iij}(3\omega_j^4-24\omega_i^2\omega_j^2+32\omega_i^4)}{16\omega_i\omega_j(\omega_j^2-4\omega_i^2)^2}+\frac{3B^a_e a^{ac}_j B^b_e a^{bc}_i B^c_e \phi_{iij}\phi_{ijj}(3\omega_j^2-8\omega_i^2)}{8(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)}+\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_j B^c_e \phi_{iij}\phi_{ijj}(3\omega_j^2-8\omega_i^2)}{8(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)}+\frac{3B^a_e a^{ac}_j B^b_e a^{bc}_k B^c_e \phi_{iik}\phi_{ijk}\omega_j(3\omega_k^2-8\omega_i^2)}{8\omega_k(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)}+\frac{3B^a_e a^{ac}_k B^b_e a^{bc}_j B^c_e \phi_{iik}\phi_{ijk}\omega_j(3\omega_k^2-8\omega_i^2)}{8\omega_k(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)}+\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_j B^c_e \phi_{iii}\phi_{iij}(5\omega_j^4-23\omega_i^2\omega_j^2+8\omega_i^4)}{16\omega_i\omega_j(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)}$$

$$+\frac{3B^a_e a^{ac}_j B^b_e a^{bc}_i B^c_e \phi_{iii}\phi_{iij}(5\omega_j^4-23\omega_i^2\omega_j^2+8\omega_i^4)}{16\omega_i\omega_j(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)}+\frac{3B^a_e a^{ac}_j B^b_e a^{bc}_k B^c_e \phi_{iij}\phi_{iik}(3\omega_j^2\omega_k^2-8\omega_i^2\omega_k^2-8\omega_i^2\omega_j^2+32\omega_i^4)}{16\omega_j\omega_k(\omega_j^2-4\omega_i^2)(\omega_k^2-4\omega_i^2)}$$

$$-\frac{12B^a_e a^{ac}_i B^b_e \zeta^b_{ij} B^c_e \zeta^c_{ij}\phi_{ijj}\omega_i\omega_j}{(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)}-\frac{12B^a_e \zeta^a_{ij} B^b_e a^{bc}_i B^c_e \zeta^c_{ij}\phi_{ijj}\omega_i\omega_j}{(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)}-\frac{B^a_e a^{ac}_i B^b_e \zeta^b_{ij} B^c_e \zeta^c_{ij}\phi_{iii}(5\omega_j^4-17\omega_i^2\omega_j^2-24\omega_i^4)}{\omega_i^2(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)}-\frac{B^a_e \zeta^a_{ij} B^b_e a^{bc}_i B^c_e \zeta^c_{ij}\phi_{iii}(5\omega_j^4-17\omega_i^2\omega_j^2-24\omega_i^4)}{\omega_i^2(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)}-\frac{B^a_e a^{ac}_j B^b_e \zeta^b_{ij} B^c_e \zeta^c_{ij}\phi_{iij}(5\omega_j^4-13\omega_i^2\omega_j^2-40\omega_i^4)}{\omega_i\omega_j(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)}-\frac{B^a_e \zeta^a_{ij} B^b_e a^{bc}_j B^c_e \zeta^c_{ij}\phi_{iij}(5\omega_j^4-13\omega_i^2\omega_j^2-40\omega_i^4)}{\omega_i\omega_j(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)}-\frac{2B^a_e a^{ac}_i B^b_e \zeta^b_{ik} B^c_e \zeta^c_{jk}\phi_{iii}\sqrt{\omega_j}}{\sqrt{\omega_i}(\omega_k^2-\omega_i^2)}+\frac{2B^a_e \zeta^a_{ij} B^b_e a^{bc}_k B^c_e \zeta^c_{jk}\phi_{iii}\sqrt{\omega_k}}{\sqrt{\omega_i}(\omega_j^2-\omega_i^2)}-\frac{2B^a_e a^{ac}_i B^b_e \zeta^b_{ik} B^c_e \zeta^c_{ij}\phi_{iik}\sqrt{\omega_j}\sqrt{\omega_k}}{\omega_i(\omega_k^2-\omega_i^2)}$$

$$-\frac{2B^a_e \zeta^a_{ij} B^b_e a^{bc}_k B^c_e \zeta^c_{ik}\phi_{iij}\sqrt{\omega_j}\sqrt{\omega_k}}{\omega_i(\omega_j^2-\omega_i^2)}-\frac{12B^a_e a^{ac}_i B^b_e \zeta^b_{ij} B^c_e \zeta^c_{ik}\phi_{ijk}\omega_i\sqrt{\omega_j}\sqrt{\omega_k}}{(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)}-\frac{12B^a_e \zeta^a_{ij} B^b_e a^{bc}_i B^c_e \zeta^c_{ik}\phi_{ijk}\omega_i\sqrt{\omega_j}\sqrt{\omega_k}}{(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)}+\frac{2B^a_e a^{ac}_j B^b_e \zeta^b_{ij} B^c_e \zeta^c_{ik}\phi_{iik}\omega_i\sqrt{\omega_j}(\omega_k^2+8\omega_i^2)}{\omega_k^{\frac{3}{2}}(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)}+\frac{2B^a_e \zeta^a_{ij} B^b_e a^{bc}_j B^c_e \zeta^c_{ik}\phi_{iik}\omega_i\sqrt{\omega_j}(\omega_k^2+8\omega_i^2)}{\omega_k^{\frac{3}{2}}(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)}+\frac{B^a_e a^{ac}_i B^b_e \zeta^b_{jk} B^c_e \zeta^c_{ij}\phi_{iik}(3\omega_j^2\omega_k^2-8\omega_i^2\omega_j^2+32\omega_i^4)}{\sqrt{\omega_i}\omega_k^{\frac{3}{2}}(\omega_j^2-4\omega_i^2)(\omega_k^2-4\omega_i^2)}+\frac{B^a_e \zeta^a_{jk} B^b_e a^{bc}_i B^c_e \zeta^c_{ij}\phi_{iik}(3\omega_j^2\omega_k^2-8\omega_i^2\omega_j^2+32\omega_i^4)}{\sqrt{\omega_i}\omega_k^{\frac{3}{2}}(\omega_j^2-4\omega_i^2)(\omega_k^2-4\omega_i^2)}-\frac{B^a_e a^{ac}_j B^b_e \zeta^b_{ik} B^c_e \zeta^c_{ik}\phi_{iij}(3\omega_j^2\omega_k^2-8\omega_i^2\omega_k^2+5\omega_i^2\omega_j^2-24\omega_i^4)}{\omega_i\omega_j(\omega_j^2-4\omega_i^2)(\omega_k^2-\omega_i^2)}$$

$$-\frac{B^a_e \zeta^a_{ij} B^b_e a^{bc}_k B^c_e \zeta^c_{ij}\phi_{iik}(3\omega_j^2\omega_k^2+5\omega_i^2\omega_k^2-8\omega_i^2\omega_j^2-24\omega_i^4)}{\omega_i\omega_k(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)}+\frac{B^a_e a^{ac}_i B^b_e \zeta^b_{ij} B^c_e \zeta^c_{jk}\phi_{iik}(3\omega_j^2\omega_k^2+7\omega_i^2\omega_k^2-8\omega_i^2\omega_j^2-8\omega_i^4)}{\sqrt{\omega_j}\omega_k^{\frac{3}{2}}(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)}+\frac{B^a_e \zeta^a_{ij} B^b_e a^{bc}_i B^c_e \zeta^c_{jk}\phi_{iik}(3\omega_j^2\omega_k^2+7\omega_i^2\omega_k^2-8\omega_i^2\omega_j^2-8\omega_i^4)}{\sqrt{\omega_j}\omega_k^{\frac{3}{2}}(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)}$$

$$-\frac{13B^a_e a^{ab}_i B^b_e \phi_{iii}\phi_{iiii}}{24\omega_i^2}-\frac{B^a_e a^{ab}_j B^b_e \phi_{iiii}\phi_{iij}}{8\omega_i\omega_j}-\frac{5B^a_e a^{ab}_i B^b_e \phi_{iii}\phi_{iiij}}{12\omega_i\omega_j}-\frac{B^a_e a^{ab}_j B^b_e \phi_{iiij}\phi_{ijj}}{2(\omega_j^2-\omega_i^2)}-\frac{B^a_e a^{ab}_j B^b_e \phi_{iiik}\phi_{ijk}\omega_k}{2\omega_j(\omega_k^2-\omega_i^2)}-\frac{B^a_e a^{ab}_j B^b_e \phi_{iij}\phi_{iijj}(3\omega_j^2-8\omega_i^2)}{4\omega_j^2(\omega_j^2-4\omega_i^2)}-\frac{B^a_e a^{ab}_j B^b_e \phi_{iijk}\phi_{iik}(3\omega_k^2-8\omega_i^2)}{4\omega_j\omega_k(\omega_k^2-4\omega_i^2)}+\frac{B^a_e a^{ab}_j B^b_e B^c_e(\zeta^c_{ij})^2\phi_{iij}\omega_i(\omega_j^2+8\omega_i^2)}{\omega_j^3(\omega_j^2-4\omega_i^2)}+\frac{B^a_e a^{ab}_j B^b_e B^c_e\zeta^c_{ij}\zeta^c_{ik}\phi_{iik}\omega_i(\omega_k^2+8\omega_i^2)}{\omega_j^{\frac{3}{2}}\omega_k^{\frac{3}{2}}(\omega_k^2-4\omega_i^2)}-\frac{B^a_e a^{ab}_i B^b_e \phi_{iiij}\phi_{iij}(5\omega_j^4-19\omega_i^2\omega_j^2+8\omega_i^4)}{4\omega_i\omega_j(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)}$$

$$+\frac{4B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ij}\phi_{iijj}\omega_i\omega_j}{(\omega_j^2-\omega_i^2)^2}+\frac{B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ij}\phi_{iiii}(\omega_j^4-6\omega_i^2\omega_j^2-3\omega_i^4)}{2\omega_i^2(\omega_j^2-\omega_i^2)^2}+\frac{4B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ij}B^c_e(\zeta^c_{ij})^2(3\omega_j^4+10\omega_i^2\omega_j^2+19\omega_i^4)}{\omega_i^2(\omega_j^2-\omega_i^2)^2}+\frac{4B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ik}\phi_{iijk}\omega_i\sqrt{\omega_j}\sqrt{\omega_k}}{(\omega_j^2-\omega_i^2)(\omega_k^2-\omega_i^2)}-\frac{B^a_e\zeta^a_{ij}B^b_e\zeta^b_{jk}\phi_{iiik}(\omega_j^2+3\omega_i^2)\sqrt{\omega_k}}{\sqrt{\omega_i}(\omega_j^2-\omega_i^2)(\omega_k^2-\omega_i^2)}-\frac{B^a_e\zeta^a_{jk}B^b_e\zeta^b_{ij}\phi_{iiik}(\omega_j^2+3\omega_i^2)\sqrt{\omega_k}}{\sqrt{\omega_i}(\omega_j^2-\omega_i^2)(\omega_k^2-\omega_i^2)}+\frac{4B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ik}B^c_e\zeta^c_{ij}\zeta^c_{ik}(3\omega_j^2\omega_k^2+5\omega_i^2\omega_k^2+5\omega_i^2\omega_j^2+19\omega_i^4)}{\omega_i^2(\omega_j^2-\omega_i^2)(\omega_k^2-\omega_i^2)}$$

$$+\frac{5B^a_e a^{ab}_i B^b_e \phi^3_{iii}}{12\omega_i^3}+\frac{5B^a_e a^{ab}_j B^b_e \phi^2_{iii}\phi_{ijj}}{12\omega_i^2\omega_j}+\frac{B^a_e a^{ab}_j B^b_e \phi^3_{iij}(3\omega_j^4-24\omega_i^2\omega_j^2+32\omega_i^4)}{8\omega_i\omega_j^2(\omega_j^2-4\omega_i^2)^2}+\frac{B^a_e a^{ab}_j B^b_e \phi^2_{iij}\phi_{jjj}(3\omega_j^4-16\omega_i^2\omega_j^2+32\omega_i^4)}{8\omega_j^3(\omega_j^2-4\omega_i^2)^2}+\frac{B^a_e a^{ab}_j B^b_e \phi_{iij}\phi^2_{ijj}(3\omega_j^2-8\omega_i^2)}{2\omega_j(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)}+\frac{B^a_e a^{ab}_j B^b_e \phi_{iii}\phi_{iij}\phi_{ijj}(5\omega_j^4-23\omega_i^2\omega_j^2+8\omega_i^4)}{4\omega_i\omega_j^2(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)}+\frac{B^a_e a^{ab}_i B^b_e \phi^2_{iij}\phi_{ijj}(15\omega_j^6-99\omega_i^2\omega_j^4+176\omega_i^4\omega_j^2-32\omega_i^6)}{8\omega_i\omega_j^2(\omega_j^2-4\omega_i^2)^2(\omega_j^2-\omega_i^2)}+\frac{B^a_e a^{ab}_i B^b_e \phi_{iii}\phi^2_{iij}(13\omega_j^6-113\omega_i^2\omega_j^4+256\omega_i^4\omega_j^2-96\omega_i^6)}{8\omega_i^2\omega_j(\omega_j^2-4\omega_i^2)^2(\omega_j^2-\omega_i^2)}+\frac{B^a_e a^{ab}_j B^b_e \phi^2_{iik}\phi_{jkk}(3\omega_k^4-16\omega_i^2\omega_k^2+32\omega_i^4)}{8\omega_j\omega_k^2(\omega_k^2-4\omega_i^2)^2}$$

$$+\frac{B^a_e a^{ab}_j B^b_e \phi_{iij}\phi^2_{iik}(3\omega_k^4-24\omega_i^2\omega_k^2+32\omega_i^4)}{8\omega_i\omega_j\omega_k(\omega_k^2-4\omega_i^2)^2}+\frac{B^a_e a^{ab}_j B^b_e \phi_{iik}\phi_{ijj}\phi_{ijk}(3\omega_k^2-8\omega_i^2)}{2\omega_k(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)}+\frac{B^a_e a^{ab}_j B^b_e \phi_{iik}\phi_{ijk}\phi_{ikk}(3\omega_k^2-8\omega_i^2)}{2\omega_j(\omega_k^2-4\omega_i^2)(\omega_k^2-\omega_i^2)}+\frac{B^a_e a^{ab}_j B^b_e \phi_{iij}\phi^2_{ijk}\omega_k(3\omega_j^2-8\omega_i^2)}{2\omega_j^2(\omega_j^2-4\omega_i^2)(\omega_k^2-\omega_i^2)}+\frac{B^a_e a^{ab}_j B^b_e \phi_{iii}\phi_{iik}\phi_{ijk}(5\omega_k^4-23\omega_i^2\omega_k^2+8\omega_i^4)}{4\omega_i\omega_j\omega_k(\omega_k^2-4\omega_i^2)(\omega_k^2-\omega_i^2)}+\frac{B^a_e a^{ab}_j B^b_e \phi_{iij}\phi_{iik}\phi_{jjk}(3\omega_j^2\omega_k^2-8\omega_i^2\omega_k^2-8\omega_i^2\omega_j^2+32\omega_i^4)}{4\omega_j^2\omega_k(\omega_j^2-4\omega_i^2)(\omega_k^2-4\omega_i^2)}$$

$$+\frac{B^a_e a^{ab}_i B^b_e \phi_{iij}\phi_{iik}\phi_{ijk}(15\omega_j^4\omega_k^4-57\omega_i^2\omega_j^2\omega_k^4+24\omega_i^4\omega_k^4-57\omega_i^2\omega_j^4\omega_k^2+227\omega_i^4\omega_j^2\omega_k^2-104\omega_i^6\omega_k^2+24\omega_i^4\omega_j^4-104\omega_i^6\omega_j^2+32\omega_i^8)}{8\omega_i\omega_j\omega_k(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)(\omega_k^2-\omega_i^2)}$$

$$-\frac{8B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ij}\phi^2_{ijj}\omega_i\omega_j^2}{(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)^2}-\frac{8B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ij}\phi^2_{ijk}\omega_i\omega_j\omega_k}{(\omega_j^2-\omega_i^2)^2(\omega_k^2-4\omega_i^2)}-\frac{4B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ij}\phi_{iii}\phi_{ijj}\omega_j(\omega_j^2-8\omega_i^2)}{(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)^2}-\frac{4B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ij}\phi_{iij}\phi_{jjj}\omega_i(\omega_j^2-8\omega_i^2)}{(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)^2}-\frac{4B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ij}\phi_{iik}\phi_{jjk}\omega_i\omega_j(\omega_k^2-8\omega_i^2)}{(\omega_j^2-\omega_i^2)^2\omega_k(\omega_k^2-4\omega_i^2)}-\frac{8B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ik}\phi_{ijj}\phi_{ijk}\omega_i\omega_j^{\frac{3}{2}}\sqrt{\omega_k}}{(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)(\omega_k^2-\omega_i^2)}-\frac{8B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ik}\phi_{ijk}\phi_{ikk}\omega_i\sqrt{\omega_j}\omega_k^{\frac{3}{2}}}{(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)(\omega_k^2-\omega_i^2)}+\frac{8B^a_e\zeta^a_{ij}B^b_e\zeta^b_{jk}\omega_i^{\frac{3}{2}}\phi_{iik}\phi_{ijj}\omega_j\sqrt{\omega_k}}{(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)}+\frac{8B^a_e\zeta^a_{jk}B^b_e\zeta^b_{ij}\omega_i^{\frac{3}{2}}\phi_{iik}\phi_{ijj}\omega_j\sqrt{\omega_k}}{(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)}-\frac{B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ij}\phi^2_{iii}(5\omega_j^6-40\omega_i^2\omega_j^4+83\omega_i^4\omega_j^2+12\omega_i^6)}{3\omega_i^3(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)^2}$$

$$-\frac{4B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ik}\phi_{iij}\phi_{jjk}\omega_i(\omega_j^2-8\omega_i^2)\sqrt{\omega_k}}{\sqrt{\omega_j}(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)(\omega_k^2-\omega_i^2)}-\frac{4B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ik}\phi_{iik}\omega_i\sqrt{\omega_j}\phi_{jkk}(\omega_k^2-8\omega_i^2)}{(\omega_j^2-\omega_i^2)\sqrt{\omega_k}(\omega_k^2-4\omega_i^2)(\omega_k^2-\omega_i^2)}+\frac{B^a_e\zeta^a_{ij}B^b_e\zeta^b_{jk}\phi_{iik}\phi_{ikk}(\omega_j^2+3\omega_i^2)(3\omega_k^2-8\omega_i^2)}{\sqrt{\omega_i}(\omega_j^2-\omega_i^2)\sqrt{\omega_k}(\omega_k^2-4\omega_i^2)(\omega_k^2-\omega_i^2)}+\frac{B^a_e\zeta^a_{jk}B^b_e\zeta^b_{ij}\phi_{iik}\phi_{ikk}(\omega_j^2+3\omega_i^2)(3\omega_k^2-8\omega_i^2)}{\sqrt{\omega_i}(\omega_j^2-\omega_i^2)\sqrt{\omega_k}(\omega_k^2-4\omega_i^2)(\omega_k^2-\omega_i^2)}-\frac{B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ij}\phi^2_{iij}(3\omega_j^{10}-39\omega_i^2\omega_j^8+113\omega_i^4\omega_j^6-101\omega_i^6\omega_j^4-176\omega_i^8\omega_j^2+32\omega_i^{10})}{2\omega_i^2\omega_j^3(\omega_j^2-4\omega_i^2)^2(\omega_j^2-\omega_i^2)^2}-\frac{B^a_e\zeta^a_{jk}B^b_e\zeta^b_{jk}\phi^2_{iij}(3\omega_j^4\omega_k^2-16\omega_i^2\omega_j^2\omega_k^2+32\omega_i^4\omega_k^2+4\omega_i^2\omega_j^4+64\omega_i^4\omega_j^2-128\omega_i^6)}{2\omega_j^3(\omega_j^2-4\omega_i^2)^2(\omega_k^2-4\omega_i^2)}$$

$$-\frac{4B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ik}\phi_{iii}\phi_{ijk}\sqrt{\omega_j}\sqrt{\omega_k}(\omega_j^2\omega_k^2-6\omega_i^2\omega_k^2-6\omega_i^2\omega_j^2+32\omega_i^4)}{(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)(\omega_k^2-\omega_i^2)}-\frac{B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ij}\phi^2_{iik}(3\omega_j^4\omega_k^4-18\omega_i^2\omega_j^2\omega_k^4-9\omega_i^4\omega_k^4-24\omega_i^2\omega_j^4\omega_k^2+112\omega_i^4\omega_j^2\omega_k^2+72\omega_i^6\omega_k^2+32\omega_i^4\omega_j^4-192\omega_i^6\omega_j^2-96\omega_i^8)}{2\omega_i^2(\omega_j^2-\omega_i^2)^2\omega_k(\omega_k^2-4\omega_i^2)^2}+\frac{B^a_e\zeta^a_{ij}B^b_e\zeta^b_{jk}\phi_{iij}\phi_{ijk}\sqrt{\omega_k}(3\omega_j^4\omega_k^2-7\omega_i^2\omega_j^2\omega_k^2-24\omega_i^4\omega_k^2-12\omega_i^2\omega_j^4+4\omega_i^4\omega_j^2+96\omega_i^6)}{\sqrt{\omega_i}\omega_j(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)(\omega_k^2-\omega_i^2)}+\frac{B^a_e\zeta^a_{jk}B^b_e\zeta^b_{ij}\phi_{iij}\phi_{ijk}\sqrt{\omega_k}(3\omega_j^4\omega_k^2-7\omega_i^2\omega_j^2\omega_k^2-24\omega_i^4\omega_k^2-12\omega_i^2\omega_j^4+4\omega_i^4\omega_j^2+96\omega_i^6)}{\sqrt{\omega_i}\omega_j(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)(\omega_k^2-\omega_i^2)}$$

$$-\frac{B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ik}\phi_{iij}\phi_{iik}(3\omega_j^4\omega_k^4-11\omega_i^2\omega_j^2\omega_k^4+8\omega_i^4\omega_k^4-11\omega_i^2\omega_j^4\omega_k^2+3\omega_i^4\omega_j^2\omega_k^2-40\omega_i^6\omega_k^2+8\omega_i^4\omega_j^4-40\omega_i^6\omega_j^2+32\omega_i^8)}{2\omega_j^{\frac{3}{2}}\omega_k^{\frac{3}{2}}(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)(\omega_k^2-\omega_i^2)}+\frac{B^a_e\zeta^a_{ij}B^b_e\zeta^b_{jk}\phi_{iii}\phi_{iik}(5\omega_j^4\omega_k^4-17\omega_i^2\omega_j^2\omega_k^4-28\omega_i^4\omega_k^4-23\omega_i^2\omega_j^4\omega_k^2+67\omega_i^4\omega_j^2\omega_k^2+116\omega_i^6\omega_k^2+8\omega_i^4\omega_j^4-40\omega_i^6\omega_j^2+32\omega_i^8)}{2\omega_i^{\frac{3}{2}}\omega_k^{\frac{3}{2}}(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)(\omega_k^2-\omega_i^2)}+\frac{B^a_e\zeta^a_{jk}B^b_e\zeta^b_{ij}\phi_{iii}\phi_{iik}(5\omega_j^4\omega_k^4-17\omega_i^2\omega_j^2\omega_k^4-28\omega_i^4\omega_k^4-23\omega_i^2\omega_j^4\omega_k^2+67\omega_i^4\omega_j^2\omega_k^2+116\omega_i^6\omega_k^2+8\omega_i^4\omega_j^4-40\omega_i^6\omega_j^2+32\omega_i^8)}{2\omega_i^{\frac{3}{2}}\omega_k^{\frac{3}{2}}(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)(\omega_k^2-4\omega_i^2)(\omega_k^2-\omega_i^2)}$$

$\sigma_{\alpha\beta}(i,j)$

$$-\frac{B^a_e a^{ab}_i B^b_e \phi_{iij}\phi_{iik}\phi_{ijk}(9\omega_j^2\omega_k^8 - 18\omega_i^2\omega_k^8 - 21\omega_j^4\omega_k^6 + 13\omega_i^2\omega_j^2\omega_k^6 + 50\omega_i^4\omega_k^6 + 15\omega_j^6\omega_k^4 - 7\omega_i^2\omega_j^4\omega_k^4 - 27\omega_i^4\omega_j^2\omega_k^4 - 38\omega_i^6\omega_k^4 - 3\omega_j^8\omega_k^2 + 3\omega_i^2\omega_j^6\omega_k^2 + \omega_i^4\omega_j^4\omega_k^2 + 17\omega_i^6\omega_j^2\omega_k^2 + 6\omega_i^8\omega_k^2 + \omega_i^2\omega_j^8 - 6\omega_i^4\omega_j^6 + 9\omega_i^6\omega_j^4 - 4\omega_i^8\omega_j^2)}{\omega_k(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)(\omega_k^2-\omega_i^2)(\omega_k^2-\omega_j^2)\Delta(\omega_k,\omega_j,\omega_i)}$$

$$-\frac{B^a_e\zeta^a_{ij}B^b_e\zeta^b_{ik}\phi_{iik}\phi_{jjj}}{\omega_j^{\frac{3}{2}}\omega_k^{\frac{1}{2}}} - \frac{B^a_e\zeta^a_{ik}B^b_e\zeta^b_{ij}\phi_{iik}\phi_{jjj}}{\omega_j^{\frac{3}{2}}\omega_k^{\frac{3}{2}}} - \frac{B^a_e\zeta^a_{ik}B^b_e\zeta^b_{jk}\phi_{iii}\phi_{jjj}}{\omega_i^{\frac{1}{2}}\omega_j^{\frac{3}{2}}} - \frac{8B^a_e\zeta^a_{ik}B^b_e\zeta^b_{ik}\phi_{ijj}\phi_{ikk}\omega_k}{(\omega_k^2-\omega_i^2)^2} - \frac{8B^a_e\zeta^a_{ik}B^b_e\zeta^b_{ik}\phi_{jjk}\phi_{kkk}\omega_i}{(\omega_k^2-\omega_i^2)^2} - \frac{8B^a_e\zeta^a_{ik}B^b_e\zeta^b_{ik}\phi_{jjj}\phi_{jkk}\omega_i\omega_k}{\omega_j(\omega_k^2-\omega_i^2)^2} - \frac{B^a_e\zeta^a_{ik}B^b_e\zeta^b_{ik}\phi_{iii}\phi_{ijj}(3\omega_k^4-10\omega_i^2\omega_k^2-\omega_i^4)}{\omega_i^3(\omega_k^2-\omega_i^2)^2} - \dots$$

[illegible]

$$+\frac{5B^a_e a^{ac}_i B^b_e a^{bc}_i B^c_e a^{cd}_j B^d_e a^{dc}_k B^c_e}{8}+\frac{5B^a_e a^{ac}_k B^b_e a^{bc}_j B^c_e a^{cd}_i B^d_e a^{dc}_j B^c_e}{8}+\frac{5B^a_e a^{ac}_i B^b_e a^{bc}_k B^c_e a^{cd}_j B^d_e a^{dc}_i B^c_e}{8}$$

$$+\frac{B^a_e a^{ac}_j B^b_e a^{bd}_k B^c_e a^{cd}_i B^d_e \phi_{iii}}{2\omega_i}+\frac{B^a_e a^{ac}_j B^b_e a^{bd}_k B^c_e a^{cd}_i B^d_e \phi_{iii}}{2\omega_i}+\frac{B^a_e a^{ac}_i B^b_e a^{bd}_k B^c_e a^{cd}_j B^d_e \phi_{iii}}{2\omega_i}+\frac{B^a_e a^{ac}_j B^b_e a^{bd}_k B^c_e a^{cd}_i B^d_e \phi_{jjk}}{2\omega_k}+\frac{B^a_e a^{ac}_j B^b_e a^{bd}_k B^c_e a^{cd}_i B^d_e \phi_{jjk}}{2\omega_k}+\frac{B^a_e a^{ac}_k B^b_e a^{bd}_i B^c_e a^{cd}_j B^d_e \phi_{jjk}}{2\omega_k}+\frac{3B^a_e a^{ac}_j B^b_e a^{bd}_k B^c_e a^{cd}_i B^d_e \phi_{ijj}}{2\omega_i}-\frac{2B^a_e a^{ac}_k B^b_e a^{bd}_i B^c_e a^{cd}_j B^d_e \phi_{iij}\omega_i}{(\omega_j^2-4\omega_i^2)}-\frac{2B^a_e a^{ac}_i B^b_e a^{bd}_j B^c_e a^{cd}_k B^d_e \phi_{iij}\omega_i}{(\omega_j^2-4\omega_i^2)}+\frac{2B^a_e a^{ac}_i B^b_e a^{bd}_j B^c_e a^{cd}_k B^d_e \phi_{iij}\omega_j}{(4\omega_j^2-\omega_i^2)}+\frac{B^a_e a^{ac}_j B^b_e a^{bd}_k B^c_e a^{cd}_i B^d_e \phi_{ijk}\omega_k(\omega_k^2-\omega_j^2-\omega_i^2)}{\Delta(\omega_k,\omega_j,\omega_i)}$$

$$+\frac{B^a_e a^{ac}_k B^b_e a^{bd}_i B^c_e a^{cd}_j B^d_e \phi_{ijk}\omega_k(\omega_k^2-\omega_j^2-\omega_i^2)}{\Delta(\omega_k,\omega_j,\omega_i)}+\frac{B^a_e a^{ac}_i B^b_e a^{bd}_k B^c_e a^{cd}_j B^d_e \phi_{ijk}\omega_k(\omega_k^2-\omega_j^2-\omega_i^2)}{\Delta(\omega_k,\omega_j,\omega_i)}$$

$$+\frac{3B^a_e a^{ad}_k B^b_e \zeta^b_{ij} B^c_e a^{cd}_i \zeta^c_{ij} B^d_e(5\omega_j^2-\omega_i^2)}{\omega_j(\omega_j^2-\omega_i^2)}-\frac{3B^a_e a^{ad}_k B^b_e \zeta^b_{jk} B^c_e a^{cd}_i \zeta^c_{jk} B^d_e(\omega_k^2+3\omega_j^2)}{\omega_j(\omega_k^2-\omega_j^2)}+\frac{3B^a_e \zeta^a_{ij} B^b_e a^{bd}_k B^c_e a^{cd}_i \zeta^c_{ij} B^d_e(5\omega_j^2-\omega_i^2)}{\omega_j(\omega_j^2-\omega_i^2)}-\frac{3B^a_e \zeta^a_{jk} B^b_e a^{bd}_k B^c_e a^{cd}_i \zeta^c_{jk} B^d_e(\omega_k^2+3\omega_j^2)}{\omega_i(\omega_k^2-\omega_j^2)}+\frac{6B^a_e a^{ad}_k B^b_e \zeta^b_{jk} B^c_e a^{cd}_i \zeta^c_{ij} B^d_e \omega_j\sqrt{\omega_k}}{\sqrt{\omega_i}(\omega_k^2-\omega_j^2)}-\frac{6B^a_e a^{ad}_k B^b_e \zeta^b_{ik} B^c_e a^{cd}_j \zeta^c_{ij} B^d_e \omega_i\sqrt{\omega_k}}{\sqrt{\omega_j}(\omega_k^2-\omega_i^2)}-\frac{6B^a_e \zeta^a_{ik} B^b_e a^{bd}_k B^c_e a^{cd}_j \zeta^c_{ij} B^d_e \omega_i\sqrt{\omega_k}}{\sqrt{\omega_j}(\omega_k^2-\omega_i^2)}-\frac{6B^a_e \zeta^a_{jk} B^b_e a^{bd}_k B^c_e a^{cd}_i \zeta^c_{ij} B^d_e \omega_j\sqrt{\omega_k}}{\sqrt{\omega_i}(\omega_k^2-\omega_j^2)}-\frac{3B^a_e a^{ad}_k B^b_e \zeta^b_{ik} B^c_e a^{cd}_j \zeta^c_{jk} B^d_e(\omega_k^2+\omega_i^2)}{\sqrt{\omega_i}\sqrt{\omega_j}(\omega_k^2-\omega_i^2)}$$

$$-\frac{3B^a_e a^{ad}_k B^b_e \zeta^b_{jk} B^c_e a^{cd}_j \zeta^c_{ik} B^d_e(\omega_k^2+\omega_j^2)}{\sqrt{\omega_i}\sqrt{\omega_j}(\omega_k^2-\omega_j^2)}-\frac{3B^a_e \zeta^a_{jk} B^b_e a^{bd}_k B^c_e a^{cd}_j \zeta^c_{ik} B^d_e(\omega_k^2+\omega_j^2)}{\sqrt{\omega_i}\sqrt{\omega_j}(\omega_k^2-\omega_j^2)}-\frac{3B^a_e \zeta^a_{ik} B^b_e a^{bd}_k B^c_e a^{cd}_j \zeta^c_{jk} B^d_e(\omega_k^2+\omega_i^2)}{\sqrt{\omega_i}\sqrt{\omega_j}(\omega_k^2-\omega_i^2)}+\frac{3B^a_e a^{ad}_k B^b_e \zeta^b_{ij} B^c_e a^{cd}_i \zeta^c_{jk} B^d_e(\omega_j^2+\omega_i^2)\sqrt{\omega_k}}{\sqrt{\omega_i}\omega_j(\omega_j^2-\omega_i^2)}+\frac{3B^a_e a^{ad}_k B^b_e \zeta^b_{ij} B^c_e a^{cd}_i \zeta^c_{jk} B^d_e(\omega_j^2+\omega_i^2)\sqrt{\omega_k}}{\sqrt{\omega_i}\omega_j(\omega_j^2-\omega_i^2)}+\frac{3B^a_e \zeta^a_{ij} B^b_e a^{bd}_k B^c_e a^{cd}_i \zeta^c_{jk} B^d_e(\omega_j^2+\omega_i^2)\sqrt{\omega_k}}{\sqrt{\omega_i}\omega_j(\omega_j^2-\omega_i^2)}+\frac{3B^a_e \zeta^a_{ij} B^b_e a^{bd}_k B^c_e a^{cd}_i \zeta^c_{jk} B^d_e(\omega_j^2+\omega_i^2)\sqrt{\omega_k}}{\sqrt{\omega_i}\omega_j(\omega_j^2-\omega_i^2)}$$

$$+\frac{B^a_e a^{ab}_i B^b_e \phi_{iijj}}{2\omega_i}+\frac{B^a_e a^{ab}_k B^b_e \phi_{iijjk}}{4\omega_k}-\frac{6B^a_e a^{ab}_i B^b_e B^c_e a^{cd}_i \zeta^c_{ij} B^d_e \zeta^d_{ij}(\omega_j^2+\omega_i^2)}{\omega_i^2\omega_j}-\frac{2B^a_e a^{ab}_k B^b_e B^c_e a^{cd}_i \zeta^c_{ij} B^d_e \zeta^d_{ij}(\omega_j^2+\omega_i^2)}{2\omega_i\omega_j\omega_k}+\frac{4B^a_e a^{ab}_i B^b_e B^c_e a^{cd}_k \zeta^c_{ij} B^d_e \zeta^d_{jk}\sqrt{\omega_k}}{\sqrt{\omega_i}\omega_j}+\frac{4B^a_e a^{ab}_k B^b_e B^c_e a^{cd}_i \zeta^c_{jk} B^d_e \zeta^d_{ij}\omega_j}{\sqrt{\omega_i}\omega_k^{\frac{3}{2}}}$$

$$-\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_i B^c_e \phi_{iijj}}{8\omega_i}+\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_j B^c_e \phi_{iij}\omega_i}{4(\omega_j^2-\omega_i^2)}-\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_j B^c_e \phi_{ijj}\omega_j}{4(\omega_j^2-\omega_i^2)}-\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_i B^c_e B^d_e(\zeta^d_{ij})^2(\omega_j^2-\omega_i^2)}{2\omega_i^2\omega_j}-\frac{3B^a_e a^{ac}_k B^b_e a^{bc}_i B^c_e \phi_{ijk}\omega_k}{4(\omega_k^2-\omega_i^2)}-\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_k B^c_e \phi_{ijk}\omega_k}{4(\omega_k^2-\omega_i^2)}+\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_k B^c_e B^d_e \zeta^d_{ij}\zeta^d_{jk}(\omega_j^2+\omega_i^2)\sqrt{\omega_k}}{\sqrt{\omega_i}\omega_j(\omega_j^2-\omega_i^2)}+\frac{3B^a_e a^{ac}_k B^b_e a^{bc}_i B^c_e B^d_e \zeta^d_{ij}\zeta^d_{jk}(\omega_j^2+\omega_i^2)\sqrt{\omega_k}}{\sqrt{\omega_i}\omega_j(\omega_j^2-\omega_i^2)}$$

$$-\frac{4B^a_e a^{ac}_k B^b_e a^{bd}_k B^c_e \zeta^c_{ik} B^d_e \zeta^d_{jk}}{\sqrt{\omega_i}\sqrt{\omega_j}}-\frac{4B^a_e a^{ac}_i B^b_e a^{bd}_k B^c_e \zeta^c_{ij} B^d_e \zeta^d_{jk}\sqrt{\omega_k}}{\sqrt{\omega_i}\omega_j}+\frac{4B^a_e a^{ac}_k B^b_e a^{bd}_i B^c_e \zeta^c_{jk} B^d_e \zeta^d_{ij}\sqrt{\omega_k}}{\sqrt{\omega_i}\omega_j}+\frac{16B^a_e a^{ac}_i B^b_e a^{bd}_i B^c_e \zeta^c_{ij} B^d_e \zeta^d_{ij}(2\omega_j^2+\omega_i^2)}{\omega_j(\omega_j^2-4\omega_i^2)}-\frac{4B^a_e a^{ac}_i B^b_e a^{bd}_i B^c_e \zeta^c_{jk} B^d_e \zeta^d_{jk}(\omega_k^2-\omega_j^2)(\omega_k^2+3\omega_j^2-\omega_i^2)}{\omega_j\Delta(\omega_k,\omega_j,\omega_i)}-\frac{4B^a_e a^{ac}_i B^b_e a^{bd}_j B^c_e \zeta^c_{ik} B^d_e \zeta^d_{jk}(\omega_k^4-\omega_j^4+4\omega_i^2\omega_j^2-\omega_i^4)}{\sqrt{\omega_i}\sqrt{\omega_j}\Delta(\omega_k,\omega_j,\omega_i)}-\frac{2B^a_e a^{ac}_k B^b_e a^{bd}_k B^c_e \zeta^c_{ij} B^d_e \zeta^d_{ij}\omega_k(\omega_j^2\omega_k^2+\omega_i^2\omega_k^2-\omega_j^4-6\omega_i^2\omega_j^2-\omega_i^4)}{\omega_i\omega_j\Delta(\omega_k,\omega_j,\omega_i)}$$

$$+\frac{4B^a_e a^{ac}_k B^b_e a^{bd}_k B^c_e \zeta^c_{jk} B^d_e \zeta^d_{ij}\sqrt{\omega_k}(2\omega_j^2\omega_k^2-\omega_i^2\omega_k^2-2\omega_j^4-3\omega_i^2\omega_j^2+\omega_i^4)}{\sqrt{\omega_i}\omega_j\Delta(\omega_k,\omega_j,\omega_i)}+\frac{4B^a_e a^{ac}_k B^b_e a^{bd}_k B^c_e \zeta^c_{ij} B^d_e \zeta^d_{jk}\sqrt{\omega_k}(2\omega_j^2\omega_k^2-\omega_i^2\omega_k^2-2\omega_j^4-3\omega_i^2\omega_j^2+\omega_i^4)}{\sqrt{\omega_i}\omega_j\Delta(\omega_k,\omega_j,\omega_i)}$$

$$+\frac{3B^a_e a^{ac}_k B^b_e a^{bc}_k B^c_e \phi_{iik}\phi_{jjk}}{8\omega_k^2}+\frac{9B^a_e a^{ac}_i B^b_e a^{bc}_i B^c_e \phi_{iii}\phi_{ijj}}{8\omega_i^2}+\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_i B^c_e \phi_{iij}\phi_{jjj}}{8\omega_i\omega_j}+\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_j B^c_e \phi_{iii}\phi_{jjj}}{8\omega_i\omega_j}+\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_k B^c_e \phi_{iik}\phi_{jjk}}{8\omega_i\omega_k}+\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_k B^c_e \phi_{iii}\phi_{jjk}}{8\omega_i\omega_k}+\frac{3B^a_e a^{ac}_k B^b_e a^{bc}_i B^c_e \phi_{iii}\phi_{jjk}}{8\omega_i\omega_k}+\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_k B^c_e \phi_{ikk}\phi_{jjk}}{4(\omega_k^2-\omega_i^2)}+\frac{3B^a_e a^{ac}_k B^b_e a^{bc}_i B^c_e \phi_{ikk}\phi_{jjk}}{4(\omega_k^2-\omega_i^2)}+\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_k B^c_e \phi_{ijk}\phi_{jjj}\omega_k}{4\omega_j(\omega_k^2-\omega_i^2)}+\frac{3B^a_e a^{ac}_k B^b_e a^{bc}_i B^c_e \phi_{ijk}\phi_{jjj}\omega_k}{4\omega_j(\omega_k^2-\omega_i^2)}-\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_i B^c_e \phi_{iij}^2(\omega_j^2-12\omega_i^2)}{4(\omega_j^2-4\omega_i^2)^2}$$

$$+\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_k B^c_e \phi_{iik}\phi_{ijj}(3\omega_k^2-\omega_i^2)}{8\omega_i\omega_k(\omega_k^2-\omega_i^2)}+\frac{3B^a_e a^{ac}_k B^b_e a^{bc}_i B^c_e \phi_{iik}\phi_{ijj}(3\omega_k^2-\omega_i^2)}{8\omega_i\omega_k(\omega_k^2-\omega_i^2)}+\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_i B^c_e \phi_{ijj}^2\omega_j(4\omega_j^2-3\omega_i^2)}{4\omega_i(4\omega_j^2-\omega_i^2)^2}+\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_k B^c_e \phi_{ijj}\phi_{jjk}\omega_j\omega_k}{2(4\omega_j^2-\omega_i^2)(\omega_k^2-\omega_i^2)}+\frac{3B^a_e a^{ac}_k B^b_e a^{bc}_i B^c_e \phi_{ijj}\phi_{jjk}\omega_j\omega_k}{2(4\omega_j^2-\omega_i^2)(\omega_k^2-\omega_i^2)}-\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_j B^c_e \phi_{iii}\phi_{iij}(\omega_j^2-6\omega_i^2)}{4(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)}+\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_j B^c_e \phi_{ijj}\phi_{jjj}(6\omega_j^2-\omega_i^2)}{4(\omega_j^2-\omega_i^2)(4\omega_j^2-\omega_i^2)}+\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_j B^c_e \phi_{iij}\phi_{ijj}(12\omega_j^4-91\omega_i^2\omega_j^2+12\omega_i^4)}{8\omega_i\omega_j(\omega_j^2-4\omega_i^2)(4\omega_j^2-\omega_i^2)}+\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_k B^c_e \phi_{ijk}\phi_{jkk}\omega_k^2(\omega_k^2-\omega_j^2-\omega_i^2)}{2(\omega_k^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}$$

$$+\frac{3B^a_e a^{ac}_k B^b_e a^{bc}_i B^c_e \phi_{ijk}\phi_{jkk}\omega_k^2(\omega_k^2-\omega_j^2-\omega_i^2)}{2(\omega_k^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}+\frac{3B^a_e a^{ac}_k B^b_e a^{bc}_k B^c_e \phi_{ijk}^2\omega_k^2(\omega_k^4-2\omega_j^2\omega_k^2-2\omega_i^2\omega_k^2+\omega_j^4+6\omega_i^2\omega_j^2+\omega_i^4)}{4\Delta(\omega_k,\omega_j,\omega_i)^2}-\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_j B^c_e \phi_{iik}\phi_{ijk}\omega_i(3\omega_k^4-4\omega_j^2\omega_k^2-4\omega_i^2\omega_k^2+\omega_j^4-2\omega_i^2\omega_j^2+\omega_i^4)}{4\omega_k(\omega_j^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}+\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_j B^c_e \phi_{ijk}\phi_{jjk}\omega_j(3\omega_k^4-4\omega_j^2\omega_k^2-4\omega_i^2\omega_k^2+\omega_j^4-2\omega_i^2\omega_j^2+\omega_i^4)}{4\omega_k(\omega_j^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}-\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_k B^c_e \phi_{iij}\phi_{ijk}\omega_i\omega_k(2\omega_k^4-3\omega_j^2\omega_k^2-8\omega_i^2\omega_k^2+\omega_j^4-\omega_i^2\omega_j^2+6\omega_i^4)}{2(\omega_j^2-4\omega_i^2)(\omega_k^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}$$

$$-\frac{3B^a_e a^{ac}_k B^b_e a^{bc}_i B^c_e \phi_{iij}\phi_{ijk}\omega_i\omega_k(2\omega_k^4-3\omega_j^2\omega_k^2-8\omega_i^2\omega_k^2+\omega_j^4-\omega_i^2\omega_j^2+6\omega_i^4)}{2(\omega_j^2-4\omega_i^2)(\omega_k^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}+\frac{3B^a_e a^{ac}_i B^b_e a^{bc}_i B^c_e \phi_{ijk}^2\omega_k(\omega_k^6-3\omega_j^2\omega_k^4-5\omega_i^2\omega_k^4+3\omega_j^4\omega_k^2-2\omega_i^2\omega_j^2\omega_k^2+7\omega_i^4\omega_k^2-\omega_j^6+7\omega_i^2\omega_j^4-3\omega_i^4\omega_j^2-3\omega_i^6)}{4\omega_i\Delta(\omega_k,\omega_j,\omega_i)^2}$$

$$+\frac{2B^a_e a^{ac}_i B^b_e \zeta^b_{ij} B^c_e \zeta^c_{ij}\phi_{iii}(5\omega_j^4-29\omega_i^2\omega_j^2-12\omega_i^4)}{\omega_i\omega_j(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)}+\frac{2B^a_e \zeta^a_{ij} B^b_e a^{bc}_i B^c_e \zeta^c_{ij}\phi_{iii}(5\omega_j^4-29\omega_i^2\omega_j^2-12\omega_i^4)}{\omega_i\omega_j(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)}-\frac{6B^a_e a^{ac}_i B^b_e \zeta^b_{ij} B^c_e \zeta^c_{ij}\phi_{ijj}(4\omega_j^6-17\omega_i^2\omega_j^4-59\omega_i^4\omega_j^2+12\omega_i^6)}{\omega_i^2(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)(4\omega_j^2-\omega_i^2)}-\frac{6B^a_e \zeta^a_{ij} B^b_e a^{bc}_i B^c_e \zeta^c_{ij}\phi_{ijj}(4\omega_j^6-17\omega_i^2\omega_j^4-59\omega_i^4\omega_j^2+12\omega_i^6)}{\omega_i^2(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)(4\omega_j^2-\omega_i^2)}+\frac{2B^a_e a^{ac}_i B^b_e \zeta^b_{jk} B^c_e \zeta^c_{ij}\phi_{jjk}}{\sqrt{\omega_i}\omega_k^{\frac{3}{2}}}-\frac{2B^a_e a^{ac}_i B^b_e \zeta^b_{jk} B^c_e \zeta^c_{ik}\phi_{jjj}}{\sqrt{\omega_i}\omega_j^{\frac{3}{2}}}-\frac{2B^a_e \zeta^a_{ik} B^b_e a^{bc}_j B^c_e \zeta^c_{ij}\phi_{iik}}{\sqrt{\omega_j}\omega_k^{\frac{3}{2}}}-\frac{2B^a_e \zeta^a_{ik} B^b_e a^{bc}_j B^c_e \zeta^c_{jk}\phi_{iii}}{\omega_i^{\frac{3}{2}}\sqrt{\omega_j}}+\frac{4B^a_e a^{ac}_k B^b_e \zeta^b_{ij} B^c_e \zeta^c_{ik}\phi_{iij}\sqrt{\omega_k}}{\sqrt{\omega_j}(\omega_j^2-\omega_i^2)}$$

$$+\frac{4B^a_e \zeta^a_{ij} B^b_e a^{bc}_k B^c_e \zeta^c_{ik}\phi_{iij}\sqrt{\omega_k}}{\sqrt{\omega_j}(\omega_j^2-\omega_i^2)}-\frac{2B^a_e a^{ac}_i B^b_e \zeta^b_{ik} B^c_e \zeta^c_{ik}\phi_{ijj}(3\omega_k^2+5\omega_i^2)}{\omega_i^2(\omega_k^2-\omega_i^2)}-\frac{2B^a_e \zeta^a_{ik} B^b_e a^{bc}_i B^c_e \zeta^c_{ik}\phi_{ijj}(3\omega_k^2+5\omega_i^2)}{\omega_i^2(\omega_k^2-\omega_i^2)}-\frac{4B^a_e a^{ac}_k B^b_e \zeta^b_{ij} B^c_e \zeta^c_{ik}\sqrt{\omega_j}\phi_{jjj}\sqrt{\omega_k}}{\omega_i(\omega_j^2-\omega_i^2)}-\frac{4B^a_e \zeta^a_{ij} B^b_e a^{bc}_k B^c_e \zeta^c_{ik}\sqrt{\omega_j}\phi_{jjj}\sqrt{\omega_k}}{\omega_i(\omega_j^2-\omega_i^2)}-\frac{4B^a_e a^{ac}_k B^b_e \zeta^b_{ik} B^c_e \zeta^c_{ij}\phi_{jjj}\omega_i\sqrt{\omega_k}}{\omega_j^{\frac{3}{2}}(\omega_k^2-\omega_i^2)}-\frac{4B^a_e \zeta^a_{ik} B^b_e a^{bc}_k B^c_e \zeta^c_{ij}\phi_{jjj}\omega_i\sqrt{\omega_k}}{\omega_j^{\frac{3}{2}}(\omega_k^2-\omega_i^2)}-\frac{2B^a_e a^{ac}_i B^b_e \zeta^b_{jk} B^c_e \zeta^c_{jk}\phi_{iii}(\omega_k^2+3\omega_j^2)}{\omega_i\omega_j(\omega_k^2-\omega_j^2)}-\frac{2B^a_e \zeta^a_{ik} B^b_e a^{bc}_j B^c_e \zeta^c_{ik}\phi_{jjj}(\omega_k^2+3\omega_i^2)}{\omega_i\omega_j(\omega_k^2-\omega_i^2)}-\frac{2B^a_e a^{ac}_k B^b_e \zeta^b_{ik} B^c_e \zeta^c_{ik}\phi_{jjk}(3\omega_k^2+5\omega_i^2)}{\omega_i\omega_k(\omega_k^2-\omega_i^2)}$$

$$-\frac{2B^a_e \zeta^a_{ik} B^b_e a^{bc}_k B^c_e \zeta^c_{ik}\phi_{jjk}(3\omega_k^2+5\omega_i^2)}{\omega_i\omega_k(\omega_k^2-\omega_i^2)}-\frac{2B^a_e a^{ac}_i B^b_e \zeta^b_{ik} B^c_e \zeta^c_{jk}\phi_{jjj}(\omega_k^2+\omega_i^2)}{\sqrt{\omega_i}\omega_j^{\frac{3}{2}}(\omega_k^2-\omega_i^2)}-\frac{2B^a_e \zeta^a_{ik} B^b_e a^{bc}_i B^c_e \zeta^c_{jk}\phi_{jjj}(\omega_k^2+\omega_i^2)}{\sqrt{\omega_i}\omega_j^{\frac{3}{2}}(\omega_k^2-\omega_i^2)}-\frac{16B^a_e a^{ac}_i B^b_e \zeta^b_{jk} B^c_e \zeta^c_{jk}\phi_{ikk}\omega_j\omega_k}{\Delta(\omega_k,\omega_j,\omega_i)}-\frac{16B^a_e \zeta^a_{ik} B^b_e a^{bc}_j B^c_e \zeta^c_{ik}\phi_{jkk}\omega_i\omega_k}{\Delta(\omega_k,\omega_j,\omega_i)}-\frac{4B^a_e a^{ac}_i B^b_e \zeta^b_{jk} B^c_e \zeta^c_{ij}\sqrt{\omega_i}\phi_{iik}(5\omega_j^2+4\omega_i^2)\sqrt{\omega_k}}{\omega_j(\omega_j^2-4\omega_i^2)(\omega_k^2-\omega_j^2)}-\frac{4B^a_e \zeta^a_{ik} B^b_e a^{bc}_j B^c_e \zeta^c_{ij}\sqrt{\omega_j}\phi_{jjk}(4\omega_j^2+5\omega_i^2)\sqrt{\omega_k}}{\omega_i(4\omega_j^2-\omega_i^2)(\omega_k^2-\omega_i^2)}+\frac{4B^a_e a^{ac}_i B^b_e \zeta^b_{jk} B^c_e \zeta^c_{ik}\sqrt{\omega_i}\phi_{iij}(2\omega_k^2+3\omega_j^2+4\omega_i^2)}{\sqrt{\omega_j}(\omega_j^2-4\omega_i^2)(\omega_k^2-\omega_j^2)}-\frac{4B^a_e \zeta^a_{ik} B^b_e a^{bc}_j B^c_e \zeta^c_{jk}\phi_{ijj}\sqrt{\omega_j}(2\omega_k^2+4\omega_j^2+3\omega_i^2)}{\sqrt{\omega_i}(4\omega_j^2-\omega_i^2)(\omega_k^2-\omega_i^2)}$$

$$-\frac{8B^a_e a^{ac}_i B^b_e \zeta^b_{ik} B^c_e \zeta^c_{jk}\omega_i^{\frac{3}{2}}\phi_{jkk}\omega_k(\omega_k^2+3\omega_j^2-\omega_i^2)}{\sqrt{\omega_j}(\omega_k^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}-\frac{8B^a_e a^{ac}_k B^b_e \zeta^b_{ik} B^c_e \zeta^c_{ij}\phi_{jkk}\omega_i\omega_k^{\frac{3}{2}}(\omega_k^2-3\omega_j^2-\omega_i^2)}{\sqrt{\omega_j}(\omega_k^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}-\frac{8B^a_e \zeta^a_{ik} B^b_e a^{bc}_i B^c_e \zeta^c_{jk}\omega_i^{\frac{3}{2}}\phi_{jkk}\omega_k(\omega_k^2+3\omega_j^2-\omega_i^2)}{\sqrt{\omega_j}(\omega_k^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}-\frac{8B^a_e \zeta^a_{ik} B^b_e a^{bc}_k B^c_e \zeta^c_{ij}\phi_{jkk}\omega_i\omega_k^{\frac{3}{2}}(\omega_k^2-3\omega_j^2-\omega_i^2)}{\sqrt{\omega_j}(\omega_k^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}-\frac{8B^a_e a^{ac}_i B^b_e \zeta^b_{jk} B^c_e \zeta^c_{jk}\phi_{ijj}(\omega_k^4-6\omega_j^2\omega_k^2-\omega_i^2\omega_k^2-3\omega_j^4+3\omega_i^2\omega_j^2)}{(4\omega_j^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}+\frac{8B^a_e \zeta^a_{ik} B^b_e a^{bc}_j B^c_e \zeta^c_{ik}\phi_{iij}(\omega_k^4-\omega_j^2\omega_k^2-6\omega_i^2\omega_k^2+3\omega_i^2\omega_j^2-3\omega_i^4)}{(\omega_j^2-4\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}$$

$$+\frac{4B^a_e a^{ac}_i B^b_e \zeta^b_{ij} B^c_e \zeta^c_{ik}\phi_{ijk}\sqrt{\omega_k}(5\omega_j^2\omega_k^2+\omega_i^2\omega_k^2-5\omega_j^4+6\omega_i^2\omega_j^2-\omega_i^4)}{\sqrt{\omega_j}(\omega_j^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}+\frac{4B^a_e \zeta^a_{ij} B^b_e a^{bc}_i B^c_e \zeta^c_{ik}\phi_{ijk}\sqrt{\omega_k}(5\omega_j^2\omega_k^2+\omega_i^2\omega_k^2-5\omega_j^4+6\omega_i^2\omega_j^2-\omega_i^4)}{\sqrt{\omega_j}(\omega_j^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}-\frac{4B^a_e a^{ac}_k B^b_e \zeta^b_{ik} B^c_e \zeta^c_{jk}\phi_{ijk}\omega_k(\omega_k^4-\omega_j^2\omega_k^2-2\omega_i^2\omega_k^2-11\omega_i^2\omega_j^2+\omega_i^4)}{\sqrt{\omega_i}\sqrt{\omega_j}(\omega_k^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}-\frac{4B^a_e \zeta^a_{ik} B^b_e a^{bc}_k B^c_e \zeta^c_{jk}\phi_{ijk}\omega_k(\omega_k^4-\omega_j^2\omega_k^2-2\omega_i^2\omega_k^2-11\omega_i^2\omega_j^2+\omega_i^4)}{\sqrt{\omega_i}\sqrt{\omega_j}(\omega_k^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}-\frac{4B^a_e a^{ac}_i B^b_e \zeta^b_{ij} B^c_e \zeta^c_{jk}\sqrt{\omega_i}\phi_{iik}\omega_j(5\omega_k^4-6\omega_j^2\omega_k^2-2\omega_i^2\omega_k^2+\omega_j^4-2\omega_i^2\omega_j^2+\omega_i^4)}{\omega_k^{\frac{3}{2}}(\omega_j^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}$$

$$-\frac{4B^a_e \zeta^a_{ij} B^b_e a^{bc}_i B^c_e \zeta^c_{jk}\sqrt{\omega_i}\phi_{iik}\omega_j(5\omega_k^4-6\omega_j^2\omega_k^2-2\omega_i^2\omega_k^2+\omega_j^4-2\omega_i^2\omega_j^2+\omega_i^4)}{\omega_k^{\frac{3}{2}}(\omega_j^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}+\frac{2B^a_e a^{ac}_k B^b_e \zeta^b_{ij} B^c_e \zeta^c_{ij}\phi_{iik}(7\omega_j^2\omega_k^4+\omega_i^2\omega_k^4-10\omega_j^4\omega_k^2-20\omega_i^2\omega_j^2\omega_k^2-2\omega_i^4\omega_k^2+3\omega_j^6-5\omega_i^2\omega_j^4+\omega_i^4\omega_j^2+\omega_i^6)}{\omega_j\omega_k(\omega_j^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}+\frac{2B^a_e \zeta^a_{ij} B^b_e a^{bc}_k B^c_e \zeta^c_{ij}\phi_{iik}(7\omega_j^2\omega_k^4+\omega_i^2\omega_k^4-10\omega_j^4\omega_k^2-20\omega_i^2\omega_j^2\omega_k^2-2\omega_i^4\omega_k^2+3\omega_j^6-5\omega_i^2\omega_j^4+\omega_i^4\omega_j^2+\omega_i^6)}{\omega_j\omega_k(\omega_j^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}$$

$$+\frac{2B^a_e a^{ac}_i B^b_e \zeta^b_{ij} B^c_e \zeta^c_{jk}\phi_{jjk}(\omega_j^2\omega_k^4-3\omega_i^2\omega_k^4-2\omega_j^4\omega_k^2-16\omega_i^2\omega_j^2\omega_k^2+2\omega_i^4\omega_k^2+\omega_j^6-\omega_i^2\omega_j^4-\omega_i^4\omega_j^2+\omega_i^6)}{\sqrt{\omega_i}\omega_k^{\frac{3}{2}}(\omega_j^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}+\frac{2B^a_e \zeta^a_{ij} B^b_e a^{bc}_i B^c_e \zeta^c_{jk}\phi_{jjk}(\omega_j^2\omega_k^4-3\omega_i^2\omega_k^4-2\omega_j^4\omega_k^2-16\omega_i^2\omega_j^2\omega_k^2+2\omega_i^4\omega_k^2+\omega_j^6-\omega_i^2\omega_j^4-\omega_i^4\omega_j^2+\omega_i^6)}{\sqrt{\omega_i}\omega_k^{\frac{3}{2}}(\omega_j^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}-\frac{8B^a_e a^{ac}_k B^b_e \zeta^b_{ik} B^c_e \zeta^c_{ij}\phi_{iij}\sqrt{\omega_k}(\omega_j^2\omega_k^4+2\omega_i^2\omega_k^4-\omega_j^4\omega_k^2-11\omega_i^2\omega_j^2\omega_k^2+4\omega_i^2\omega_j^4-2\omega_i^4\omega_j^2-2\omega_i^6)}{\sqrt{\omega_j}(\omega_j^2-4\omega_i^2)(\omega_k^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}$$

$$-\frac{8B^a_e \zeta^a_{ik} B^b_e a^{bc}_k B^c_e \zeta^c_{ij}\phi_{iij}\sqrt{\omega_k}(\omega_j^2\omega_k^4+2\omega_i^2\omega_k^4-\omega_j^4\omega_k^2-11\omega_i^2\omega_j^2\omega_k^2+4\omega_i^2\omega_j^4-2\omega_i^4\omega_j^2-2\omega_i^6)}{\sqrt{\omega_j}(\omega_j^2-4\omega_i^2)(\omega_k^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}+\frac{8B^a_e a^{ac}_i B^b_e \zeta^b_{ik} B^c_e \zeta^c_{jk}\sqrt{\omega_i}\phi_{iij}(\omega_k^6-5\omega_i^2\omega_k^4-\omega_j^4\omega_k^2-7\omega_i^2\omega_j^2\omega_k^2+3\omega_i^4\omega_k^2+4\omega_i^2\omega_j^4-5\omega_i^4\omega_j^2+\omega_i^6)}{\sqrt{\omega_j}(\omega_j^2-4\omega_i^2)(\omega_k^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}+\frac{8B^a_e \zeta^a_{ik} B^b_e a^{bc}_i B^c_e \zeta^c_{jk}\sqrt{\omega_i}\phi_{iij}(\omega_k^6-5\omega_i^2\omega_k^4-\omega_j^4\omega_k^2-7\omega_i^2\omega_j^2\omega_k^2+3\omega_i^4\omega_k^2+4\omega_i^2\omega_j^4-5\omega_i^4\omega_j^2+\omega_i^6)}{\sqrt{\omega_j}(\omega_j^2-4\omega_i^2)(\omega_k^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}$$

$$+\frac{4B^a_e a^{ac}_i B^b_e \zeta^b_{ik} B^c_e \zeta^c_{ij}\phi_{ijk}\sqrt{\omega_k}(5\omega_j^2\omega_k^4+4\omega_i^2\omega_k^4-5\omega_j^4\omega_k^2-38\omega_i^2\omega_j^2\omega_k^2-8\omega_i^4\omega_k^2+17\omega_i^2\omega_j^4-15\omega_i^4\omega_j^2+4\omega_i^6)}{\sqrt{\omega_j}(\omega_j^2-4\omega_i^2)(\omega_k^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}+\frac{4B^a_e \zeta^a_{ik} B^b_e a^{bc}_i B^c_e \zeta^c_{ij}\phi_{ijk}\sqrt{\omega_k}(5\omega_j^2\omega_k^4+4\omega_i^2\omega_k^4-5\omega_j^4\omega_k^2-38\omega_i^2\omega_j^2\omega_k^2-8\omega_i^4\omega_k^2+17\omega_i^2\omega_j^4-15\omega_i^4\omega_j^2+4\omega_i^6)}{\sqrt{\omega_j}(\omega_j^2-4\omega_i^2)(\omega_k^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}$$

$$-\frac{B^a_e a^{ab}_i B^b_e \phi_{iii}\phi_{ijj}}{2\omega_i^2}-\frac{B^a_e a^{ab}_k B^b_e \phi_{ikk}\phi_{jjk}}{2\omega_k^2}-\frac{3B^a_e a^{ab}_i B^b_e \phi_{iii}\phi_{ijj}}{4\omega_i^2}-\frac{B^a_e a^{ab}_i B^b_e \phi_{iij}\phi_{jjj}}{2\omega_i\omega_j}-\frac{B^a_e a^{ab}_i B^b_e \phi_{iik}\phi_{jjk}}{2\omega_i\omega_k}-\frac{B^a_e a^{ab}_k B^b_e \phi_{iik}\phi_{ijj}}{2\omega_i\omega_k}-\frac{B^a_e a^{ab}_k B^b_e \phi_{iii}\phi_{ijk}}{2\omega_i\omega_k}-\frac{B^a_e a^{ab}_k B^b_e \phi_{iijj}\phi_{iik}}{4\omega_i\omega_k}-\frac{2B^a_e a^{ab}_k B^b_e B^c_e(\zeta^c_{ik})^2\phi_{jjk}\omega_i}{\omega_k^3}-\frac{B^a_e a^{ab}_k B^b_e \phi_{iijk}\phi_{jkk}}{(\omega_k^2-\omega_j^2)}-\frac{B^a_e a^{ab}_i B^b_e B^c_e(\zeta^c_{ij})^2\phi_{iii}(3\omega_j^2-\omega_i^2)}{\omega_i^3\omega_j}-\frac{2B^a_e a^{ab}_k B^b_e B^c_e \zeta^c_{ij}\zeta^c_{ik}\phi_{jjj}\omega_i}{\omega_j^{\frac{3}{2}}\omega_k^{\frac{3}{2}}}+\frac{B^a_e a^{ab}_k B^b_e \phi_{iiij}\phi_{ijk}\omega_i}{\omega_k(\omega_j^2-\omega_i^2)}+\frac{2B^a_e a^{ab}_k B^b_e \phi_{iij}\phi_{iijk}\omega_i}{\omega_k(\omega_j^2-4\omega_i^2)}-\frac{B^a_e a^{ab}_k B^b_e B^c_e(\zeta^c_{ij})^2\phi_{iik}(\omega_j^2-\omega_i^2)}{\omega_i^2\omega_j\omega_k}$$

$$-\frac{B^a_e a^{ab}_i B^b_e \phi_{iij}\phi_{ijjj}(3\omega_j^2-\omega_i^2)}{2\omega_i\omega_j(\omega_j^2-\omega_i^2)}-\frac{B^a_e a^{ab}_i B^b_e \phi_{iik}\phi_{ijjk}(3\omega_k^2-\omega_i^2)}{2\omega_i\omega_k(\omega_k^2-\omega_i^2)}-\frac{B^a_e a^{ab}_i B^b_e \phi_{iijj}\phi_{ijj}(12\omega_j^2-\omega_i^2)}{4\omega_i\omega_j(4\omega_j^2-\omega_i^2)}+\frac{3B^a_e a^{ab}_i B^b_e \phi_{iiij}\phi_{iij}(\omega_j^2-2\omega_i^2)}{(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)}-\frac{4B^a_e a^{ab}_k B^b_e B^c_e \zeta^c_{ij}\zeta^c_{ik}\phi_{iij}(\omega_j^2+2\omega_i^2)}{\sqrt{\omega_j}\omega_k^{\frac{3}{2}}(\omega_j^2-4\omega_i^2)}-\frac{4B^a_e a^{ab}_k B^b_e B^c_e \zeta^c_{ij}\zeta^c_{ik}\phi_{jkk}(\omega_j^2+\omega_i^2)}{\omega_i\sqrt{\omega_j}\sqrt{\omega_k}(\omega_k^2-\omega_j^2)}+\frac{B^a_e a^{ab}_i B^b_e B^c_e(\zeta^c_{ij})^2\phi_{ijj}(12\omega_j^4-\omega_i^2\omega_j^2+\omega_i^4)}{\omega_i^2\omega_j^3(4\omega_j^2-\omega_i^2)}-\frac{B^a_e a^{ab}_k B^b_e \phi_{ijk}\phi_{ijkk}(\omega_k^2-\omega_j^2-\omega_i^2)}{\Delta(\omega_k,\omega_j,\omega_i)}+\frac{2B^a_e a^{ab}_i B^b_e B^c_e \zeta^c_{ij}\zeta^c_{jk}\phi_{iik}(3\omega_j^2\omega_k^2+2\omega_i^2\omega_k^2-\omega_i^2\omega_j^2)}{\omega_i^{\frac{3}{2}}\omega_j\omega_k^{\frac{3}{2}}(\omega_k^2-\omega_i^2)}$$

$$-\frac{B^a_e a^{ab}_i B^b_e \phi_{iijk}\phi_{ijk}\omega_k(3\omega_k^4-6\omega_j^2\omega_k^2-4\omega_i^2\omega_k^2+3\omega_j^4+\omega_i^4)}{\omega_i(\omega_k^2-\omega_j^2)\Delta(\omega_k,\omega_j,\omega_i)}+\frac{2B^a_e a^{ab}_k B^b_e B^c_e \zeta^c_{ik}\zeta^c_{jk}\phi_{ijk}(\omega_j^2\omega_k^2+\omega_i^2\omega_k^2-\omega_j^4+6\omega_i^2\omega_j^2-\omega_i^4)}{\sqrt{\omega_i}\sqrt{\omega_j}\omega_k\Delta(\omega_k,\omega_j,\omega_i)}-\frac{4B^a_e a^{ab}_i B^b_e B^c_e \zeta^c_{ij}\zeta^c_{ik}\phi_{ijk}\sqrt{\omega_k}(3\omega_j^2\omega_k^4-6\omega_j^4\omega_k^2-\omega_i^4\omega_k^2+3\omega_j^6-4\omega_i^2\omega_j^4-2\omega_i^4\omega_j^2+\omega_i^6)}{\omega_i^2\sqrt{\omega_j}(\omega_k^2-\omega_j^2)\Delta(\omega_k,\omega_j,\omega_i)}$$

$$+\frac{8B^a_e \zeta^a_{ij} B^b_e \zeta^b_{ij}\phi_{iiii}\omega_i\omega_j}{(\omega_j^2-\omega_i^2)^2}+\frac{8B^a_e \zeta^a_{ik} B^b_e \zeta^b_{ik}\phi_{jjkk}\omega_i\omega_k}{(\omega_k^2-\omega_i^2)^2}+\frac{B^a_e \zeta^a_{ik} B^b_e \zeta^b_{ik}\phi_{iijj}(\omega_k^4-6\omega_i^2\omega_k^2-3\omega_i^4)}{\omega_i^2(\omega_k^2-\omega_i^2)^2}+\frac{8B^a_e \zeta^a_{ij} B^b_e \zeta^b_{ik}\omega_i\sqrt{\omega_j}\phi_{jjjk}\sqrt{\omega_k}}{(\omega_j^2-\omega_i^2)(\omega_k^2-\omega_i^2)}+\frac{8B^a_e \zeta^a_{ik} B^b_e \zeta^b_{ij}\omega_i\sqrt{\omega_j}\phi_{jjjk}\sqrt{\omega_k}}{(\omega_j^2-\omega_i^2)(\omega_k^2-\omega_i^2)}-\frac{2B^a_e \zeta^a_{ij} B^b_e \zeta^b_{ik}\phi_{iijk}(3\omega_j^2+\omega_i^2)\sqrt{\omega_k}}{\sqrt{\omega_j}(\omega_j^2-\omega_i^2)(\omega_k^2-\omega_i^2)}-\frac{2B^a_e \zeta^a_{ik} B^b_e \zeta^b_{ij}\phi_{iijk}(3\omega_j^2+\omega_i^2)\sqrt{\omega_k}}{\sqrt{\omega_j}(\omega_j^2-\omega_i^2)(\omega_k^2-\omega_j^2)}+\frac{2B^a_e \zeta^a_{ik} B^b_e \zeta^b_{jk}\phi_{ijjj}\sqrt{\omega_j}(\omega_k^2+3\omega_i^2)}{\sqrt{\omega_i}(\omega_j^2-\omega_i^2)(\omega_k^2-\omega_i^2)}-\frac{2B^a_e \zeta^a_{ik} B^b_e \zeta^b_{jk}\sqrt{\omega_i}\phi_{iiij}(\omega_k^2+3\omega_j^2)}{\sqrt{\omega_j}(\omega_j^2-\omega_i^2)(\omega_k^2-\omega_j^2)}+\frac{B^a_e \zeta^a_{ij} B^b_e \zeta^b_{ij}\phi_{iijj}(\omega_j^2+\omega_i^2)(\omega_j^4-10\omega_i^2\omega_j^2+\omega_i^4)}{2\omega_i^2\omega_j^2(\omega_j^2-\omega_i^2)^2}$$

$$+\frac{8B^a_e \zeta^a_{ik} B^b_e \zeta^b_{ik} B^c_e(\zeta^c_{jk})^2(\omega_k^4+2\omega_j^2\omega_k^2+4\omega_i^2\omega_j^2+\omega_i^4)}{\omega_i\omega_j(\omega_k^2-\omega_i^2)^2}+\frac{2B^a_e \zeta^a_{ij} B^b_e \zeta^b_{ij} B^c_e(\zeta^c_{ij})^2(\omega_j^2+\omega_i^2)^2(\omega_j^2-6\omega_i\omega_j+\omega_i^2)(\omega_j^2+6\omega_i\omega_j+\omega_i^2)}{\omega_i^3\omega_j^3(\omega_j^2-\omega_i^2)^2}+\frac{16B^a_e \zeta^a_{ik} B^b_e \zeta^b_{jk} B^c_e \zeta^c_{ik}\zeta^c_{jk}(\omega_k^4+2\omega_j^2\omega_k^2+2\omega_i^2\omega_k^2+7\omega_i^2\omega_j^2)}{\omega_i\omega_j(\omega_k^2-\omega_i^2)(\omega_k^2-\omega_j^2)}+\frac{4B^a_e \zeta^a_{ik} B^b_e \zeta^b_{ik} B^c_e(\zeta^c_{ij})^2(\omega_j^2\omega_k^4-\omega_i^2\omega_k^4-6\omega_i^2\omega_j^2\omega_k^2-2\omega_i^4\omega_k^2-3\omega_i^4\omega_j^2-5\omega_i^6)}{\omega_i^3\omega_j(\omega_k^2-\omega_i^2)^2}-\frac{8B^a_e \zeta^a_{ij} B^b_e \zeta^b_{ik} B^c_e \zeta^c_{ij}\zeta^c_{ik}(7\omega_j^2\omega_k^4+\omega_i^2\omega_k^4-4\omega_j^4\omega_k^2+14\omega_i^2\omega_j^2\omega_k^2+6\omega_i^4\omega_k^2-14\omega_i^2\omega_j^4-9\omega_i^4\omega_j^2-\omega_i^6)}{\omega_i\omega_j(\omega_j^2-\omega_i^2)(\omega_k^2-\omega_i^2)(\omega_k^2-\omega_j^2)}$$

$$-\frac{8B^a_e \zeta^a_{ik} B^b_e \zeta^b_{ij} B^c_e \zeta^c_{ij}\zeta^c_{ik}(7\omega_j^2\omega_k^4+\omega_i^2\omega_k^4-4\omega_j^4\omega_k^2+14\omega_i^2\omega_j^2\omega_k^2+6\omega_i^4\omega_k^2-14\omega_i^2\omega_j^4-9\omega_i^4\omega_j^2-\omega_i^6)}{\omega_i\omega_j(\omega_j^2-\omega_i^2)(\omega_k^2-\omega_i^2)(\omega_k^2-\omega_j^2)}$$

$$+\frac{3B^a_e a^{ab}_i B^b_e \phi_{iii}^2\phi_{ijj}}{4\omega_i^3}+\frac{B^a_e a^{ab}_k B^b_e \phi_{iik}^2\phi_{jjk}}{4\omega_i\omega_k^2}+\frac{B^a_e a^{ab}_k B^b_e \phi_{iik}\phi_{jjk}\phi_{kkk}}{4\omega_k^3}+\frac{B^a_e a^{ab}_k B^b_e \phi_{iii}\phi_{ikk}\phi_{jjk}}{2\omega_i\omega_k^2}+\frac{3B^a_e a^{ab}_i B^b_e \phi_{iii}\phi_{iij}\phi_{jjj}}{4\omega_i^2\omega_j}+\frac{3B^a_e a^{ab}_i B^b_e \phi_{iii}\phi_{iik}\phi_{jjk}}{4\omega_i^2\omega_k}+\frac{3B^a_e a^{ab}_k B^b_e \phi_{iij}\phi_{jjj}\phi_{jjk}}{4\omega_j^2\omega_k}+\frac{B^a_e a^{ab}_k B^b_e \phi_{iii}\phi_{ijj}\phi_{jjk}}{4\omega_i\omega_j\omega_k}+\frac{B^a_e a^{ab}_k B^b_e \phi_{iii}\phi_{ijk}\phi_{jjj}}{4\omega_i\omega_j\omega_k}+\frac{B^a_e a^{ab}_i B^b_e \phi_{iik}\phi_{jkk}^2}{\omega_k(\omega_k^2-\omega_j^2)}+\frac{B^a_e a^{ab}_k B^b_e \phi_{iii}\phi_{ijk}\phi_{jkk}}{\omega_i(\omega_k^2-\omega_j^2)}+\frac{B^a_e a^{ab}_i B^b_e \phi_{ijj}^3(12\omega_j^2-\omega_i^2)}{2\omega_i(4\omega_j^2-\omega_i^2)^2}-\frac{B^a_e a^{ab}_k B^b_e \phi_{iij}^2\phi_{iik}(\omega_j^2-12\omega_i^2)}{2(\omega_j^2-4\omega_i^2)^2\omega_k}+\frac{B^a_e a^{ab}_i B^b_e \phi_{iik}^2\phi_{ijj}(3\omega_k^2-\omega_i^2)}{2\omega_i^2\omega_k(\omega_k^2-\omega_i^2)}$$

$$+\frac{B^a_e a^{ab}_i B^b_e \phi_{iik}\phi_{ikk}\phi_{jjk}(3\omega_k^2-\omega_i^2)}{2\omega_i\omega_k^2(\omega_k^2-\omega_i^2)}+\frac{B^a_e a^{ab}_k B^b_e \phi_{iij}\phi_{jjk}\phi_{jkk}(3\omega_k^2-\omega_j^2)}{2\omega_j\omega_k^2(\omega_k^2-\omega_j^2)}-\frac{2B^a_e a^{ab}_k B^b_e \phi_{iij}\phi_{iik}\phi_{jkk}\omega_i}{(\omega_j^2-4\omega_i^2)(\omega_k^2-\omega_j^2)}+\frac{B^a_e a^{ab}_i B^b_e \phi_{iik}\phi_{ijk}\phi_{jjj}(3\omega_k^2-\omega_i^2)}{2\omega_i\omega_j\omega_k(\omega_k^2-\omega_i^2)}-\frac{B^a_e a^{ab}_k B^b_e \phi_{iij}^2\phi_{jjk}\omega_i(3\omega_j^2-4\omega_i^2)}{2\omega_j(\omega_j^2-4\omega_i^2)^2\omega_k}+\frac{B^a_e a^{ab}_i B^b_e \phi_{iii}\phi_{ijj}^2(2\omega_j^2-\omega_i^2)(12\omega_j^2-\omega_i^2)}{4\omega_i^2\omega_j(4\omega_j^2-\omega_i^2)^2}-\frac{B^a_e a^{ab}_k B^b_e \phi_{iii}\phi_{iij}\phi_{ijk}(\omega_j^2-6\omega_i^2)}{\omega_k(\omega_j^2-4\omega_i^2)(\omega_j^2-\omega_i^2)}-\frac{3B^a_e a^{ab}_i B^b_e \phi_{iii}\phi_{iij}^2(\omega_j^4-11\omega_i^2\omega_j^2+20\omega_i^4)}{2\omega_i(\omega_j^2-4\omega_i^2)^2(\omega_j^2-\omega_i^2)}+\frac{B^a_e a^{ab}_i B^b_e \phi_{iij}\phi_{ijj}\phi_{jjj}(36\omega_j^4-19\omega_i^2\omega_j^2+3\omega_i^4)}{4\omega_i\omega_j^2(\omega_j^2-\omega_i^2)(4\omega_j^2-\omega_i^2)}$$

$$+\frac{B^a_e a^{ab}_k B^b_e \phi_{iij}\phi_{ijj}\phi_{ijk}(12\omega_j^4-91\omega_i^2\omega_j^2+12\omega_i^4)}{4\omega_i\omega_j\omega_k(\omega_j^2-4\omega_i^2)(4\omega_j^2-\omega_i^2)}+\frac{B^a_e a^{ab}_i B^b_e \phi_{iij}^2\phi_{ijj}(12\omega_j^6-151\omega_i^2\omega_j^4+395\omega_i^4\omega_j^2-52\omega_i^6)}{2\omega_i^2\omega_j(\omega_j^2-4\omega_i^2)^2(4\omega_j^2-\omega_i^2)}+\frac{B^a_e a^{ab}_i B^b_e \phi_{iik}\phi_{ijj}\phi_{jjk}(12\omega_j^2\omega_k^2-\omega_i^2\omega_k^2-4\omega_i^2\omega_j^2+\omega_i^4)}{4\omega_i\omega_j\omega_k(4\omega_j^2-\omega_i^2)(\omega_k^2-\omega_i^2)}+\frac{B^a_e a^{ab}_k B^b_e \phi_{ijk}^2\phi_{kkk}\omega_k(\omega_k^4-2\omega_j^2\omega_k^2-2\omega_i^2\omega_k^2+\omega_j^4+6\omega_i^2\omega_j^2+\omega_i^4)}{2\Delta(\omega_k,\omega_j,\omega_i)^2}+\frac{B^a_e a^{ab}_k B^b_e \phi_{ijk}\phi_{ikk}\phi_{jkk}\omega_k(\omega_k^2-\omega_j^2-\omega_i^2)(2\omega_k^2-\omega_j^2-\omega_i^2)}{(\omega_k^2-\omega_i^2)(\omega_k^2-\omega_j^2)\Delta(\omega_k,\omega_j,\omega_i)}-\frac{2B^a_e a^{ab}_k B^b_e \phi_{iij}\phi_{ijk}\phi_{ikk}\omega_i(2\omega_k^4-3\omega_j^2\omega_k^2-8\omega_i^2\omega_k^2+\omega_j^4-\omega_i^2\omega_j^2+6\omega_i^4)}{(\omega_j^2-4\omega_i^2)(\omega_k^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)}$$

$$+\frac{B^a_e a^{ab}_i B^b_e \phi_{ijk}^2\phi_{ikk}\omega_k^2(3\omega_k^6-9\omega_j^2\omega_k^4-8\omega_i^2\omega_k^4+9\omega_j^4\omega_k^2+12\omega_i^2\omega_j^2\omega_k^2+7\omega_i^4\omega_k^2-3\omega_j^6-4\omega_i^2\omega_j^4+\omega_i^4\omega_j^2-2\omega_i^6)}{\omega_i(\omega_k^2-\omega_j^2)\Delta(\omega_k,\omega_j,\omega_i)^2}+\frac{B^a_e a^{ab}_i B^b_e \phi_{iik}\phi_{ijk}\phi_{jkk}(3\omega_k^6-6\omega_j^2\omega_k^4-5\omega_i^2\omega_k^4+3\omega_j^4\omega_k^2+2\omega_i^2\omega_j^2\omega_k^2+3\omega_i^4\omega_k^2-\omega_i^2\omega_j^4+2\omega_i^4\omega_j^2-\omega_i^6)}{\omega_i(\omega_k^2-\omega_i^2)(\omega_k^2-\omega_j^2)\Delta(\omega_k,\omega_j,\omega_i)}$$

$$+\frac{B^a_e a^{ab}_i B^b_e \phi_{iii}\phi_{ijk}^2\omega_k(3\omega_k^8-12\omega_j^2\omega_k^6-13\omega_i^2\omega_k^6+18\omega_j^4\omega_k^4+11\omega_i^2\omega_j^2\omega_k^4+19\omega_i^4\omega_k^4-12\omega_j^6\omega_k^2+17\omega_i^2\omega_j^4\omega_k^2-2\omega_i^4\omega_j^2\omega_k^2-11\omega_i^6\omega_k^2+3\omega_j^8-15\omega_i^2\omega_j^6+15\omega_i^4\omega_j^4-5\omega_i^6\omega_j^2+2\omega_i^8)}{2\omega_i^2(\omega_k^2-\omega_j^2)\Delta(\omega_k,\omega_j,\omega_i)^2}+\frac{B^a_e a^{ab}_i B^b_e \phi_{iij}\phi_{ijk}\phi_{jjk}(9\omega_j^2\omega_k^6-3\omega_i^2\omega_k^6-21\omega_j^4\omega_k^4-7\omega_i^2\omega_j^2\omega_k^4+6\omega_i^4\omega_k^4+15\omega_j^6\omega_k^2-5\omega_i^2\omega_j^4\omega_k^2+9\omega_i^4\omega_j^2\omega_k^2-3\omega_i^6\omega_k^2-3\omega_j^8+7\omega_i^2\omega_j^6-5\omega_i^4\omega_j^4+\omega_i^6\omega_j^2)}{2\omega_i\omega_j\omega_k(\omega_j^2-\omega_i^2)(\omega_k^2-\omega_j^2)\Delta(\omega_k,\omega_j,\omega_i)}$$

$$+\frac{B^a_e a^{ab}_k B^b_e \phi_{iik}\phi_{ijk}^2(\omega_j^2\omega_k^8-7\omega_i^2\omega_k^8-3\omega_j^4\omega_k^6+18\omega_i^2\omega_j^2\omega_k^6+25\omega_i^4\omega_k^6+3\omega_j^6\omega_k^4-29\omega_i^2\omega_j^4\omega_k^4-7\omega_i^4\omega_j^2\omega_k^4-31\omega_i^6\omega_k^4-\omega_j^8\omega_k^2+20\omega_i^2\omega_j^6\omega_k^2-22\omega_i^4\omega_j^4\omega_k^2-12\omega_i^6\omega_j^2\omega_k^2+15\omega_i^8\omega_k^2-2\omega_i^2\omega_j^8+8\omega_i^4\omega_j^6-12\omega_i^6\omega_j^4+8\omega_i^8\omega_j^2-2\omega_i^{10})}{2\omega_i\omega_k^2(\omega_j^2-\omega_i^2)\Delta(\omega_k,\omega_j,\omega_i)^2}$$

$$+\frac{B^a_e a^{ab}_i B^b_e \phi_{ijj}\phi_{ijk}^2\omega_k(12\omega_j^2\omega_k^8-\omega_i^2\omega_k^8-72\omega_j^4\omega_k^6-34\omega_i^2\omega_j^2\omega_k^6+3\omega_i^4\omega_k^6+144\omega_j^6\omega_k^4+60\omega_i^2\omega_j^4\omega_k^4+47\omega_i^4\omega_j^2\omega_k^4-3\omega_i^6\omega_k^4-120\omega_j^8\omega_k^2+18\omega_i^2\omega_j^6\omega_k^2+9\omega_i^4\omega_j^4\omega_k^2-28\omega_i^6\omega_j^2\omega_k^2+\omega_i^8\omega_k^2+36\omega_j^{10}-43\omega_i^2\omega_j^8+5\omega_i^4\omega_j^6-\omega_i^6\omega_j^4+3\omega_i^8\omega_j^2)}{2\omega_i\omega_j(4\omega_j^2-\omega_i^2)(\omega_k^2-\omega_j^2)\Delta(\omega_k,\omega_j,\omega_i)^2}$$

$\tau_{\alpha\beta\gamma\delta}(i)$

$\xi_{\alpha\beta\gamma\delta\varepsilon\eta}$

$$
\begin{aligned}
&+\frac{3B^a_e a^{ab}_i B^b_e B^c_e a^{cg}_i B^d_e a^{dg}_i B^e_e a^{ef}_i B^f_e B^g_e}{8\omega_i^2}+\frac{3B^a_e a^{ab}_i B^b_e B^c_e a^{cd}_i B^d_e B^e_e a^{eg}_i B^f_e a^{fg}_i B^g_e}{16\omega_i^2}+\frac{3B^a_e a^{ag}_i B^b_e a^{bg}_i B^c_e a^{cd}_i B^d_e B^e_e a^{ef}_i B^f_e B^g_e}{16\omega_i^2}+\frac{3B^a_e a^{ab}_i B^b_e B^c_e a^{cg}_i B^d_e a^{dg}_j B^e_e a^{ef}_j B^f_e B^g_e}{16\omega_i\omega_j}\\
&+\frac{3B^a_e a^{ab}_i B^b_e B^c_e a^{cg}_j B^d_e a^{dg}_i B^e_e a^{ef}_j B^f_e B^g_e}{16\omega_i\omega_j}+\frac{3B^a_e a^{ab}_i B^b_e B^c_e a^{cd}_j B^d_e B^e_e a^{eg}_i B^f_e a^{fg}_j B^g_e}{16\omega_i(\omega_j+\omega_i)}+\frac{3B^a_e a^{ab}_i B^b_e B^c_e a^{cd}_j B^d_e B^e_e a^{eg}_j B^f_e a^{fg}_i B^g_e}{16\omega_i(\omega_j+\omega_i)}+\frac{3B^a_e a^{ag}_i B^b_e a^{bg}_j B^c_e a^{cd}_i B^d_e B^e_e a^{ef}_j B^f_e B^g_e}{16\omega_j(\omega_j+\omega_i)}\\
&+\frac{3B^a_e a^{ag}_i B^b_e a^{bg}_j B^c_e a^{cd}_j B^d_e B^e_e a^{ef}_i B^f_e B^g_e}{16\omega_i(\omega_j+\omega_i)}+\frac{B^a_e a^{ab}_i B^b_e B^c_e a^{cd}_i B^d_e B^e_e a^{ef}_i B^f_e \phi_{iii}}{6\omega_i^3}+\frac{B^a_e a^{ab}_i B^b_e B^c_e a^{cd}_j B^d_e B^e_e a^{ef}_i B^f_e \phi_{iij}(\omega_j+3\omega_i)}{4\omega_i^2(\omega_j+\omega_i)(\omega_j+2\omega_i)}+\frac{B^a_e a^{ab}_i B^b_e B^c_e a^{cd}_i B^d_e B^e_e a^{ef}_j B^f_e \phi_{iij}(\omega_j^2+3\omega_i\omega_j+4\omega_i^2)}{8\omega_i^2\omega_j(\omega_j+\omega_i)(\omega_j+2\omega_i)}\\
&+\frac{B^a_e a^{ab}_i B^b_e B^c_e a^{cd}_j B^d_e B^e_e a^{ef}_j B^f_e \phi_{ijj}(4\omega_j^2+3\omega_i\omega_j+\omega_i^2)}{8\omega_i\omega_j^2(\omega_j+\omega_i)(2\omega_j+\omega_i)}+\frac{B^a_e a^{ab}_i B^b_e B^c_e a^{cd}_j B^d_e B^e_e a^{ef}_k B^f_e \phi_{ijk}(\omega_k^2+2\omega_j\omega_k+\omega_i\omega_k+\omega_j^2+2\omega_i\omega_j+\omega_i^2)}{4\omega_i\omega_k(\omega_j+\omega_i)(\omega_k+\omega_j)(\omega_k+\omega_j+\omega_i)}-\frac{B^a_e a^{ab}_i B^b_e B^c_e a^{cd}_i B^d_e B^e_e \zeta^e_{ij} B^f_e \zeta^f_{ij}(\omega_j-\omega_i)}{2\omega_i^3\omega_j}\\
&-\frac{B^a_e \zeta^a_{ij} B^b_e \zeta^b_{ij} B^c_e a^{cd}_i B^d_e B^e_e a^{ef}_i B^f_e(\omega_j-\omega_i)}{2\omega_i^3\omega_j}+\frac{B^a_e a^{ab}_i B^b_e B^c_e \zeta^c_{ij} B^d_e \zeta^d_{jk} B^e_e a^{ef}_k B^f_e(\omega_j+\omega_i)(\omega_k+\omega_j)}{2\omega_i^{\frac{3}{2}}\omega_j^2\omega_k^{\frac{3}{2}}}-\frac{B^a_e a^{ab}_i B^b_e B^c_e \zeta^c_{jk} B^d_e a^{de}_i B^e_e B^f_e \zeta^f_{jk}(\omega_k-\omega_j)^2}{2\omega_i\omega_j\omega_k(\omega_k+\omega_j)(\omega_k+\omega_j+\omega_i)}+\frac{B^a_e \zeta^a_{ij} B^b_e a^{bc}_k B^c_e B^d_e a^{de}_k B^e_e B^f_e \zeta^f_{ij}(\omega_j-\omega_i)^2}{2\omega_i\omega_j\omega_k(\omega_j+\omega_i)(\omega_k+\omega_j+\omega_i)}\\
&-\frac{B^a_e a^{ab}_i B^b_e B^c_e \zeta^c_{ij} B^d_e a^{de}_k B^e_e B^f_e \zeta^f_{jk}(\omega_j+\omega_i)(\omega_k-\omega_j)}{2\omega_i^{\frac{3}{2}}\omega_j^2\sqrt{\omega_k}(\omega_k+\omega_j)}+\frac{B^a_e \zeta^a_{ij} B^b_e a^{bc}_i B^c_e B^d_e \zeta^d_{jk} B^e_e a^{ef}_k B^f_e(\omega_j^2-\omega_i^2)(\omega_k+\omega_j)}{2\sqrt{\omega_i}\omega_j^2\omega_k^{\frac{3}{2}}}-\frac{B^a_e a^{ab}_i B^b_e B^c_e \zeta^c_{jk} B^d_e \zeta^d_{ij} B^e_e a^{ef}_k B^f_e(\omega_j-\omega_i)(\omega_k-\omega_j)}{2\omega_i^{\frac{3}{2}}\omega_j\omega_k^{\frac{3}{2}}(\omega_k+\omega_j+\omega_i)}\\
&+\frac{B^a_e \zeta^a_{ij} B^b_e a^{bc}_i B^c_e B^d_e a^{de}_i B^e_e B^f_e \zeta^f_{ij}(\omega_j^2-4\omega_i^2)(\omega_j-\omega_i)^2}{2\omega_i^2\omega_j^2(\omega_j+\omega_i)}-\frac{B^a_e a^{ab}_i B^b_e B^c_e a^{cd}_j B^d_e B^e_e \zeta^e_{ik} B^f_e \zeta^f_{jk}(\omega_k+\omega_i)(\omega_k-\omega_j)}{2\omega_i^{\frac{3}{2}}\sqrt{\omega_j}\omega_k(\omega_j+\omega_i)(\omega_k+\omega_j)}-\frac{B^a_e a^{ab}_i B^b_e B^c_e a^{cd}_j B^d_e B^e_e \zeta^e_{jk} B^f_e \zeta^f_{ik}(\omega_k^2-\omega_i^2)(\omega_k+\omega_j)}{2\omega_i^{\frac{3}{2}}\sqrt{\omega_j}\omega_k(\omega_j+\omega_i)}\\
&+\frac{B^a_e \zeta^a_{ij} B^b_e \zeta^b_{ik} B^c_e a^{cd}_j B^d_e B^e_e a^{ef}_k B^f_e(\omega_j^2-\omega_i^2)(\omega_k+\omega_i)}{2\omega_i\sqrt{\omega_j}\omega_k^{\frac{3}{2}}(\omega_k+\omega_j)}+\frac{B^a_e \zeta^a_{ik} B^b_e \zeta^b_{ij} B^c_e a^{cd}_j B^d_e B^e_e a^{ef}_k B^f_e(\omega_j+\omega_i)(\omega_k-\omega_i)}{2\omega_i\sqrt{\omega_j}\omega_k^{\frac{3}{2}}(\omega_k+\omega_i)(\omega_k+\omega_j)}-\frac{B^a_e \zeta^a_{ij} B^b_e a^{bc}_i B^c_e B^d_e a^{de}_k B^e_e B^f_e \zeta^f_{jk}(\omega_j^2-\omega_i^2)(\omega_k-\omega_j)}{2\sqrt{\omega_i}\omega_j^2\sqrt{\omega_k}(\omega_k+\omega_j)}\\
&-\frac{B^a_e a^{ab}_i B^b_e B^c_e \zeta^c_{jk} B^d_e a^{de}_j B^e_e B^f_e \zeta^f_{ik}(\omega_k^2-\omega_i^2)(\omega_k-\omega_j)}{2\omega_i^{\frac{3}{2}}\sqrt{\omega_j}\omega_k(\omega_k+\omega_j+\omega_i)}-\frac{B^a_e \zeta^a_{ij} B^b_e a^{bc}_k B^c_e B^d_e \zeta^d_{ik} B^e_e a^{ef}_j B^f_e(\omega_j^2-\omega_i^2)(\omega_k-\omega_i)}{2\omega_i\omega_j^{\frac{3}{2}}\sqrt{\omega_k}(\omega_k+\omega_j+\omega_i)}-\frac{B^a_e a^{ab}_i B^b_e B^c_e \zeta^c_{ij} B^d_e a^{de}_i B^e_e B^f_e \zeta^f_{ij}(\omega_j^2-\omega_i^2)(3\omega_j^2+\omega_i\omega_j+2\omega_i^2)}{2\omega_i^2\omega_j^2(\omega_j+2\omega_i)}\\
&-\frac{B^a_e \zeta^a_{ij} B^b_e a^{bc}_i B^c_e B^d_e \zeta^d_{ij} B^e_e a^{ef}_i B^f_e(\omega_j^2-\omega_i^2)(3\omega_j^2+\omega_i\omega_j+2\omega_i^2)}{2\omega_i^2\omega_j^2(\omega_j+2\omega_i)}-\frac{B^a_e \zeta^a_{ij} B^b_e a^{bc}_k B^c_e B^d_e a^{de}_i B^e_e B^f_e \zeta^f_{jk}(\omega_j^2-\omega_i^2)(\omega_k-\omega_j)}{2\sqrt{\omega_i}\omega_j\sqrt{\omega_k}(\omega_k+\omega_j)(\omega_k+\omega_j+\omega_i)}\\
&-\frac{B^a_e a^{ab}_i B^b_e B^c_e \zeta^c_{ij} B^d_e \zeta^d_{ij} B^e_e a^{ef}_i B^f_e(2\omega_j^4+3\omega_i\omega_j^3+10\omega_i^2\omega_j^2+7\omega_i^3\omega_j+2\omega_i^4)}{2\omega_i^4\omega_j^2(\omega_j+\omega_i)(\omega_j+2\omega_i)}
\end{aligned}
$$

Resonances

$\sigma_{\alpha\beta}$(i,j):
$$\frac{B_e^a \zeta_{ik}^a B_e^b \zeta_{ij}^b \phi_{iij}\phi_{jjk}(20\omega_j^6\omega_k^8 - 185\omega_i^2\omega_j^4\omega_k^8 - \ldots + 4\omega_i^{12}\omega_j^2)}{\omega_j^{\frac{3}{2}}\omega_k^{\frac{3}{2}}(\omega_j^2 - 4\omega_i^2)(\omega_j^2-\omega_i^2)(4\omega_j^2-\omega_i^2)(\omega_k^2-\omega_i^2)(\omega_k^2-\omega_j^2)\Delta(\omega_k,\omega_j,\omega_i)}$$

Fermi and Darling-Dennison/Coriolis
resonance denominators

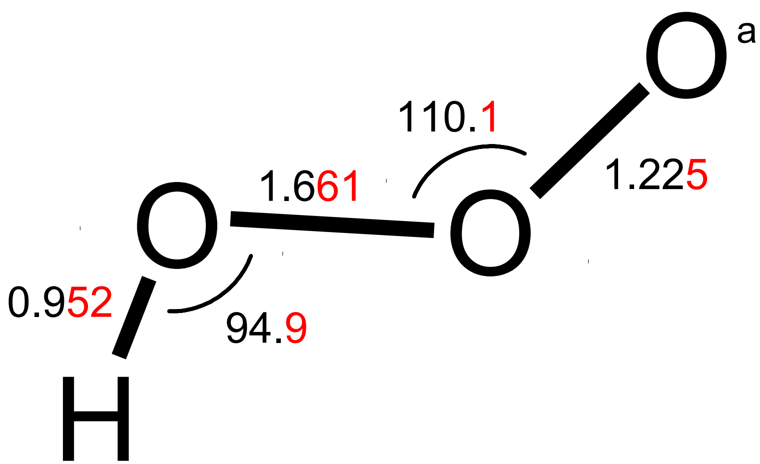
$\xi_{\alpha\beta\gamma\delta\epsilon\eta}$:
$$\frac{B_e^a a_i^{ab} B_e^b B_e^c \zeta_{ij}^c B_e^d \zeta_{ij}^d B_e^e a_i^{ef} B_e^f (2\omega_j^4 + \ldots + 2\omega_i^4)}{2\omega_i^3\omega_j^2(\omega_j+\omega_i)(\omega_j+2\omega_i)}$$

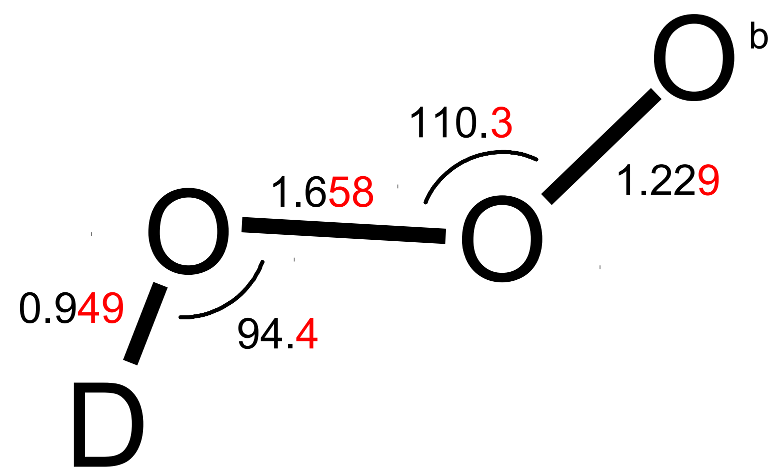
No resonance denominators!

The ground state is resonance-free. Terms with no vibrational dependence have no resonance denominators, and the denominators cancel in $\sum_{i\leq j}\sigma_{\alpha\beta}(i,j)$ and $\sum_i \tau_{\alpha\beta\gamma\delta}(i)$.

Why VPT4?

Equilibrium geometries (r_e) from $B_e^\gamma = B_0^\gamma + \frac{1}{2}\sum_i \alpha_i^\gamma$ (~VPT2).





a) M. C. McCarthy, V. Lattanzi, D. Kokkin, O. Martinez, and J. F. Stanton, *J. Chem. Phys.* **136**, 034303 (2012).
b) M. C. McCarthy, J. F. Stanton et al., *unpublished*.

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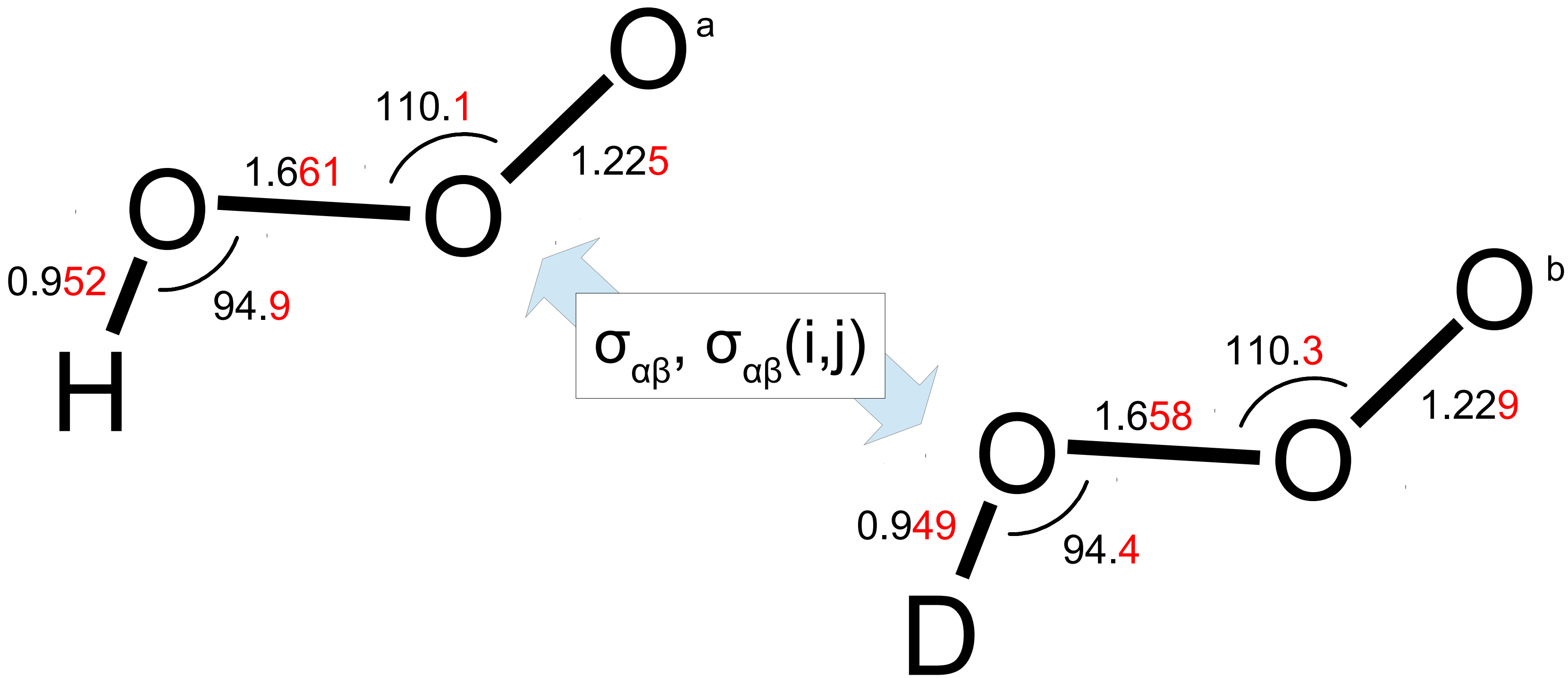

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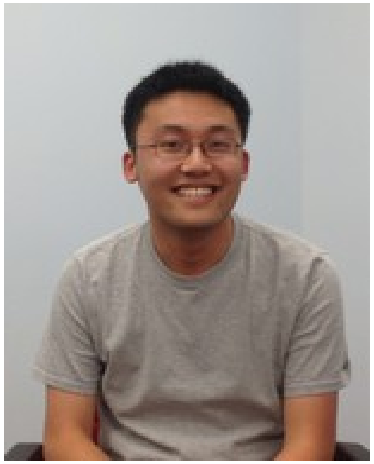
	VPT2			VPT4			
	x_{ij}	$\sigma_{\alpha\beta}(i)$	$\tau_{\alpha\beta\gamma\delta}$	y_{ijk}	$\sigma_{\alpha\beta}(i,j)$	$\tau_{\alpha\beta\gamma\delta}(i)$	$\xi_{\alpha\beta\gamma\delta\epsilon\eta}$
F_3	φ_{ijk}	φ_{iij}	0	φ_{ijk}	φ_{ijk}	φ_{ijk}	φ_{ijk}
F_4	φ_{iijj}	0	0	φ_{ijkl}	φ_{ijkl}	φ_{iijk}	0
F_5	0	0	0	φ_{iijkl}	φ_{iijjk}	0	0
F_6	0	0	0	φ_{iijjkk}	0	0	0

Future Work

- U (pseudopotential) terms missing from VPT2 may be important when going to VPT4.
- $\tilde{\sigma}_{\alpha\beta}$ equations need to be derived.
- Current constants are limited to triatomics (3 vibrational modes). The numerical implementation is not limited, however.

Acknowledgements


Dr. John Stanton


Justin Gong

	VPT2			VPT4			
	x	$\sigma_{\alpha\beta}$	$\tau_{\alpha\beta\gamma\delta}$	y	$\sigma_{\alpha\beta}$	$\tau_{\alpha\beta\gamma\delta}$	$\xi_{\alpha\beta\gamma\delta\epsilon\eta}$
const.	✓	0	✓		Not Derived		✓
$(n_i+½)$	0	✓	0	Not Derived		✓	
$(n_i+½)^2$	✓	0	0		✓		
$(n_i+½)(n_j+½)$	✓	0	0		✓		
$(n_i+½)^3$	0	0	0	✓			
$(n_i+½)^2(n_j+½)$	0	0	0	✓			
$(n_i+½)(n_j+½)(n_k+½)$	0	0	0	✓			

	VPT2			VPT4			
	x	$\sigma_{\alpha\beta}$	$\tau_{\alpha\beta\gamma\delta}$	y	$\sigma_{\alpha\beta}$	$\tau_{\alpha\beta\gamma\delta}$	$\xi_{\alpha\beta\gamma\delta\epsilon\eta}$
const.	✓	0	✓	0	Not Derived	Not Zero!	✓
$(n_i+½)$	0	✓	0	Not Derived	0	✓	Not Zero!
$(n_i+½)^2$	✓	0	0	0	✓	Not Zero!	Not Zero!
$(n_i+½)(n_j+½)$	✓	0	0	0	✓	Not Zero!	Not Zero!
$(n_i+½)^3$	0	0	0	✓	0	0	0
$(n_i+½)^2(n_j+½)$	0	0	0	✓	0	Not Zero!	0
$(n_i+½)(n_j+½)(n_k+½)$	0	0	0	✓	0	Not Zero!	0