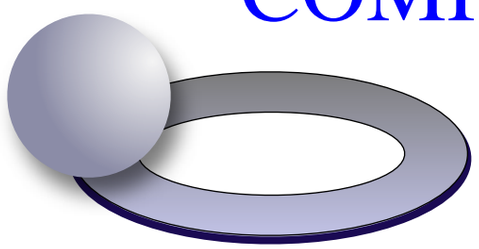
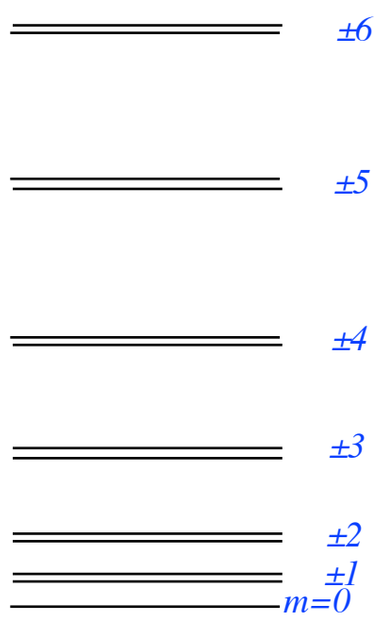
AMPLITUDE AND PHASE REVIVAL DYNAMICS FOR LEVEL-CLUSTERS: COMPARING HOMO-CYCLIC, OCTAHEDRAL and ICOSAHEDRAL ROTORS.

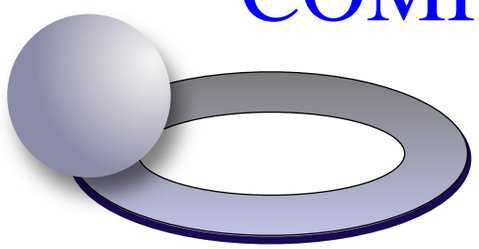


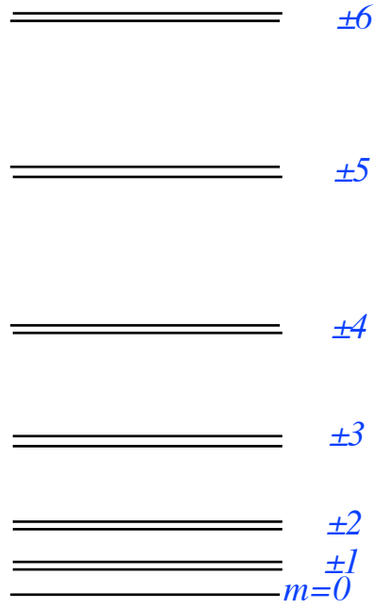
William G. Harter and Alvason Zhenhua Li
University of Arkansas - Fayetteville
Physics Department and Microelectronics-Photonics Program



AMPLITUDE AND PHASE REVIVAL DYNAMICS FOR LEVEL-CLUSTERS: COMPARING HOMO-CYCLIC, OCTAHEDRAL and ICOSAHEDRAL ROTORS.

SIMPLE



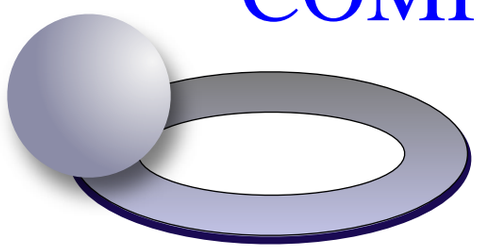


William G. Harter and Alvason Zhenhua Li
University of Arkansas - Fayetteville
Physics Department and Microelectronics-Photonics Program

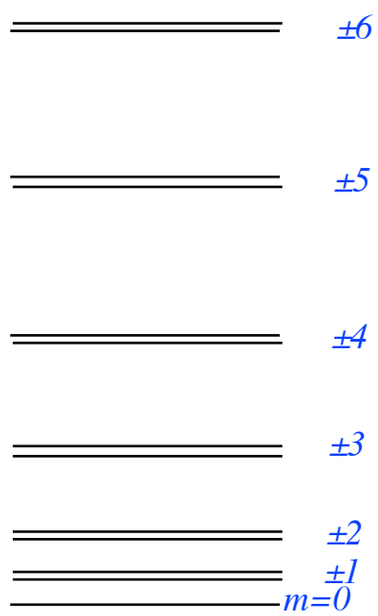
Some History of Quantum Revivals

J.H. Eberly *et. al. Phys. Rev.* **A 23**,*236 (1981)*	Laser Cavity revivals
R.S. McDowell, WGH, C.W. Patterson *LosAlmos Sci.* **3**, *38(1982)*	Symmetric-top revivals
S.I. Vetchinkin, et. al. *Chem. Phys. Lett.* **215**,*11 (1993)*	1D ∞-Square well revivals
Aronstein, Stroud, Berry, …, Schleich,.. (1995-1998)	" " " "
WGH, *J. Mol. Spectrosc.* **210**, *166 (2001)*	Bohr-rotor revivals

AMPLITUDE AND PHASE REVIVAL DYNAMICS FOR LEVEL-CLUSTERS: COMPARING HOMO-CYCLIC, OCTAHEDRAL and ICOSAHEDRAL ROTORS.



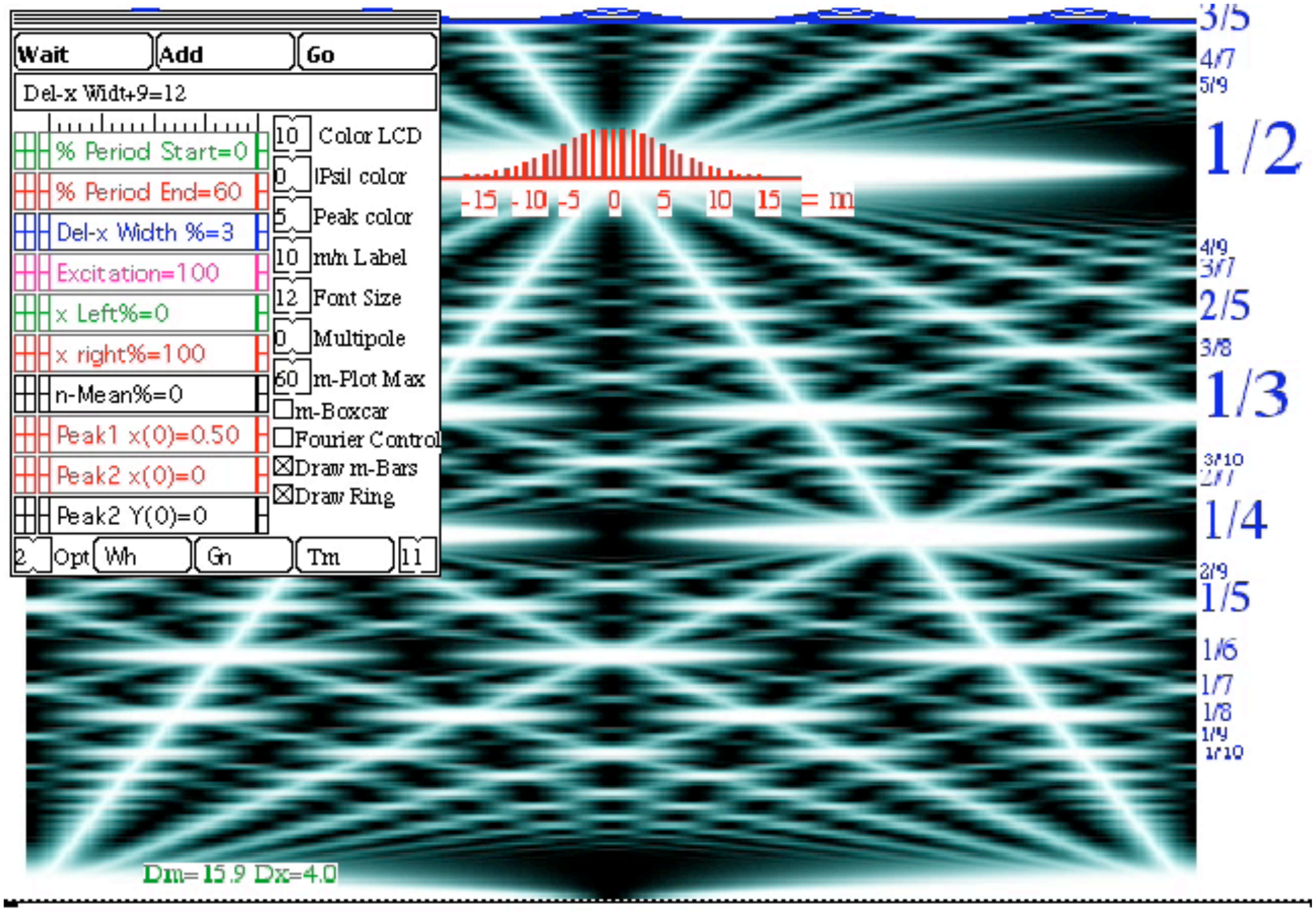
William G. Harter and Alvason Zhenhua Li
University of Arkansas - Fayetteville
Physics Department and Microelectronics-Photonics Program

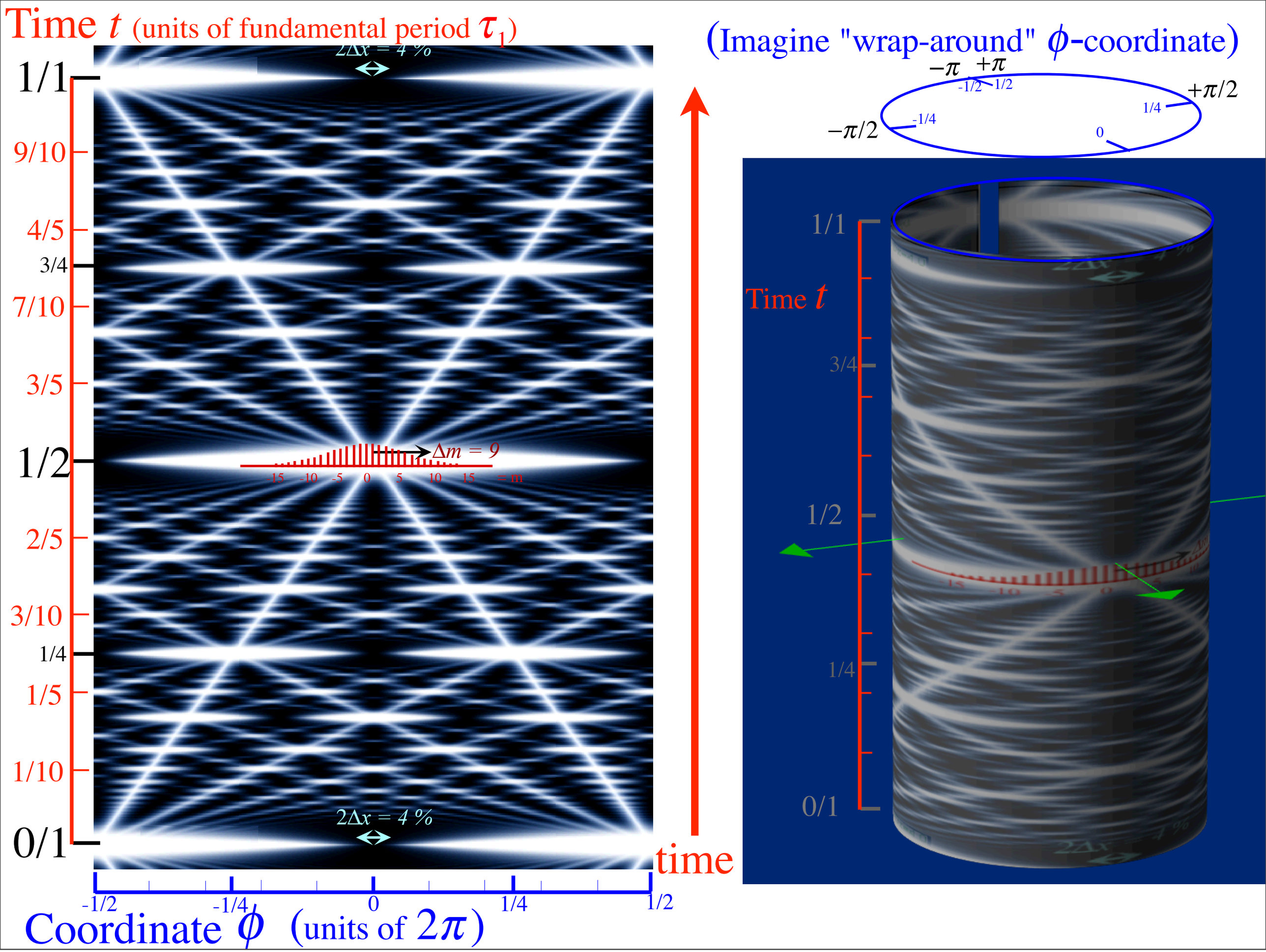


Some History of Quantum Revivals

J.H. Eberly *et. al. Phys. Rev.* **A 23**,*236 (1981)* — Laser Cavity revivals
R.S. McDowell, WGH, C.W. Patterson *LosAlmos Sci.* **3**, *38(1982)* — Symmetric-top revivals
S.I. Vetchinkin, et. al. *Chem. Phys. Lett.* **215**,*11 (1993)* — 1D ∞-Square well revivals
Aronstein, Stroud, Berry, …, Schleich,.. (1995-1998) — " " " "
WGH, *J. Mol. Spectrosc.* **210**, *166 (2001)* — Bohr-rotor revivals
Anne B. McCoy *Chem. Phys. Lett.* **501**, *603(2011)*...Anne notes Morse potential is integer-analytic.
WGH and Zhenhua A. Li, *Submitted PRL (current work)*.. Morse oscillator revivals quite like rotors & ∞-wells .

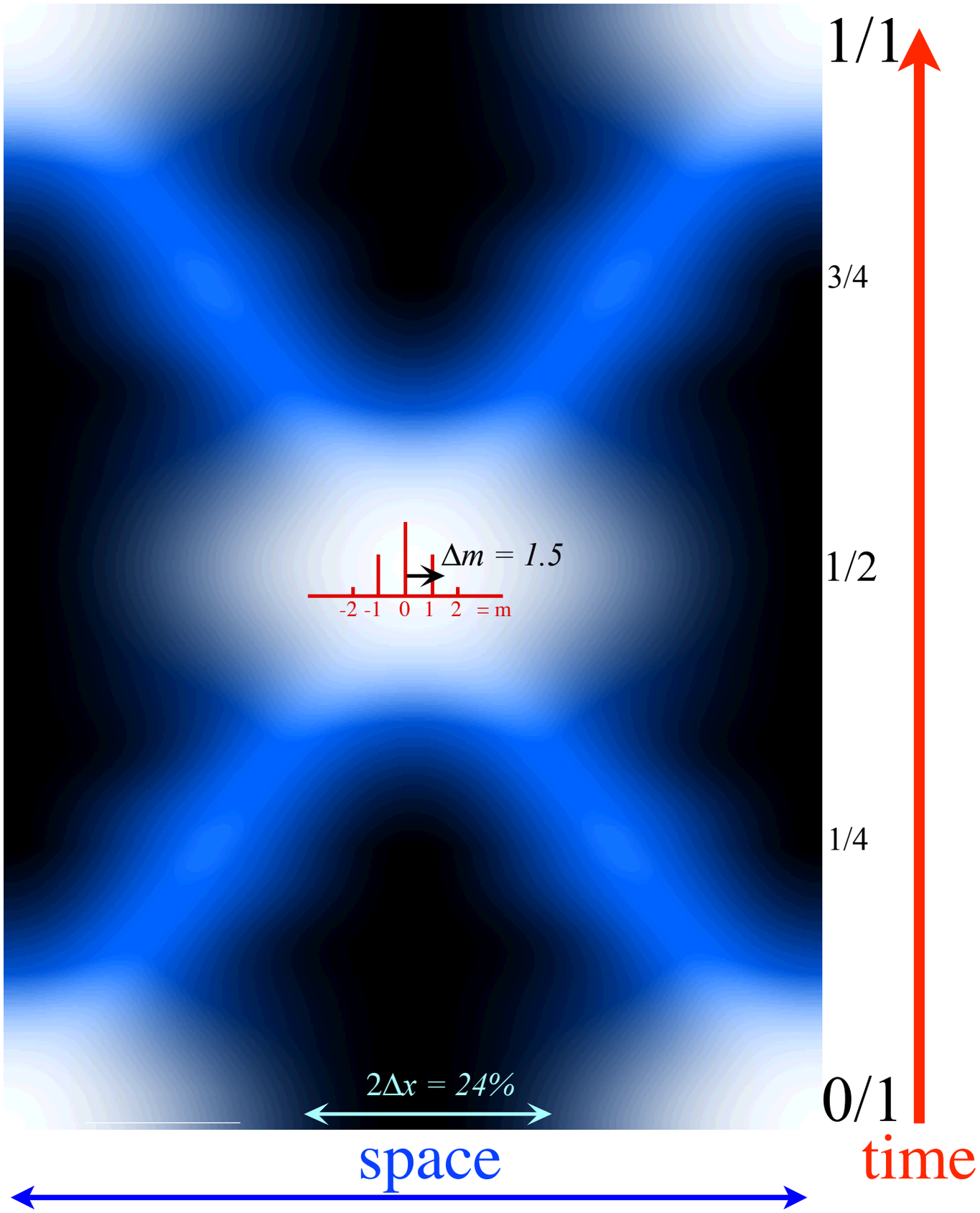
Space-time revival paths of peaks and zeros for simple rotors


Wait
Add
Go
Del-x Widt+9=12
% Period Start=0
% Period End=60
Del-x Width %=3
Excitation=100
x Left%=0
x right%=100
n-Mean%=0
Peak1 x(0)=0.50
Peak2 x(0)=0
Peak2 Y(0)=0
10 Color LCD
0 |Psi| color
5 Peak color
10 m/n Label
12 Font Size
0 Multipole
60 m-Plot Max
m-Boxcar
Fourier Control
Draw m-Bars
Draw Ring
2 Opt Wh Gn Tm 11
-15 -10 -5 0 5 10 15 = m
3/5
4/7
5/9
1/2
4/9
3/7
2/5
3/8
1/3
3/10
2/7
1/4
2/9
1/5
1/6
1/7
1/8
1/9
1/10
Dm=15.9 Dx=4.0


Time t (units of fundamental period τ_1)
1/1
9/10
4/5
3/4
7/10
3/5
1/2
2/5
3/10
1/4
1/5
1/10
0/1
$2\Delta x = 4\ \%$
$\Delta m = 9$
-15 -10 -5 0 5 10 15 = m
time
-1/2 -1/4 0 1/4 1/2
Coordinate ϕ (units of 2π)
(Imagine "wrap-around" ϕ-coordinate)
$-\pi$ $+\pi$
-1/2 1/2
$+\pi/2$
1/4
$-\pi/2$
-1/4
0
Time t
1/1
3/4
1/2
1/4
0/1

N-level-rotor system revival-beat wave dynamics

(Just 2-levels $(0, \pm 1)$ (and some ± 2) excited)

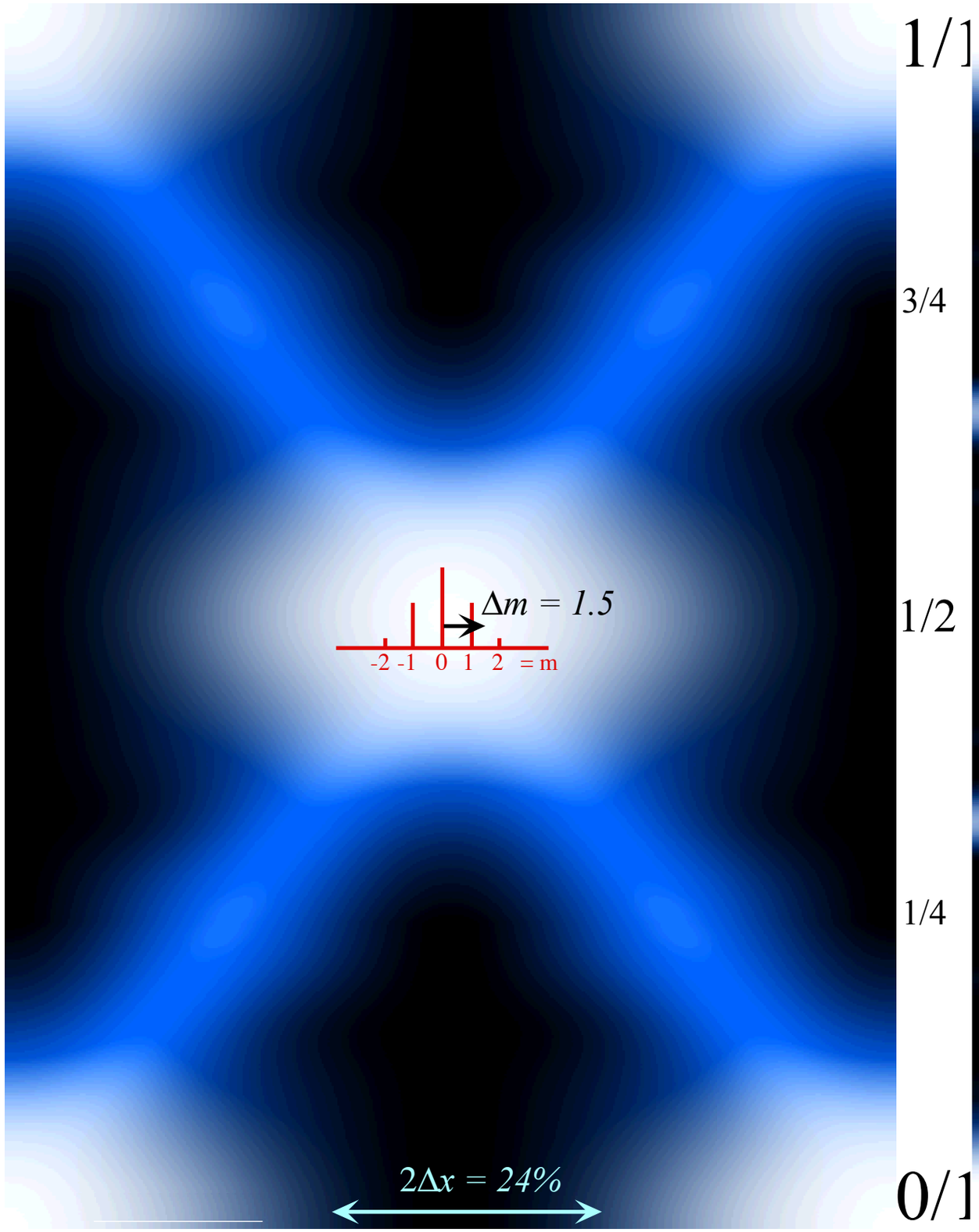


$|\Psi(x,t)|$ in space-time

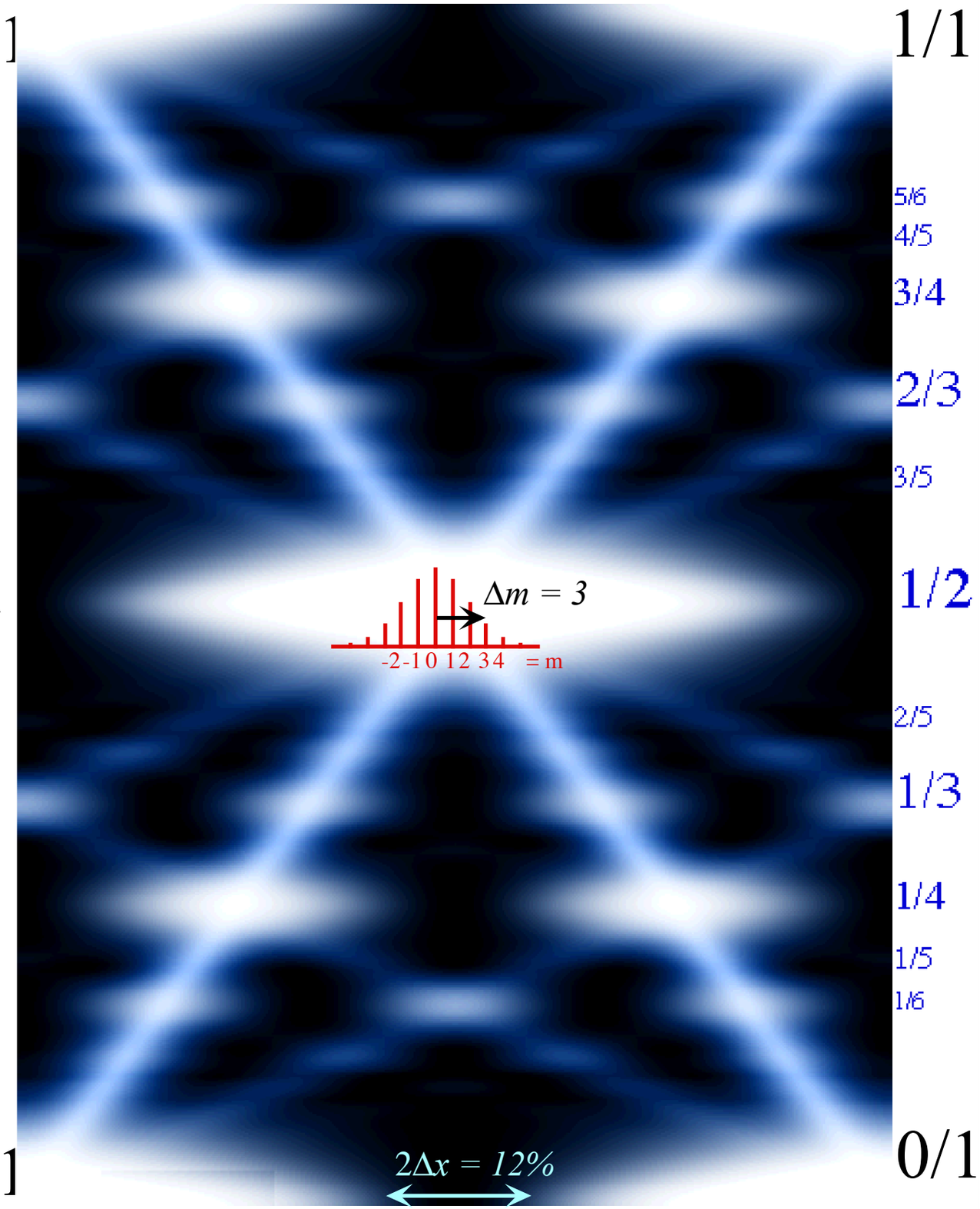
Simplest quantum revival:
Exciting first two levels
(ℓ=0 and ℓ=$\pm$1)
is like a
2-level system quantum beat
in space-time

N-level-rotor system revival-beat wave dynamics

(Just 2-levels (0, ±1) (and some ±2) excited)



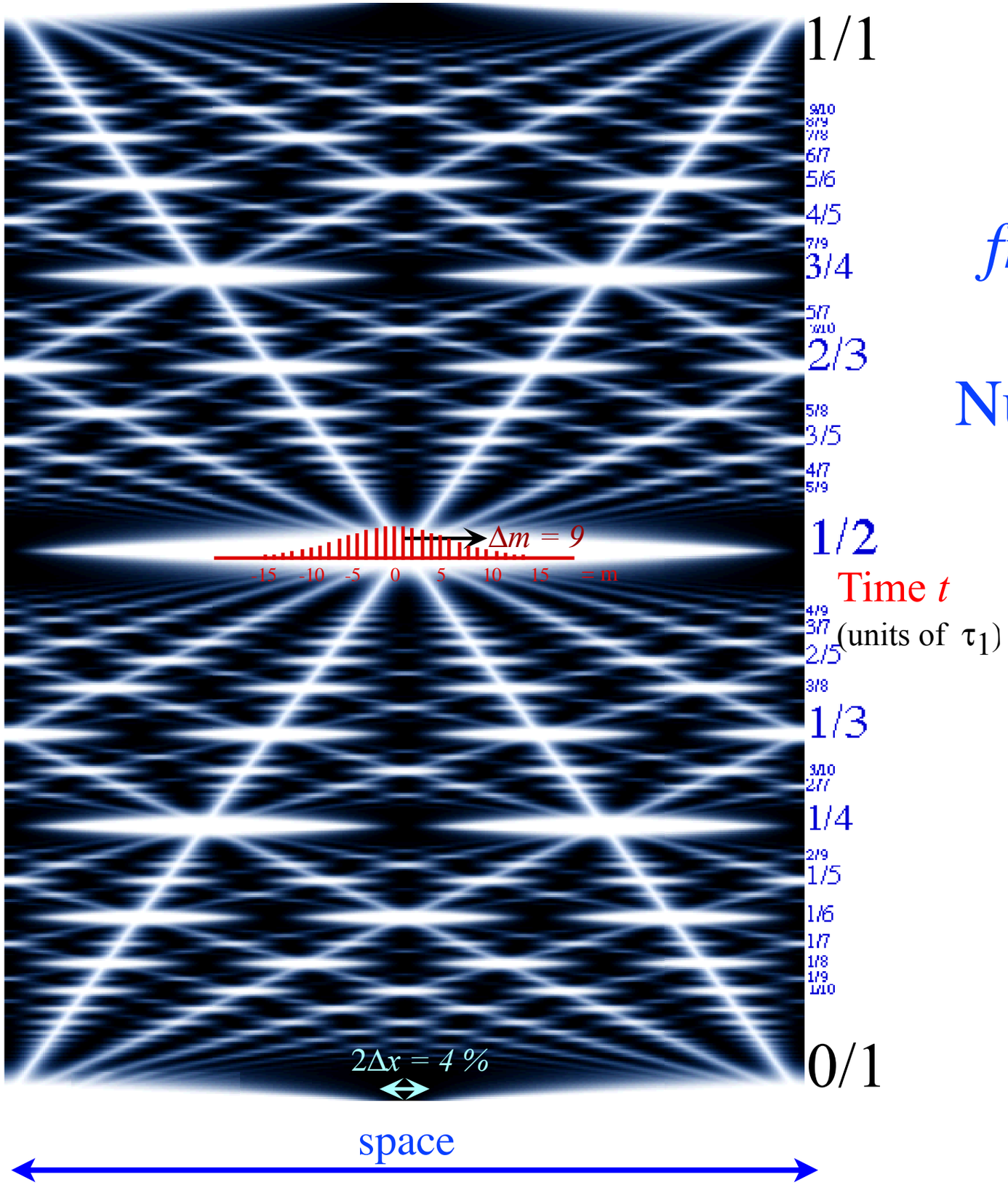
(4-levels (0, ±1,±2,±3) (and some ±4) excited)



Simplest *fractional* quantum revivals: 3,4,5-level systems

N-level-rotor system revival-beat wave dynamics

(9 or10-levels (0, ±1, ±2, ±3, ±4,..., ±9, ±10, ±11...) excited)

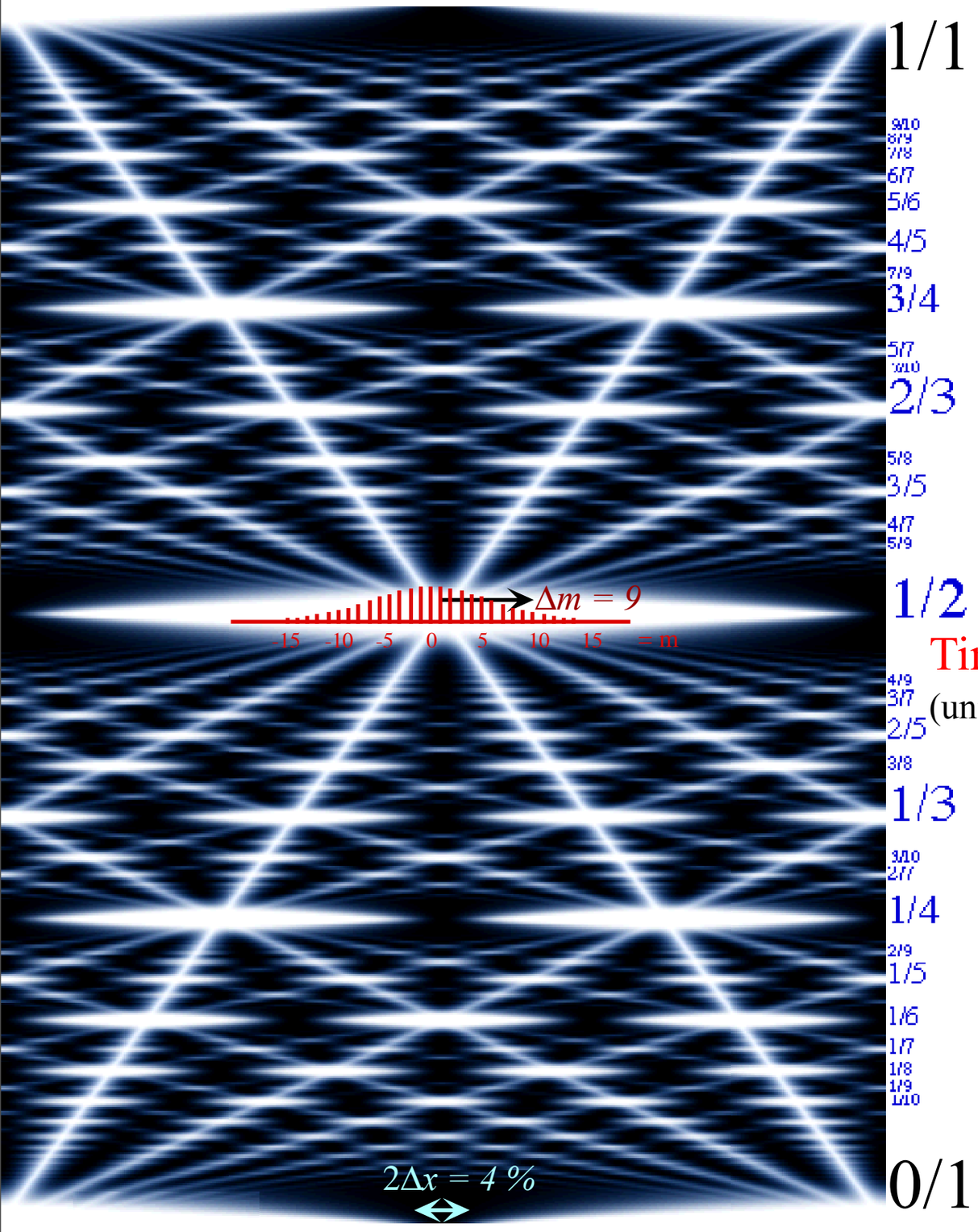


fractional quantum revivals:
in 3,4,…, N-level systems
Number increases rapidly with
number of levels
and/or bandwidth
of excitation

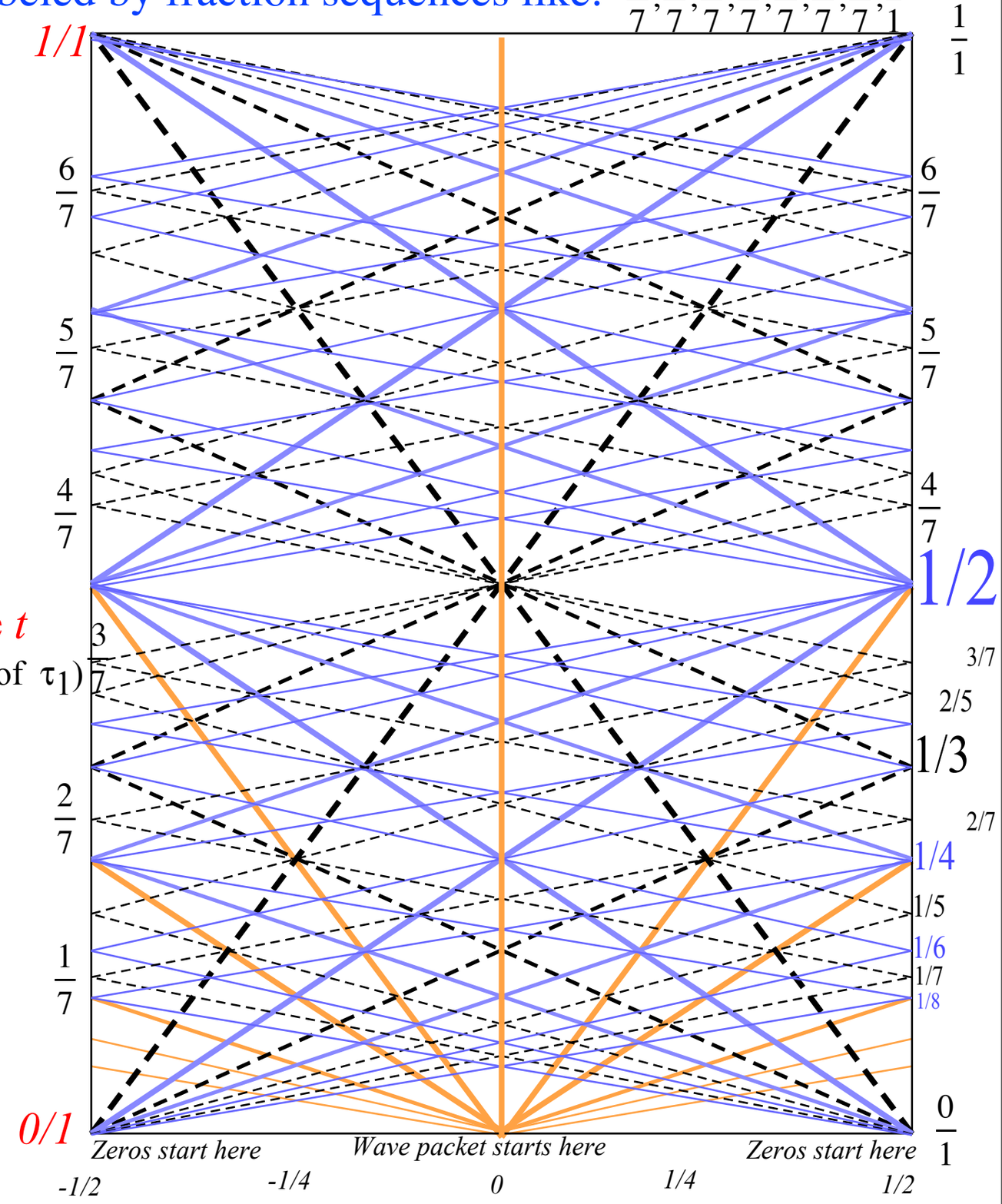
N-level-rotor system revival-beat wave dynamics

(9 or10-levels (0, ±1, ±2, ±3, ±4,..., ±9, ±10, ±11...) excited)

Zeros (clearly) and "particle-packets" (faintly) have paths labeled by fraction sequences like: $\frac{0}{7}, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{1}{1}$



Time t (units of τ_1)

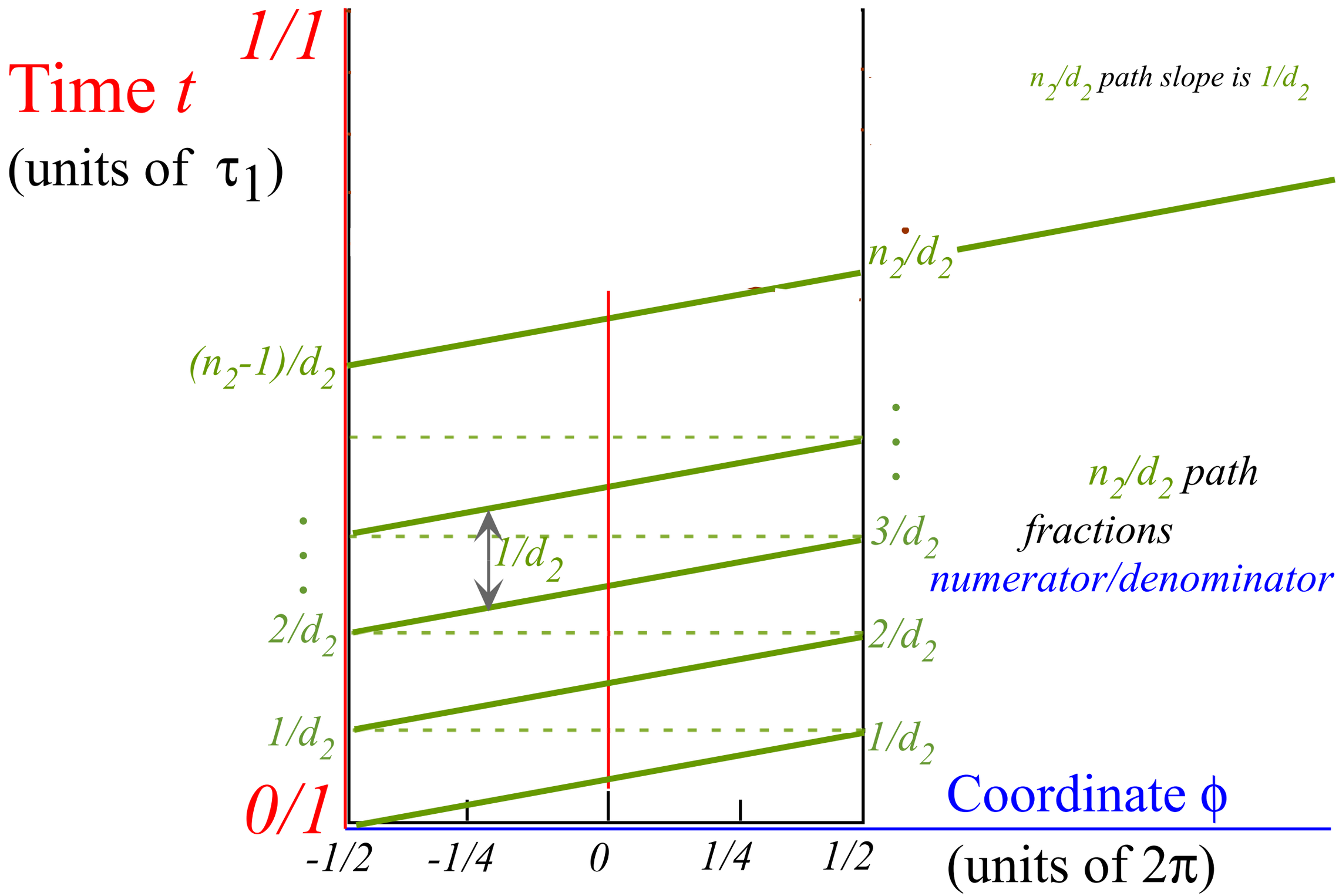


Coordinate ϕ (units of 2π)

Farey-Sum arithmetic of revival paths
(How *Rational Fractions N/D* occupy real space-time)

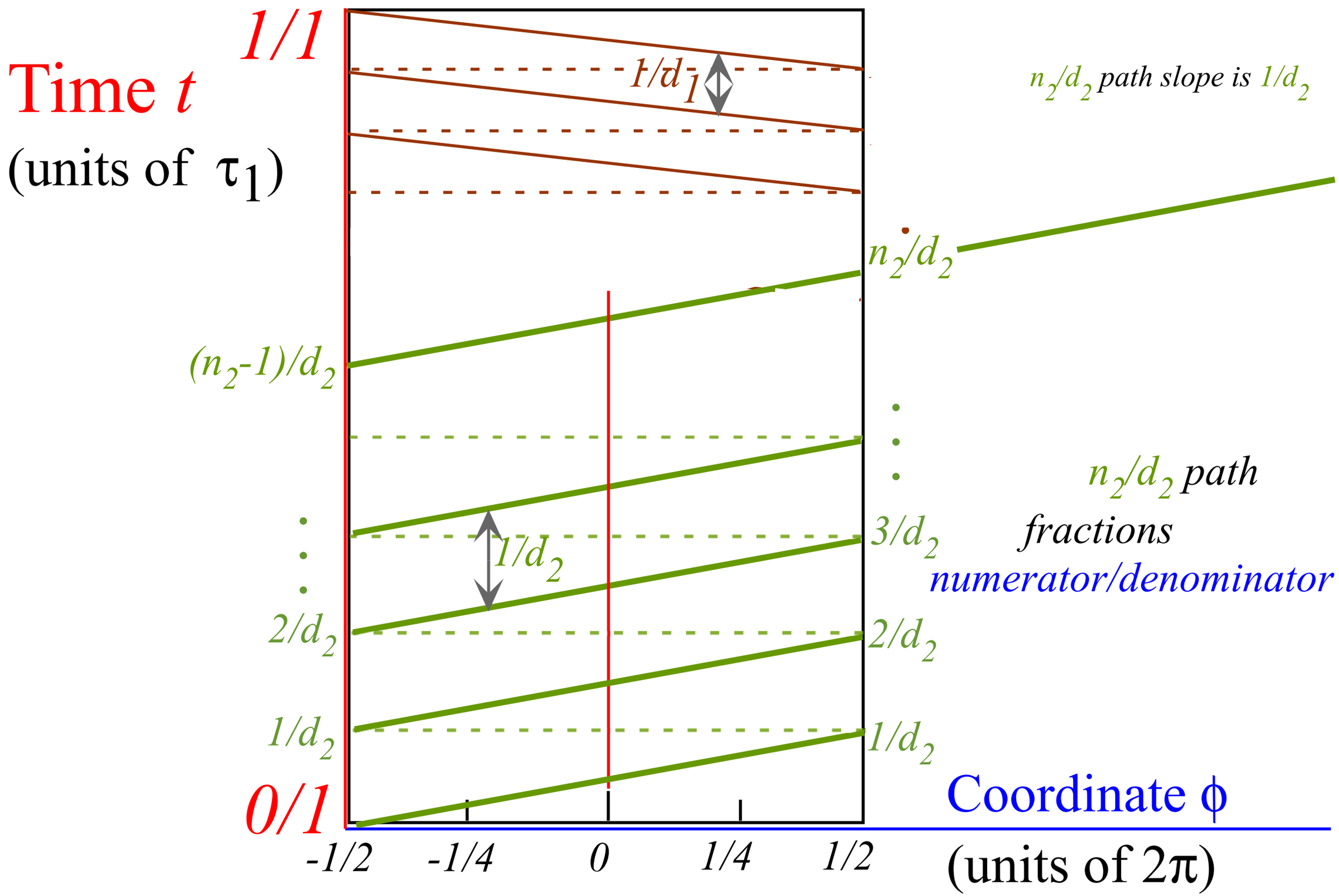
Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



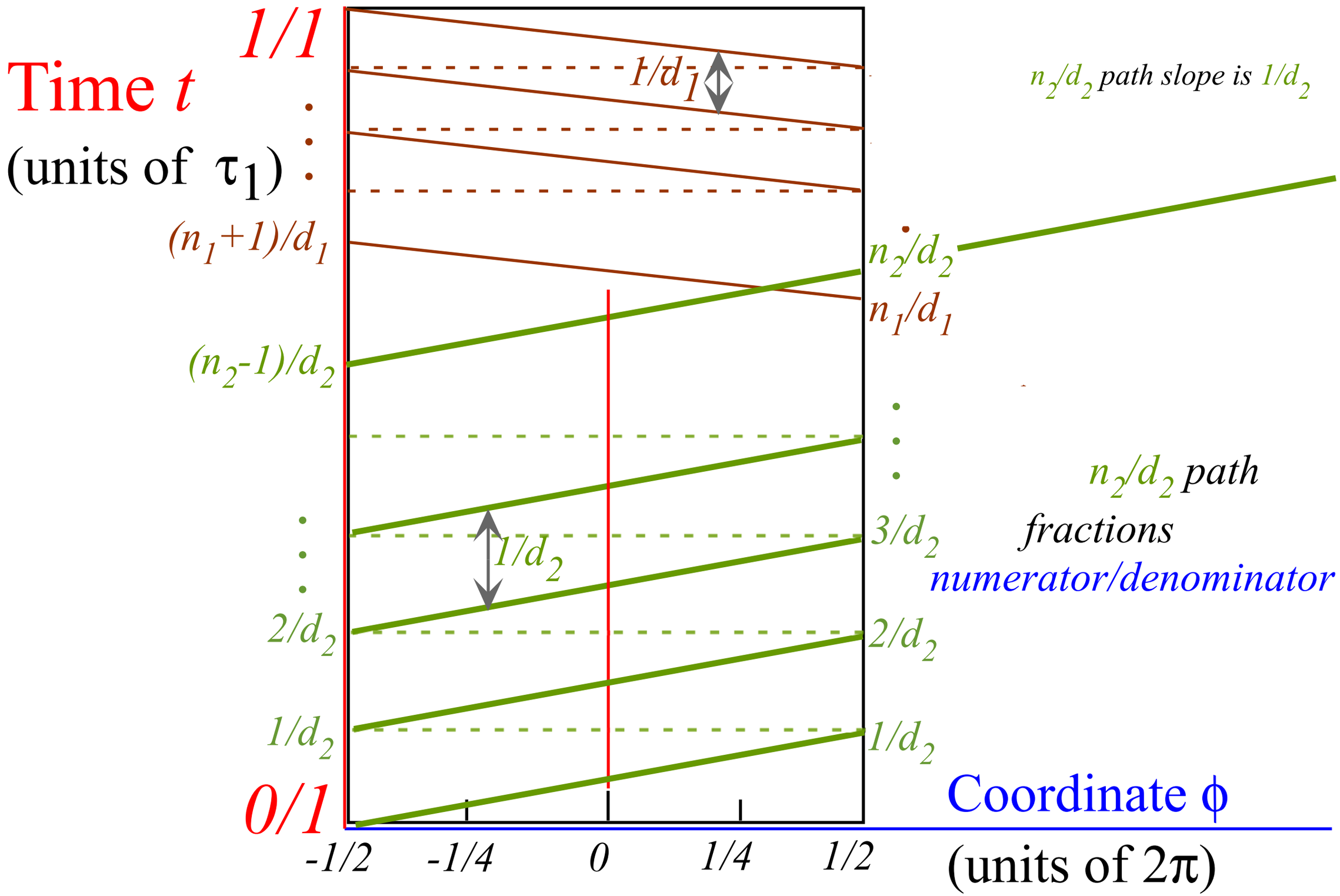
Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



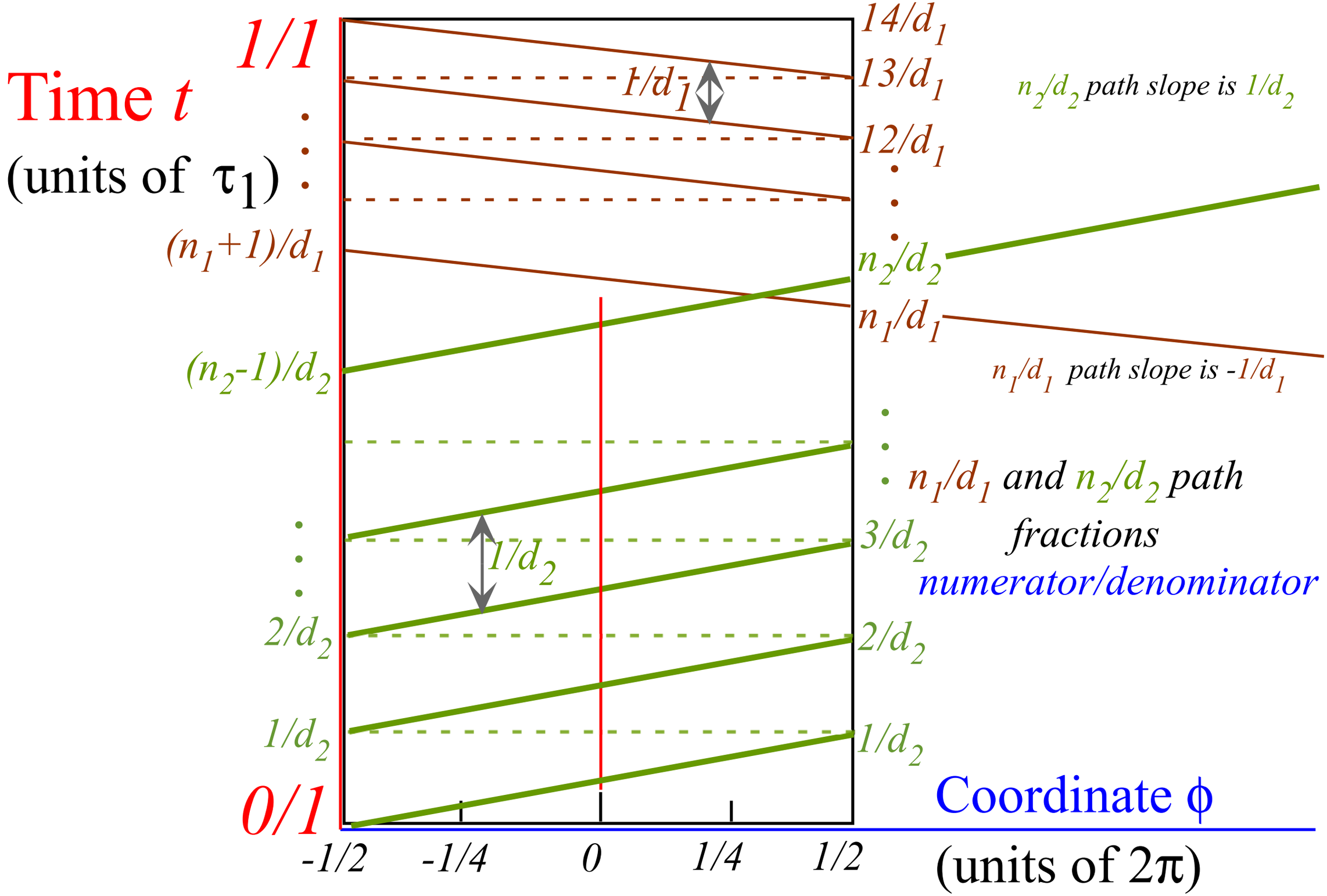
Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



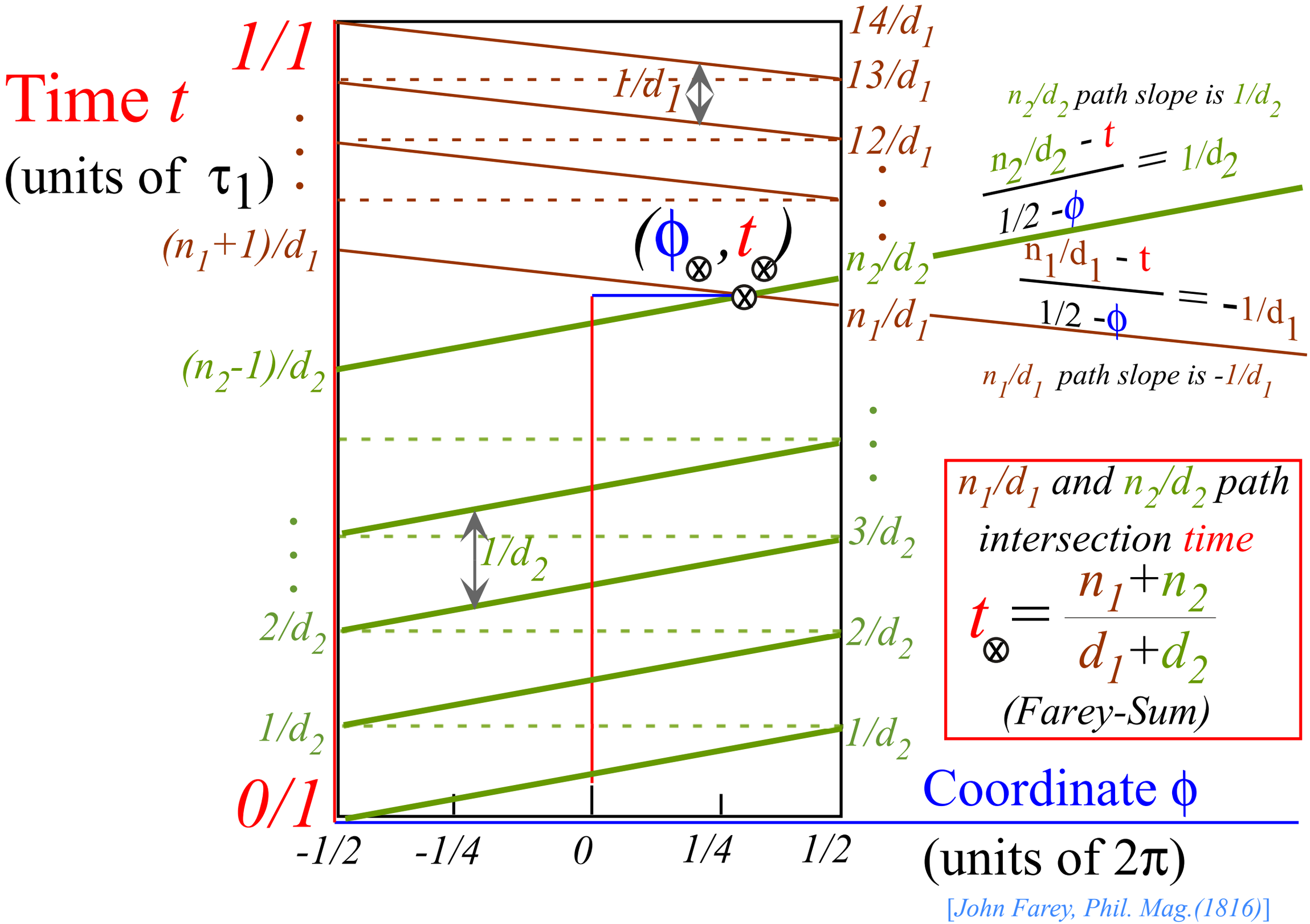
Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



Farey Sum algebra of revival-beat wave dynamics

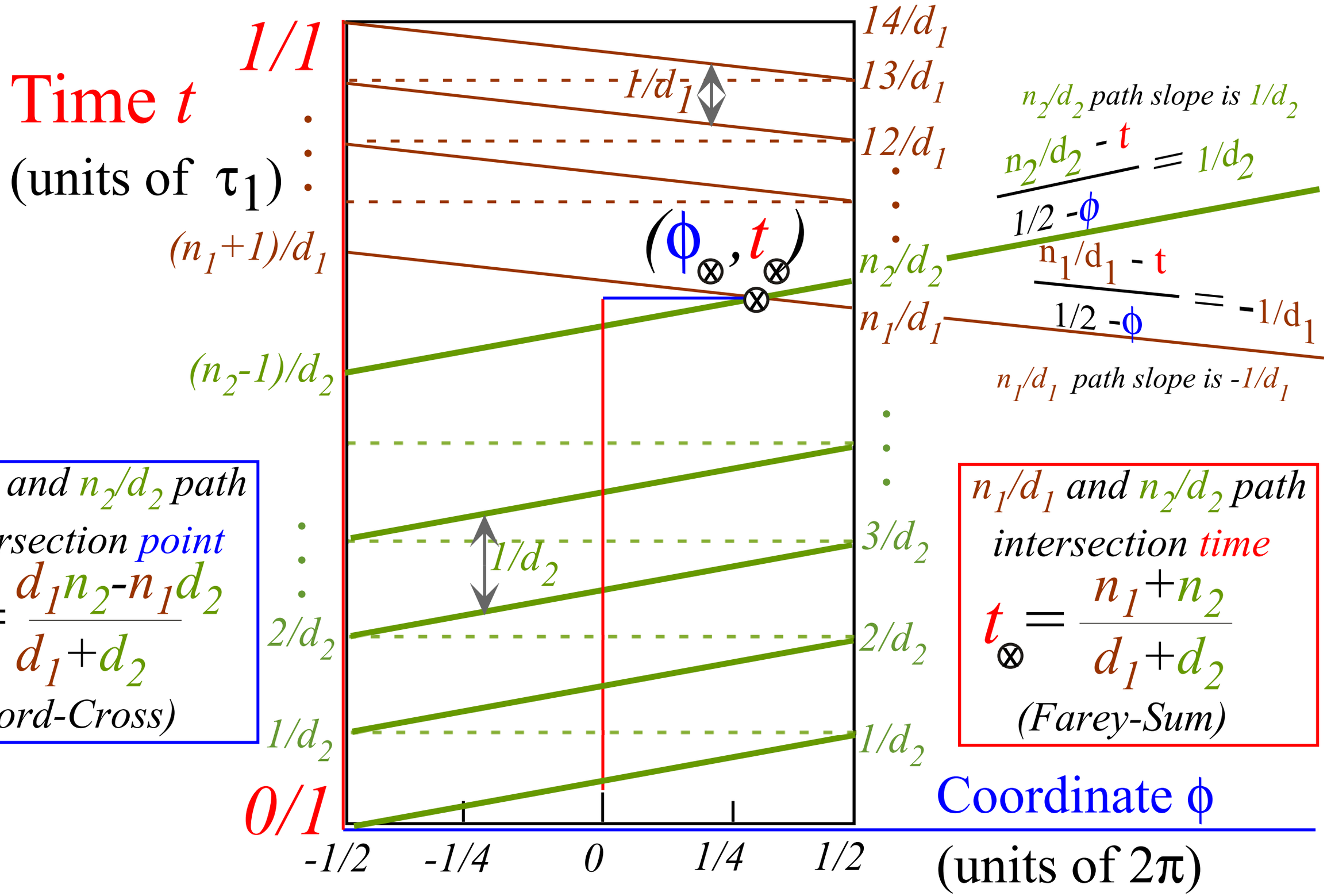
Label by *numerators N* and *denominators D* of rational fractions *N/D*



[John Farey, Phil. Mag.(1816)]

Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



n_1/d_1 *and* n_2/d_2 *path intersection point*

$$\phi_{\otimes} = \frac{d_1 n_2 - n_1 d_2}{d_1 + d_2}$$

(Ford-Cross)

n_1/d_1 *and* n_2/d_2 *path intersection time*

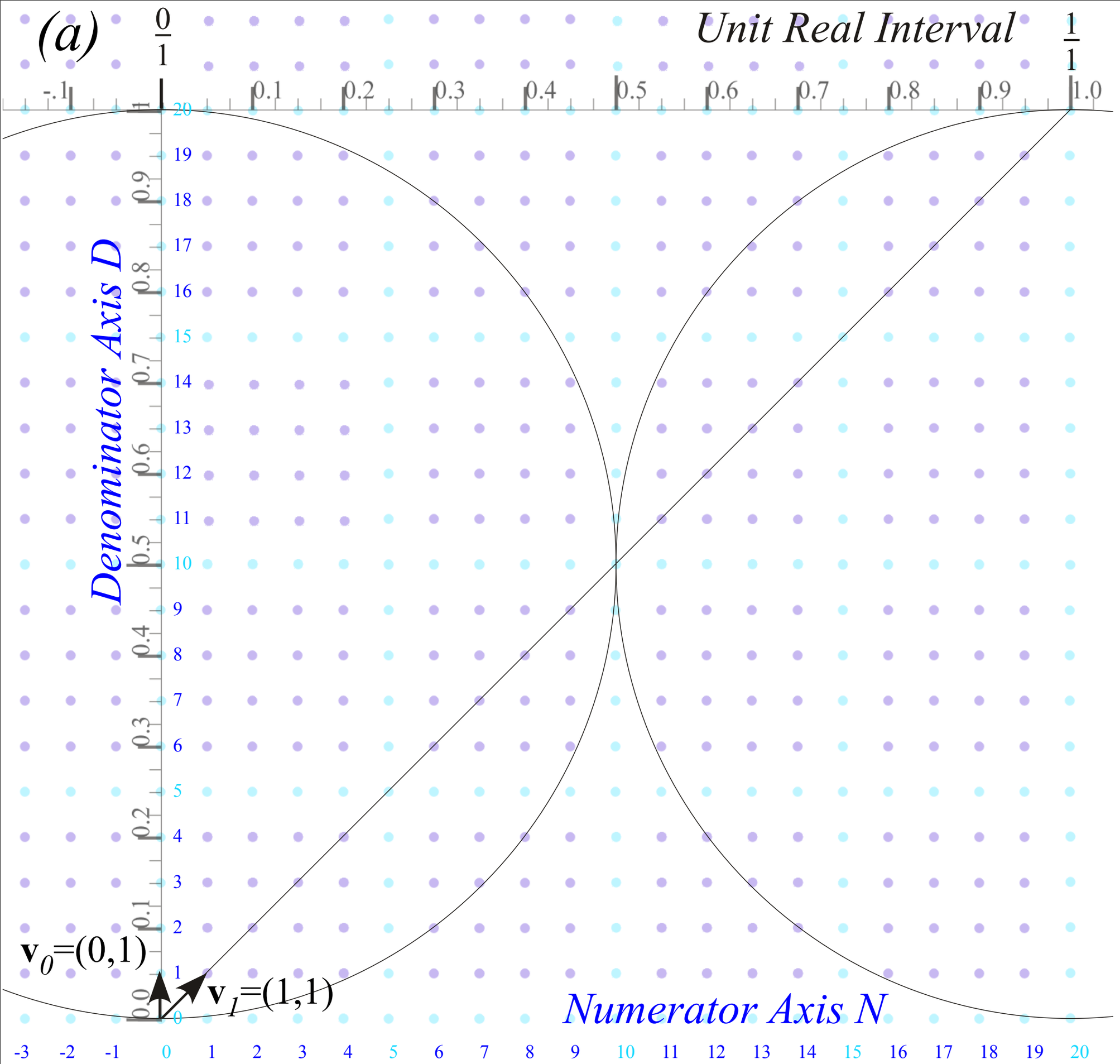
$$t_{\otimes} = \frac{n_1 + n_2}{d_1 + d_2}$$

(Farey-Sum)

[*Lester. R. Ford, Am. Math. Monthly 45,586(1938)*]

[*John Farey, Phil. Mag.(1816)*]

Ford-Circle geometry of revival paths
(How *Rational Fractions N/D* occupy real space-time)

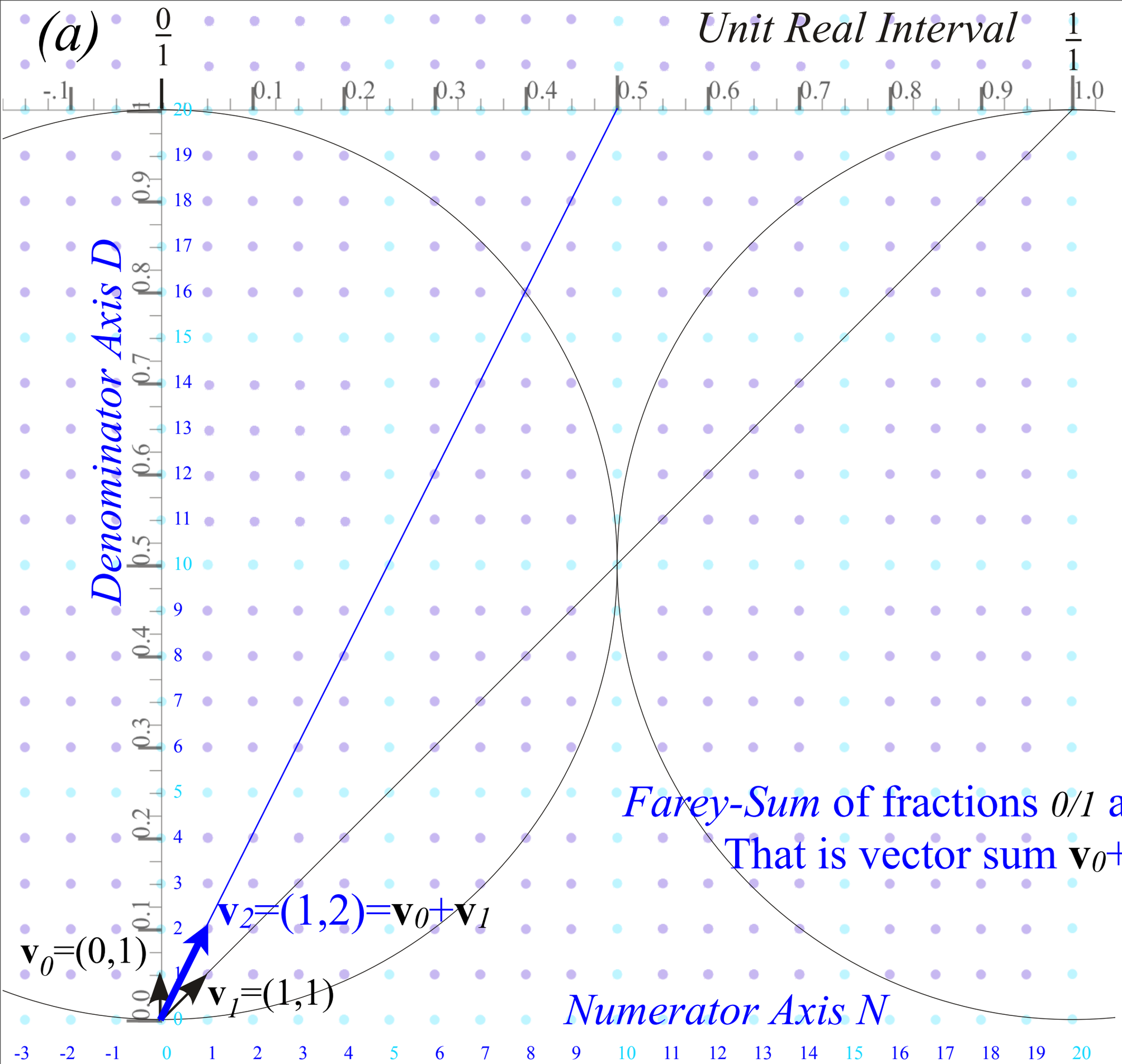


Farey Sum related to vector sum and *Ford Circles*

1/1-circle has diameter *1*

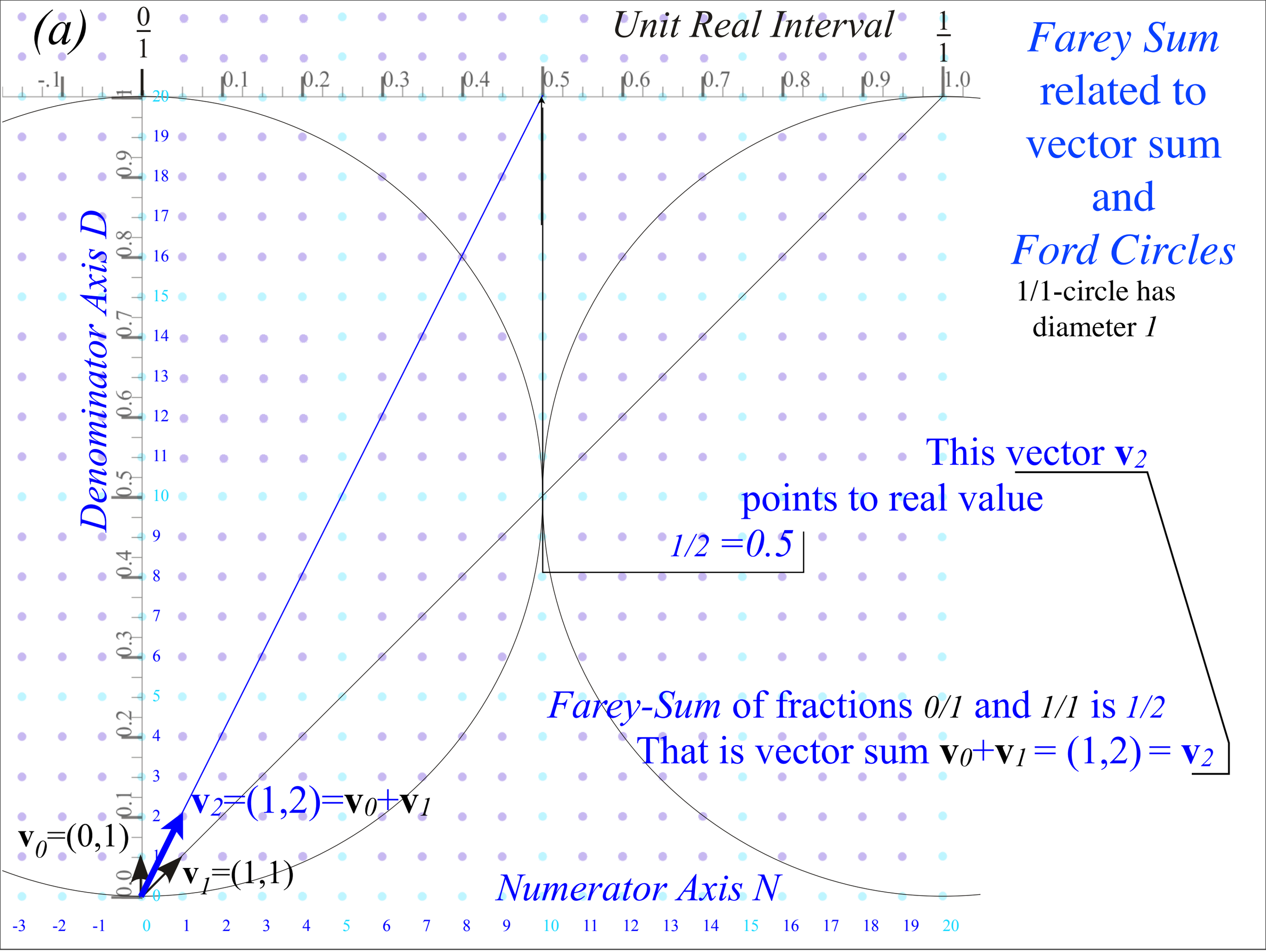
Farey Sum related to vector sum and *Ford Circles*

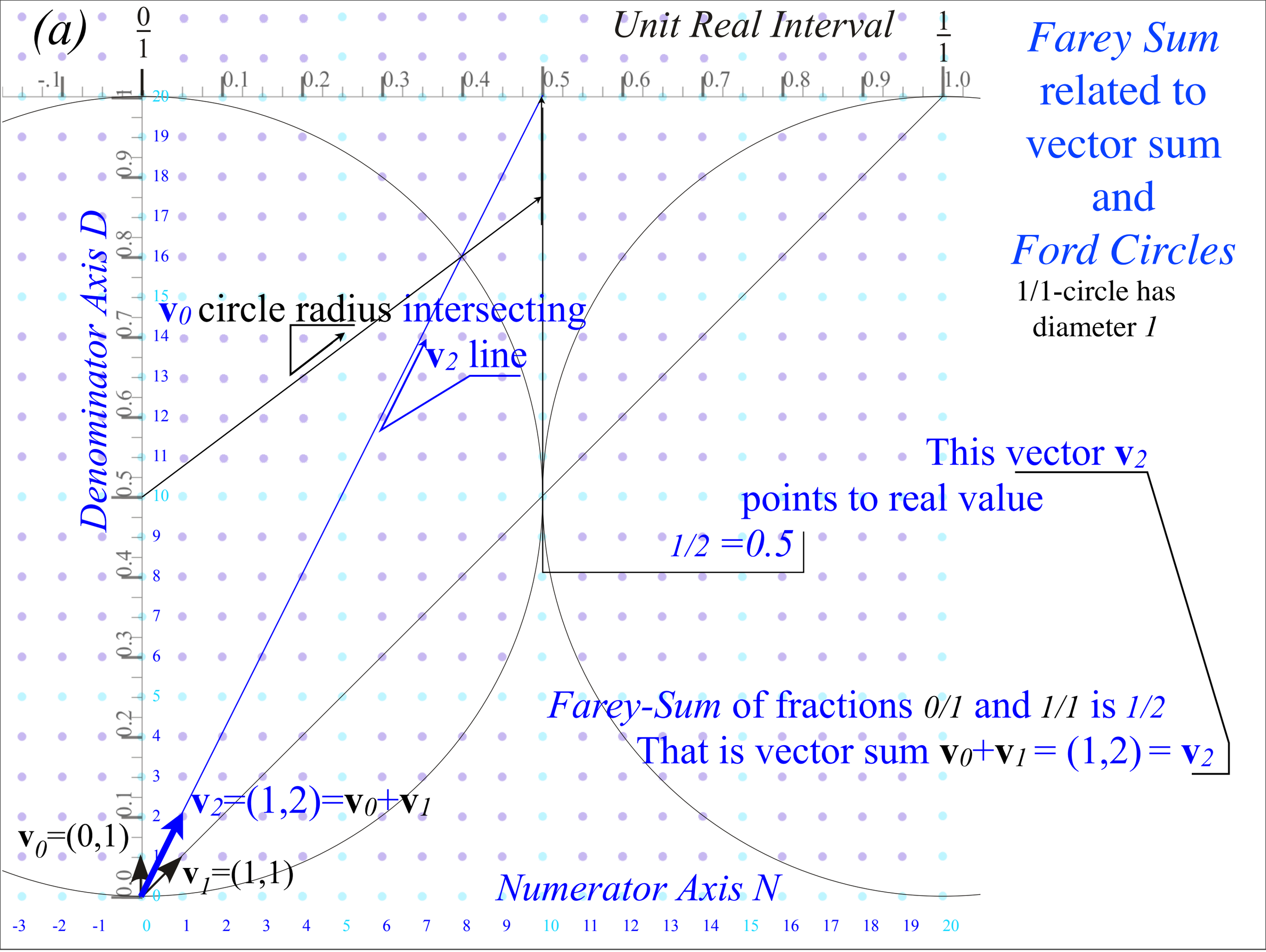
1/1-circle has diameter *1*

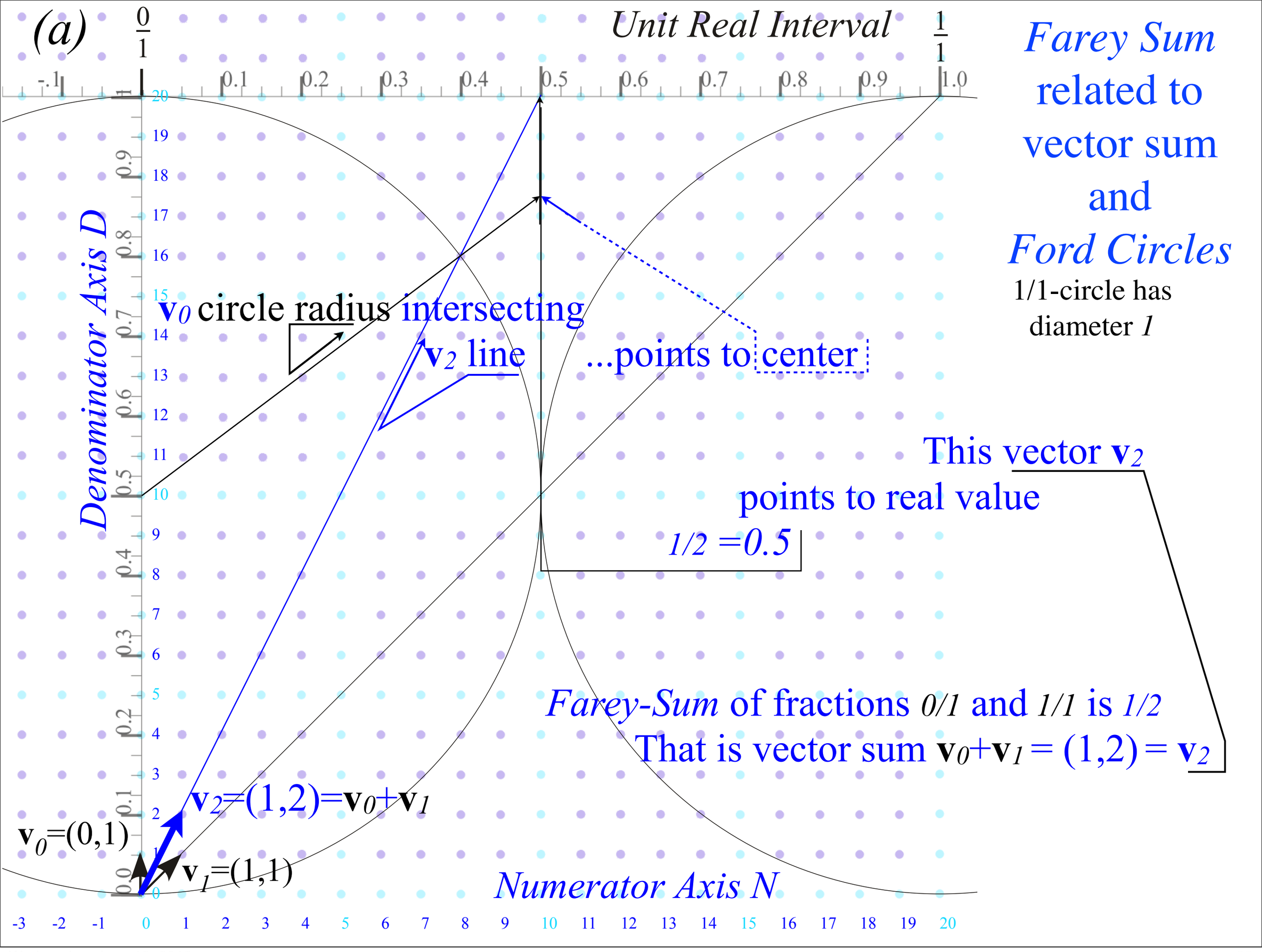


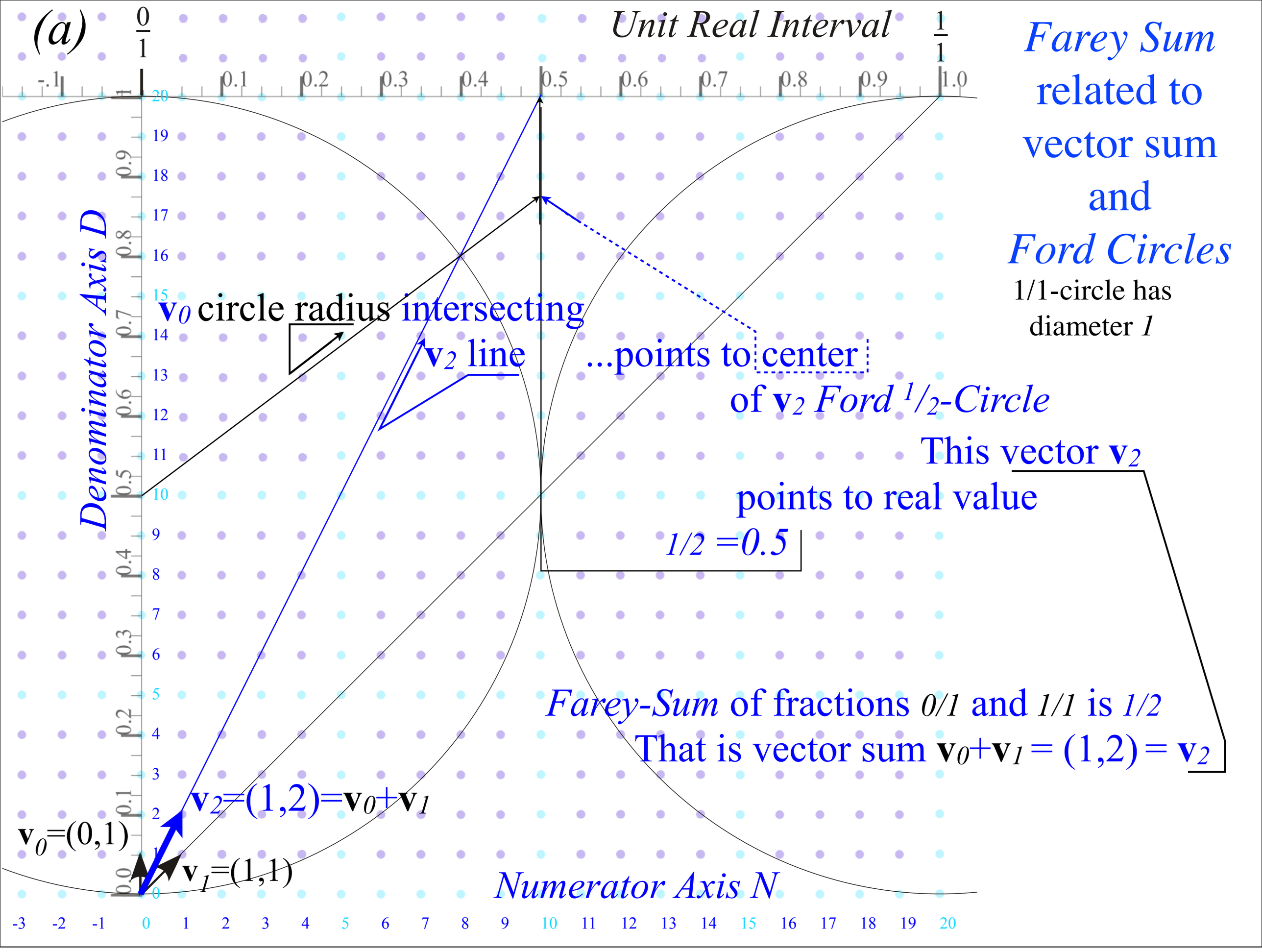
Farey-Sum of fractions *0/1* and *1/1* is *1/2*

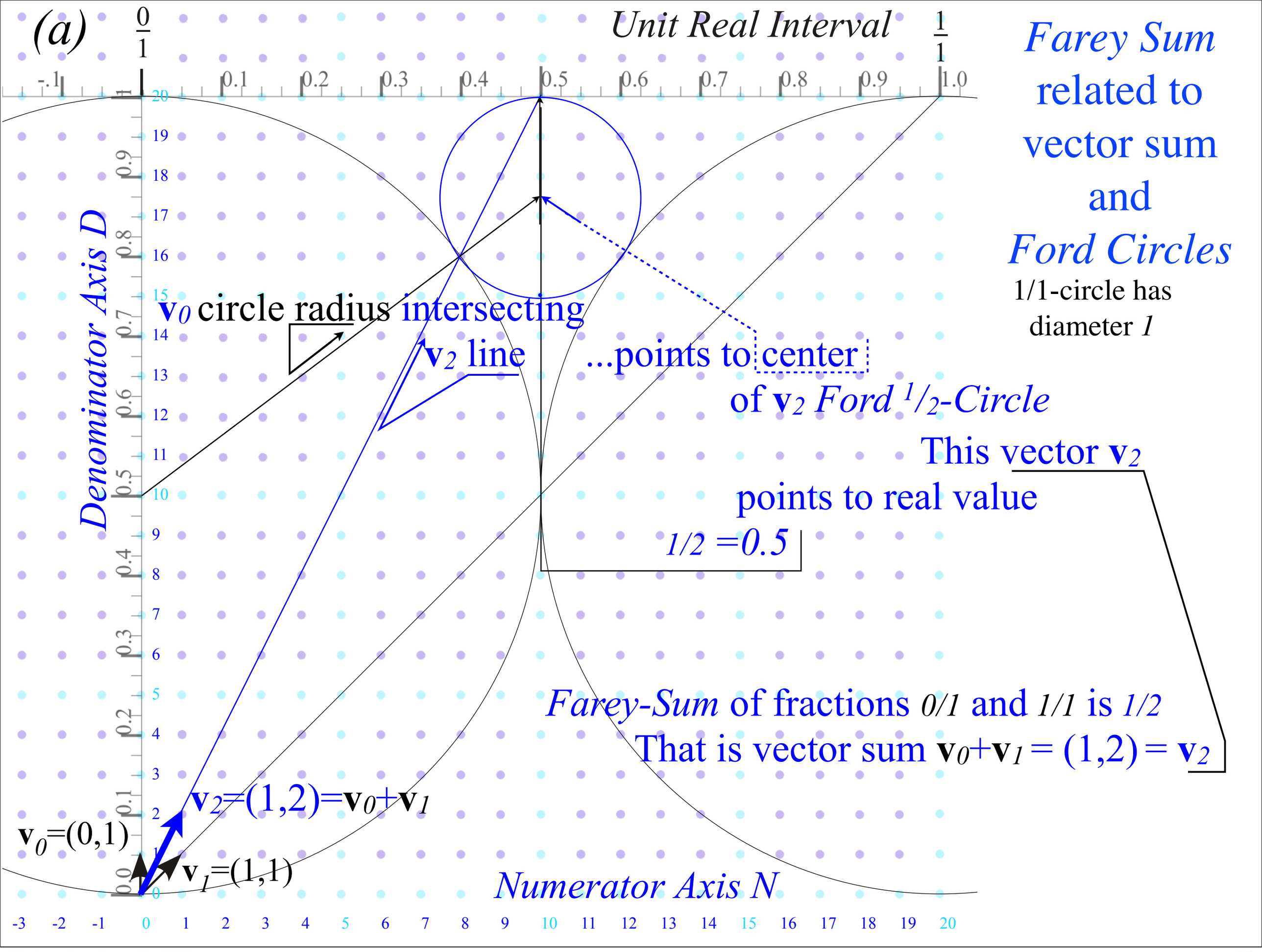
That is vector sum $\mathbf{v}_0+\mathbf{v}_1 = (1,2) = \mathbf{v}_2$

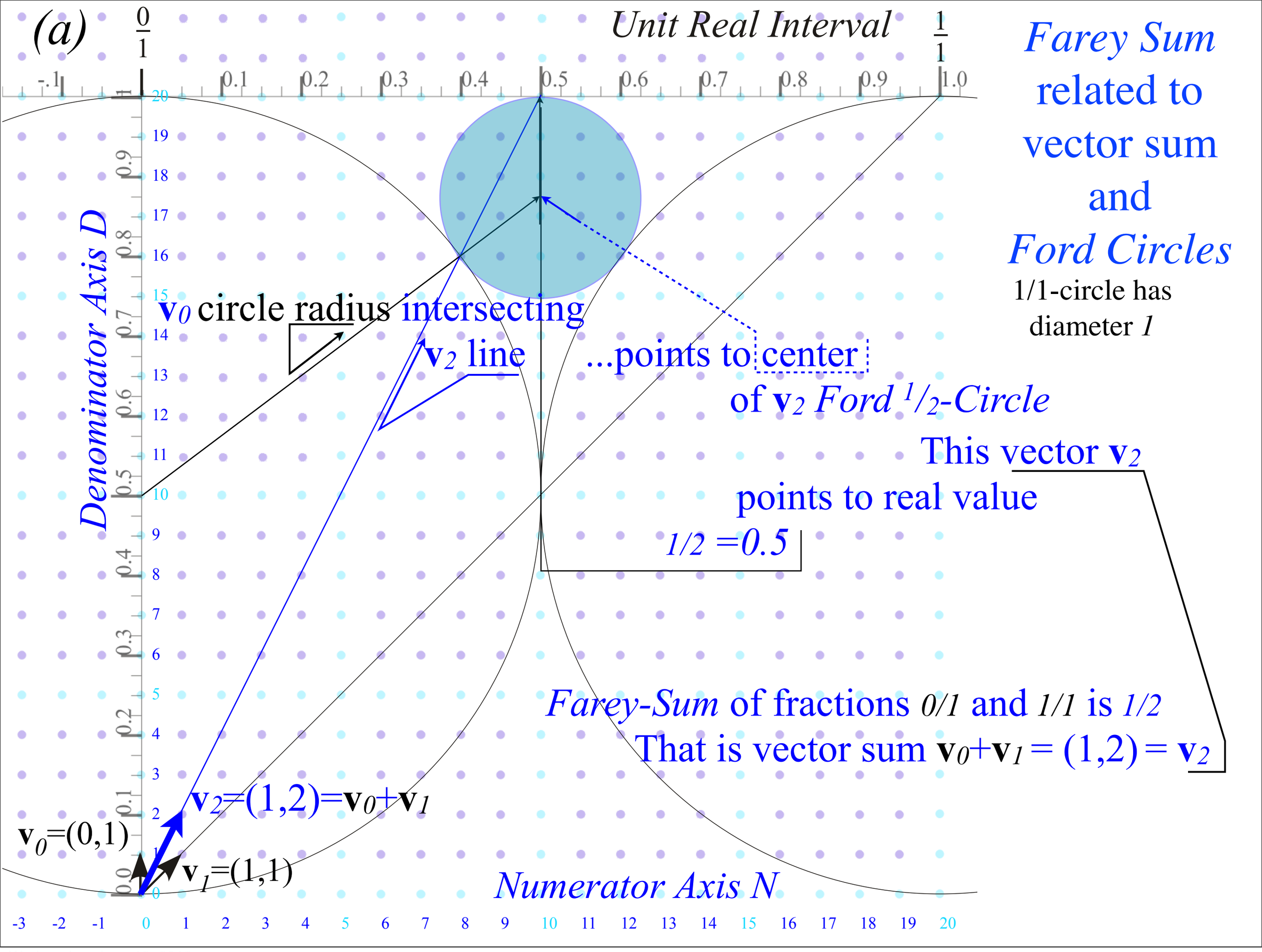

(a)
0/1
Unit Real Interval
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
This vector v2 points to real value 1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
v2=(1,2)=v0+v1
v0=(0,1)
v1=(1,1)
Denominator Axis D
Numerator Axis N

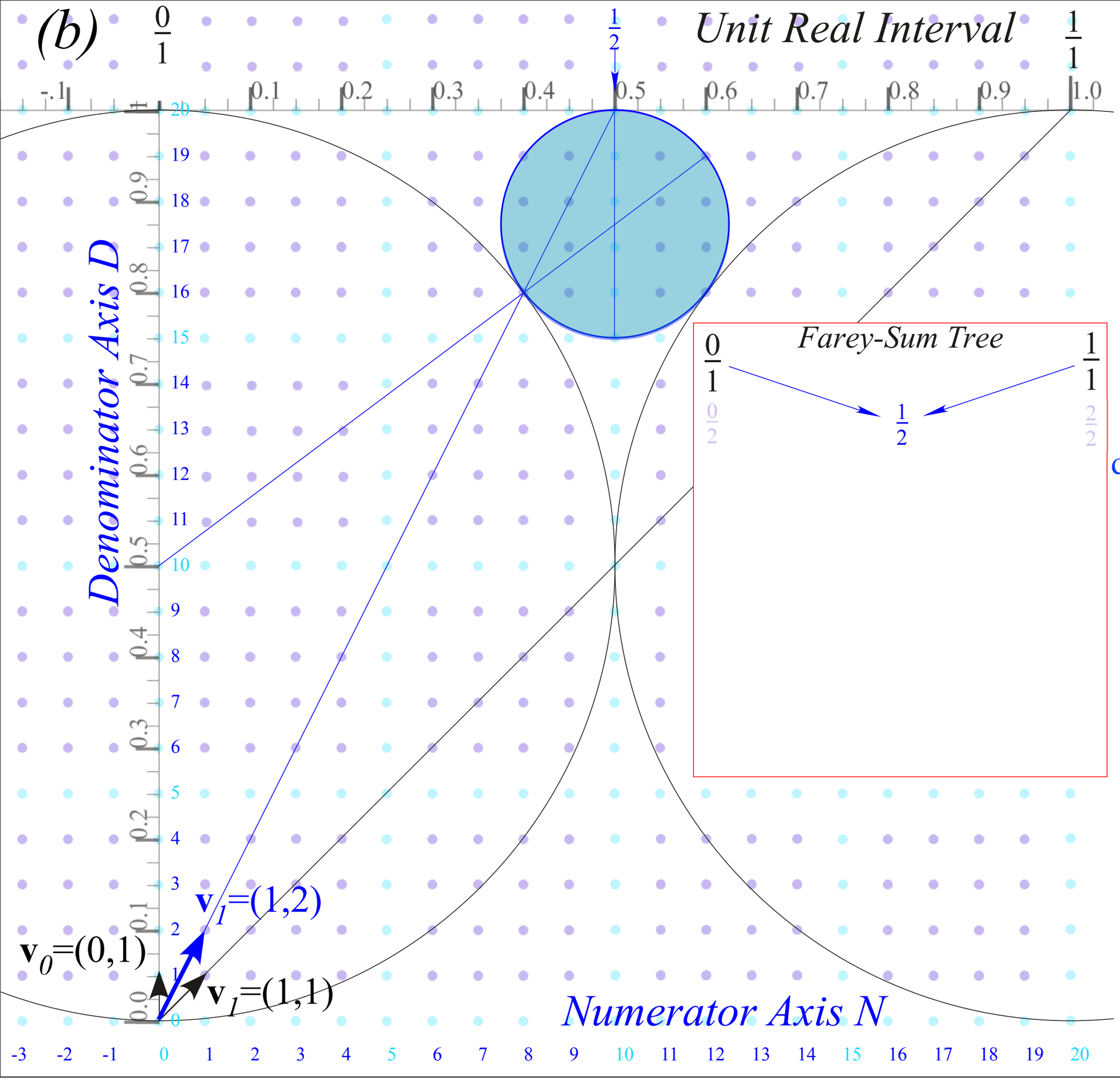

(a)
0/1
Unit Real Interval
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
v0 circle radius intersecting v2 line
This vector v2 points to real value 1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
v2=(1,2)=v0+v1
v0=(0,1)
v1=(1,1)
Numerator Axis N


(a)
0/1
Unit Real Interval
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
v0 circle radius intersecting
v2 line
...points to center
This vector v2 points to real value 1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
v2=(1,2)=v0+v1
v0=(0,1)
v1=(1,1)
Numerator Axis N


(a)
0/1
Unit Real Interval
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
v0 circle radius intersecting
v2 line
...points to center
of v2 Ford 1/2-Circle
This vector v2 points to real value
1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
v2=(1,2)=v0+v1
v0=(0,1)
v1=(1,1)
Numerator Axis N


(a)
Unit Real Interval
0/1
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
v0 circle radius intersecting
v2 line
...points to center
of v2 Ford 1/2-Circle
This vector v2 points to real value
1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
v2=(1,2)=v0+v1
v0=(0,1)
v1=(1,1)
Numerator Axis N


(a)
Unit Real Interval
0/1
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
v0 circle radius intersecting
v2 line
...points to center
of v2 Ford 1/2-Circle
This vector v2 points to real value
1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
v2=(1,2)=v0+v1
v0=(0,1)
v1=(1,1)
Numerator Axis N

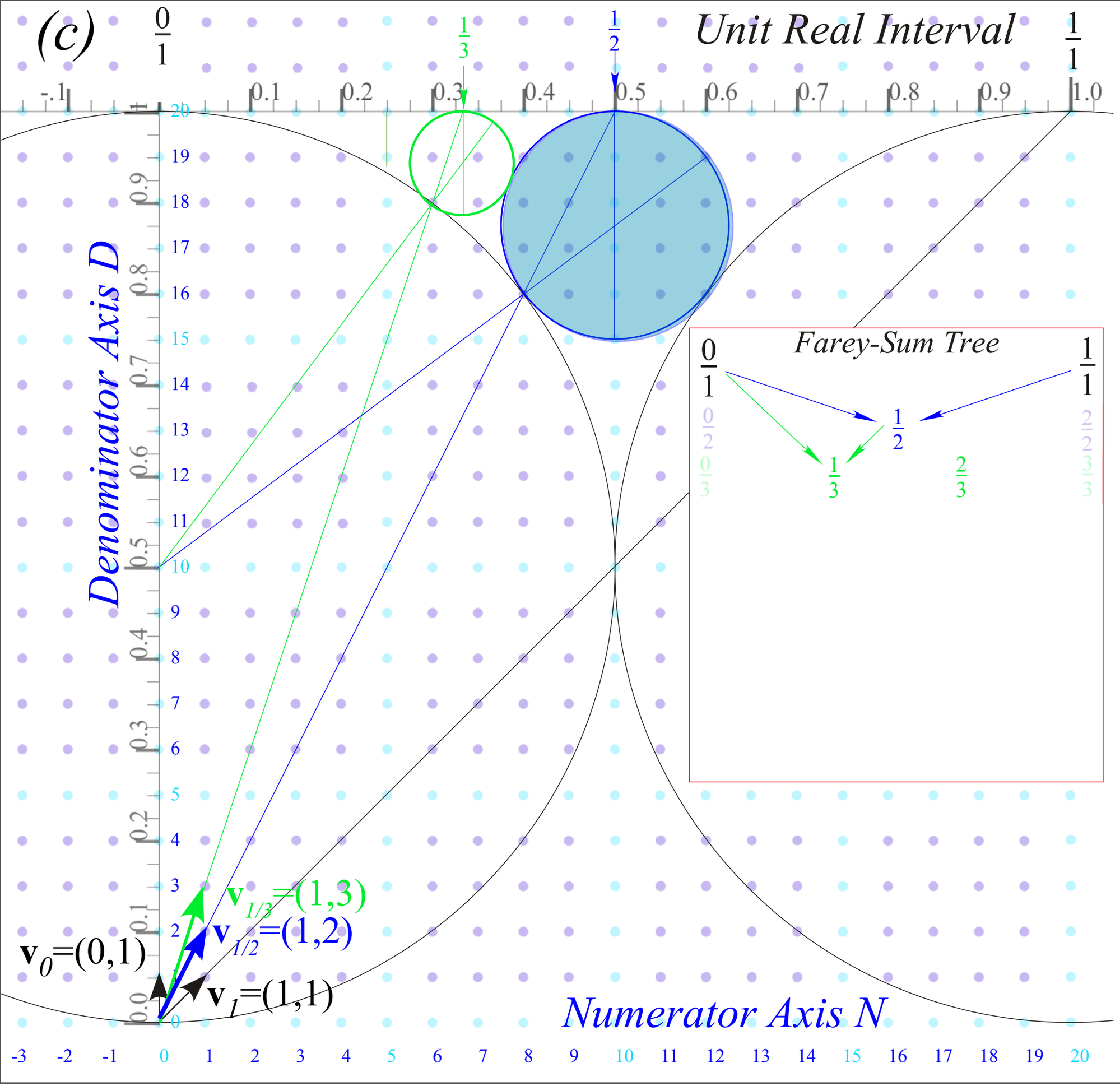


Farey Sum related to vector sum and *Ford Circles*

1/1-circle has diameter *1*

1/2-circle has diameter $1/2^2=1/4$

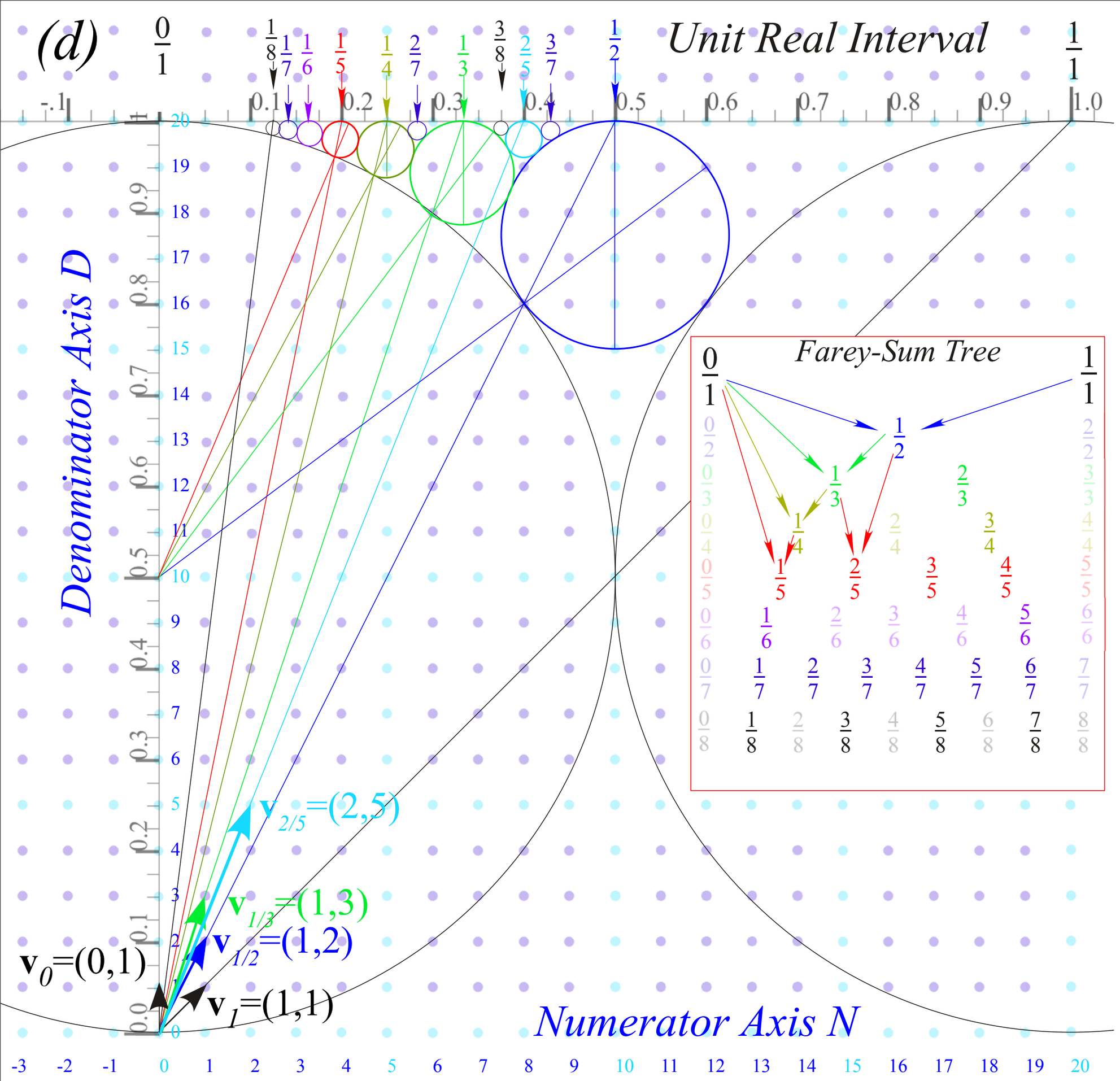
Farey Sum related to vector sum and *Ford Circles*



1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

Farey Sum related to vector sum and *Ford Circles*

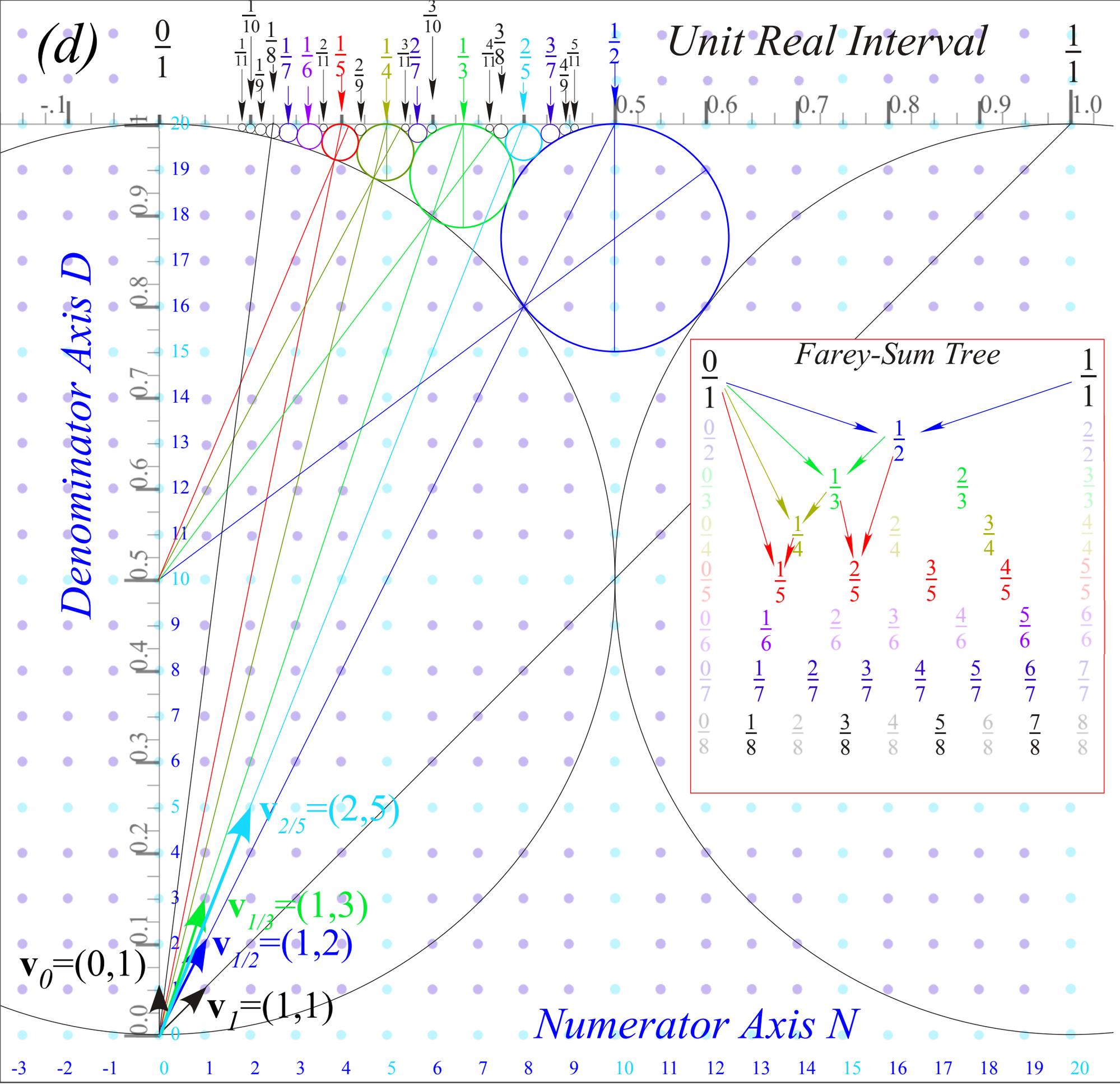


1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

n/d-circles have diameter $1/d^2$

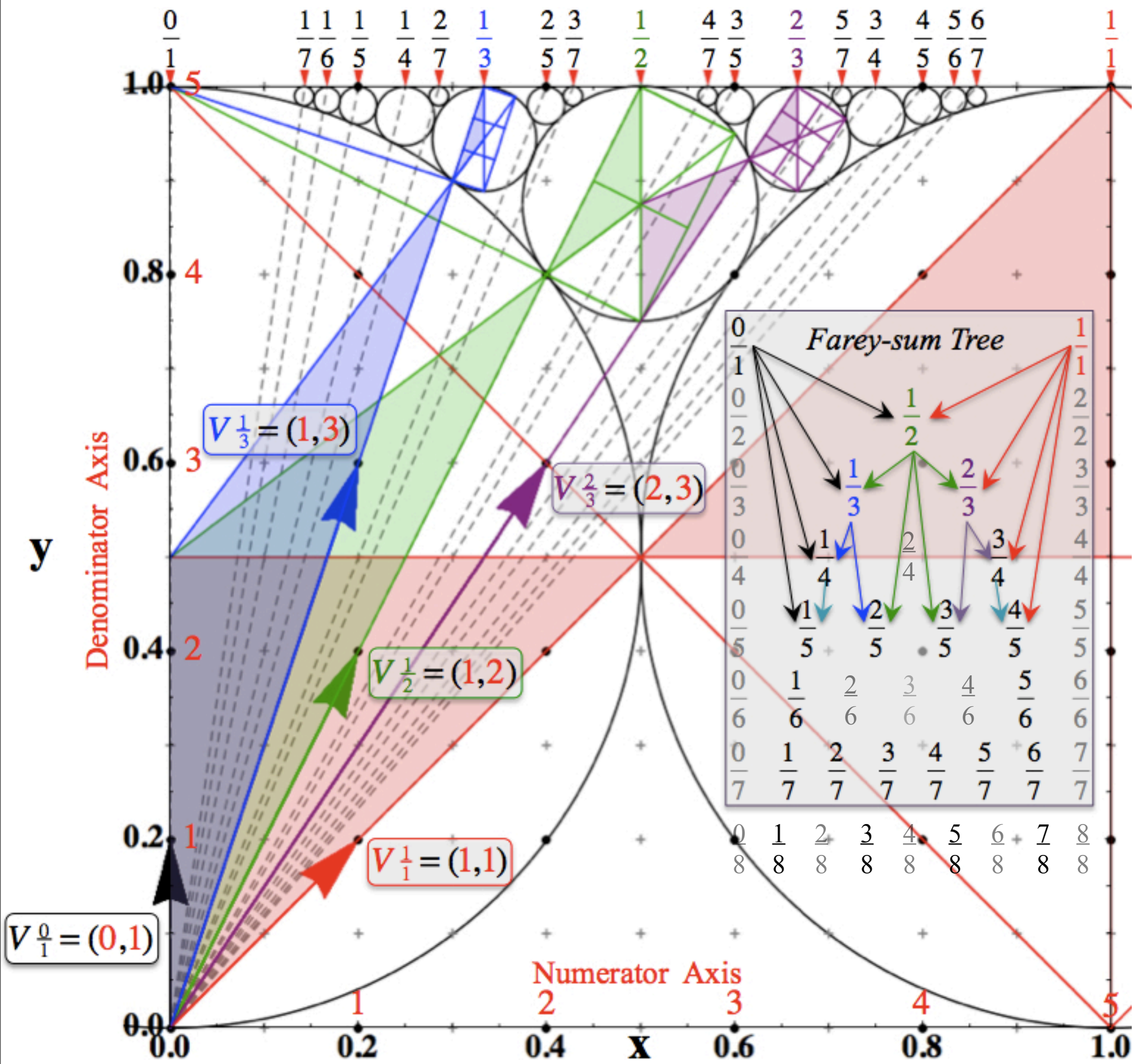
Farey Sum related to vector sum and *Ford Circles*



1/2-circle has diameter $1/2^2=1/4$

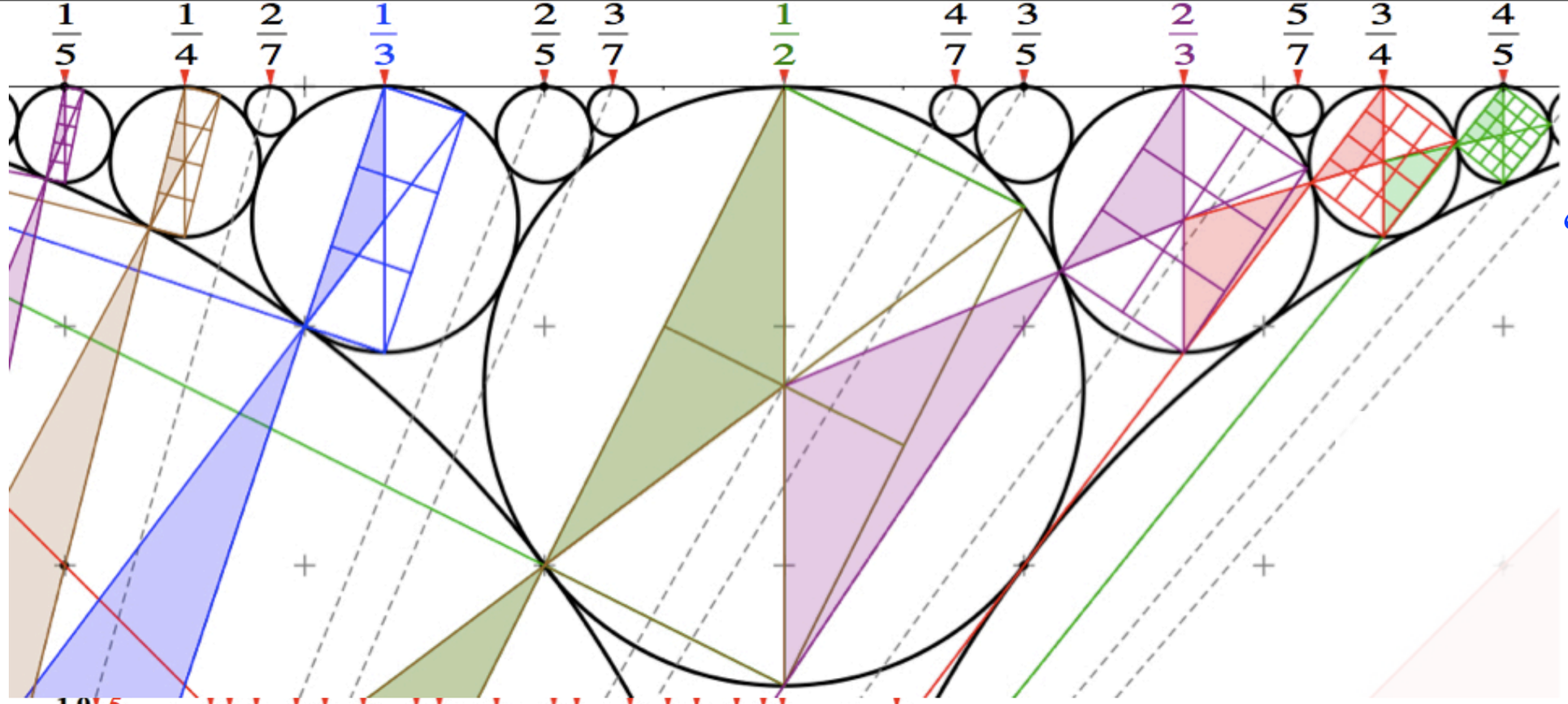
1/3-circles have diameter $1/3^2=1/9$

n/d-circles have diameter $1/d^2$



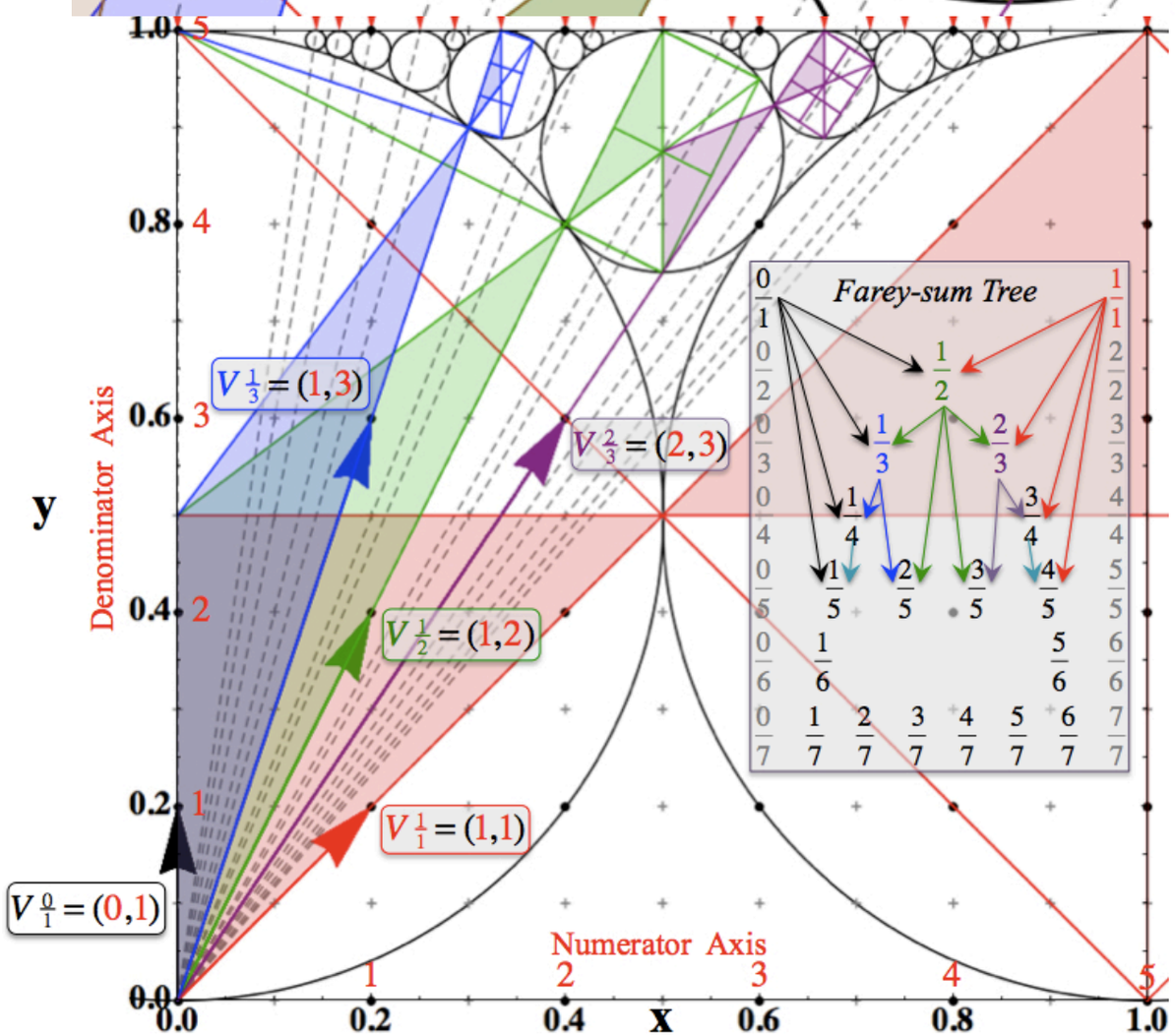
Thales Rectangles

provide
analytic geometry
of
fractal structure



"Quantized" Thales Rectangles

provide analytic geometry of fractal structure



Rotor-fractional-revivals of Ψ-phase and amplitude

(...in space-time…)

2-level-system and C_2 symmetry phase dynamics

C_2 Character Table describes eigenstates

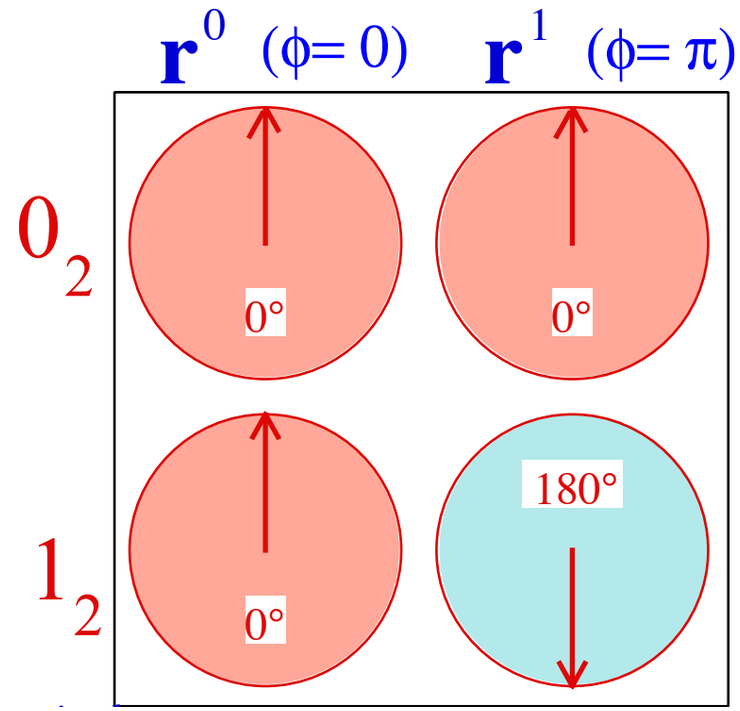
symmetric A_1

vs.

antisymmetric A_2

	$1 = r^0$	$r = r^1$
$0 \bmod 2$	1	1
$\pm 1 \bmod 2$	1	-1

C_2 *Phasor*-Character Table



2-level-system and C_2 symmetry phase dynamics

C_2 Character Table describes eigenstates

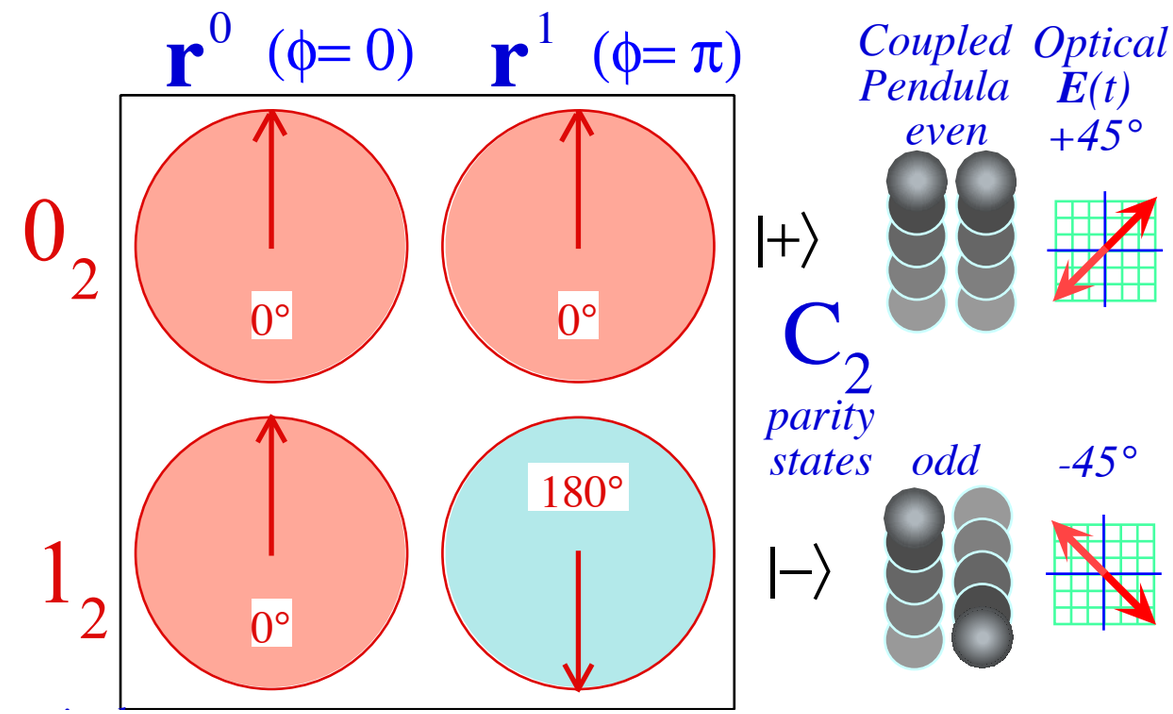
symmetric A_1

vs.

antisymmetric A_2

	$1 = r^0$	$r = r^1$
$0 \bmod 2$	1	1
$\pm 1 \bmod 2$	1	-1

C_2 *Phasor*-Character Table



2-level-system and C_2 symmetry phase dynamics

C_2 Character Table describes eigenstates

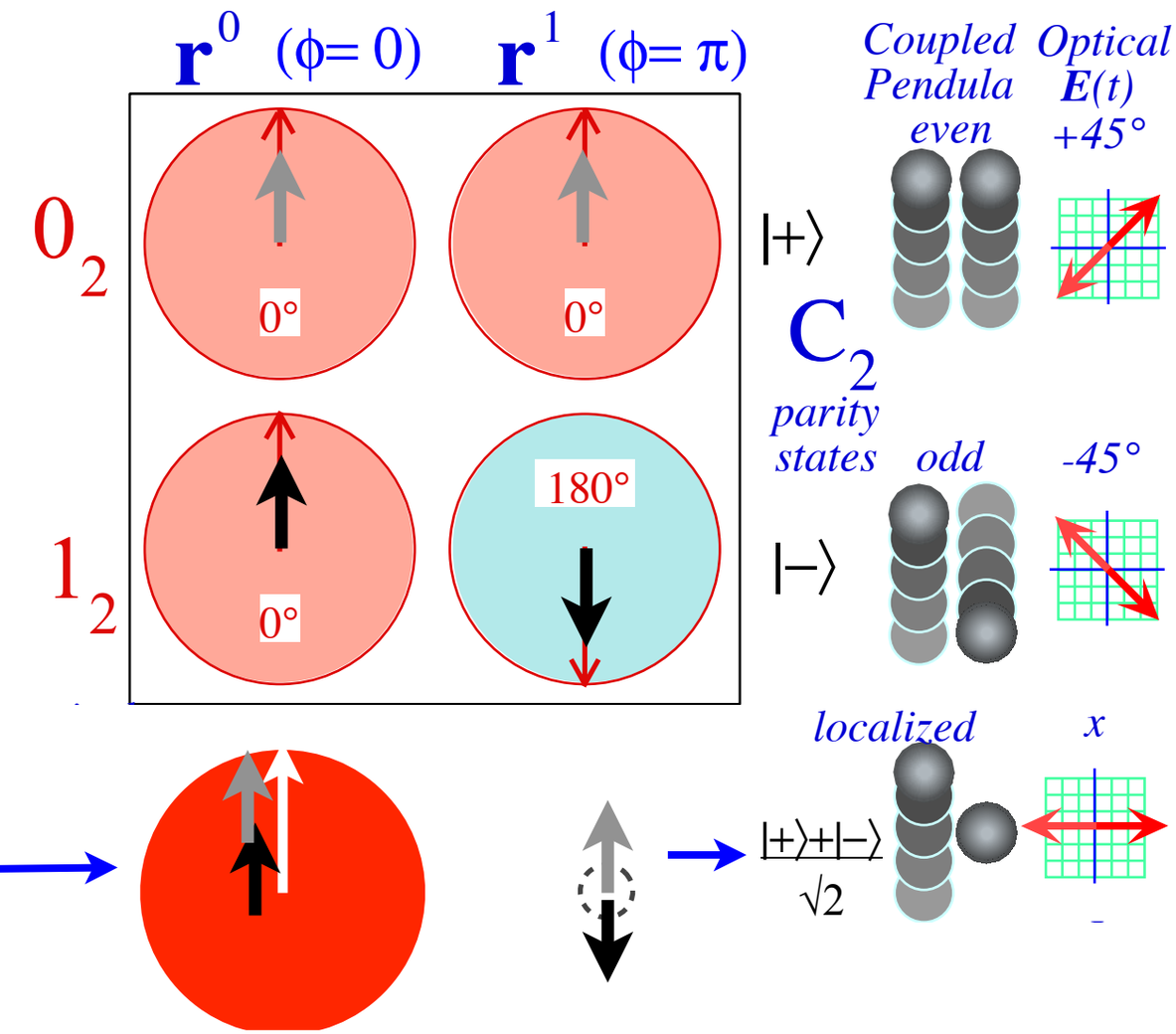
symmetric A_1

vs.

antisymmetric A_2

	$1 = r^0$	$r = r^1$
$0 \bmod 2$	1	1
$\pm 1 \bmod 2$	1	-1

C_2 *Phasor*-Character Table



Phasor C_2 Characters describe local state beats

Initial sum

2-level-system and C_2 symmetry phase dynamics

C_2 Character Table describes eigenstates

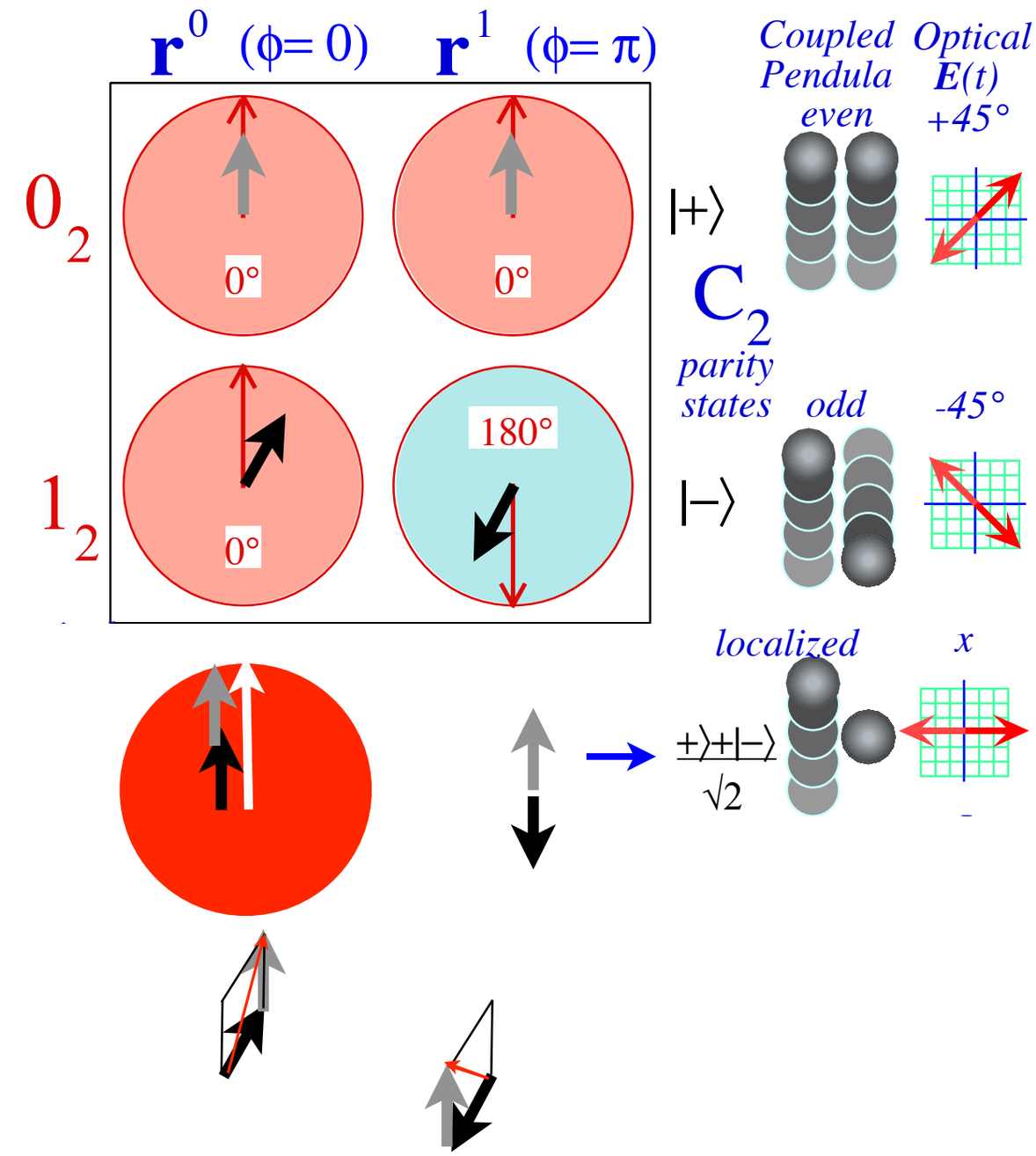
symmetric A_1

vs.

antisymmetric A_2

	$1 = r^0$	$r = r^1$
$0 \bmod 2$	1	1
$\pm 1 \bmod 2$	1	-1

C_2 *Phasor*-Character Table



Phasor C_2 Characters describe local state beats

Initial sum

2-level-system and C_2 symmetry phase dynamics

C_2 Character Table describes eigenstates

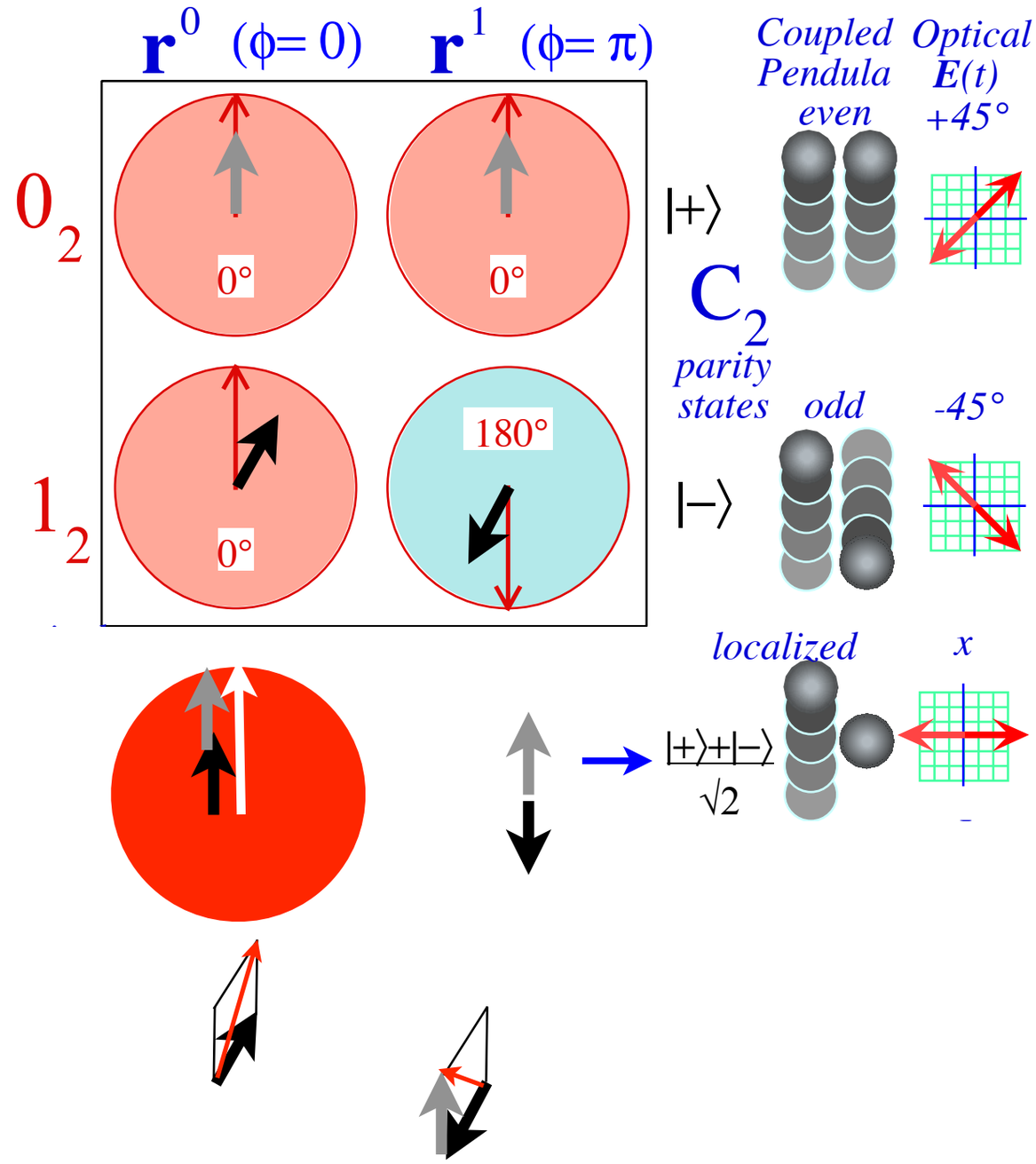
symmetric A_1

vs.

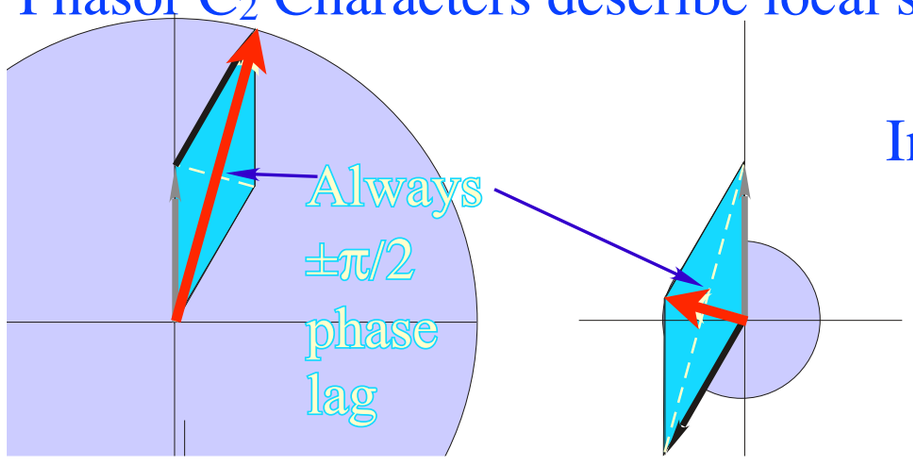
antisymmetric A_2

	$1 = r^0$	$r = r^1$
$0 \bmod 2$	1	1
$\pm 1 \bmod 2$	1	-1

C_2 *Phasor*-Character Table



Phasor C_2 Characters describe local state beats



Initial sum

2-level-system and C_2 symmetry phase dynamics

C_2 Character Table describes eigenstates

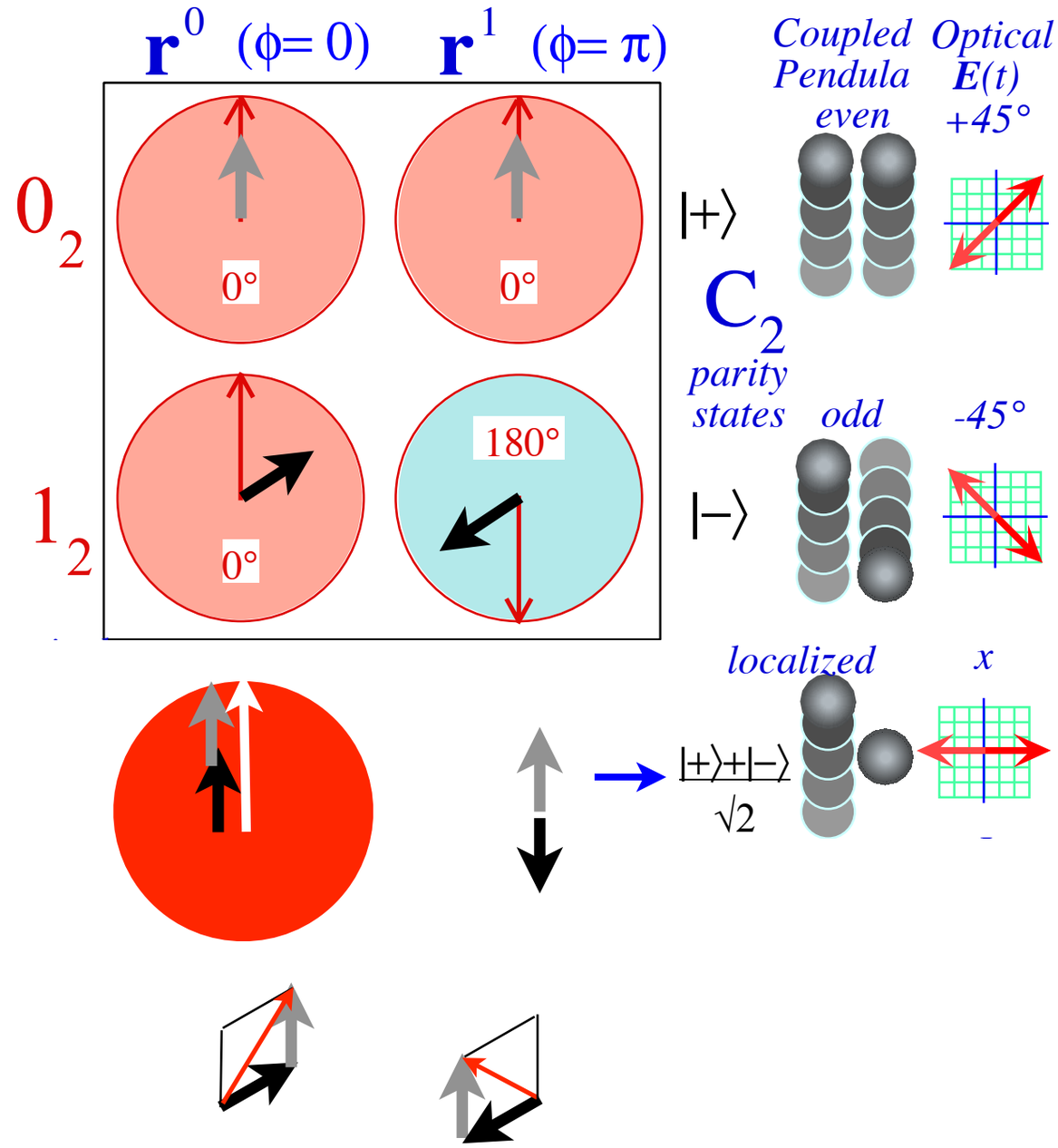
symmetric A_1

vs.

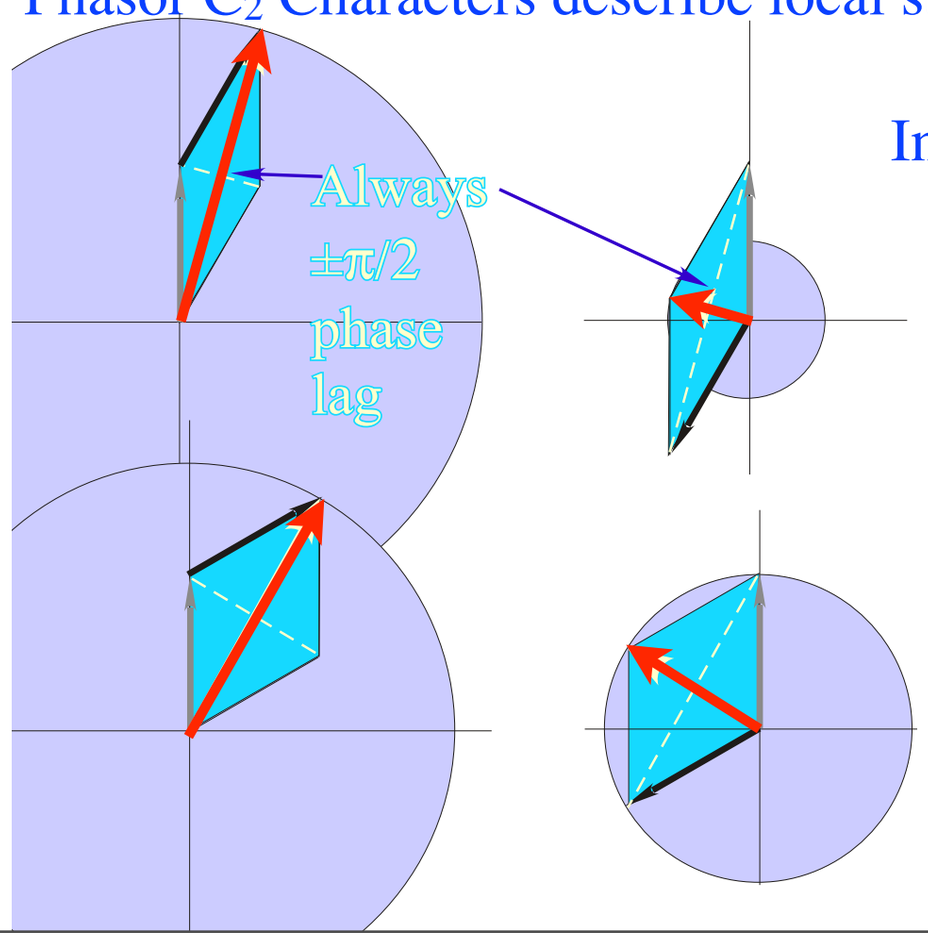
antisymmetric A_2

	$1 = r^0$	$r = r^1$
$0 \bmod 2$	1	1
$\pm 1 \bmod 2$	1	-1

C_2 *Phasor*-Character Table



Phasor C_2 Characters describe local state beats



Initial sum –

2-level-system and C_2 symmetry phase dynamics

C_2 Character Table describes eigenstates

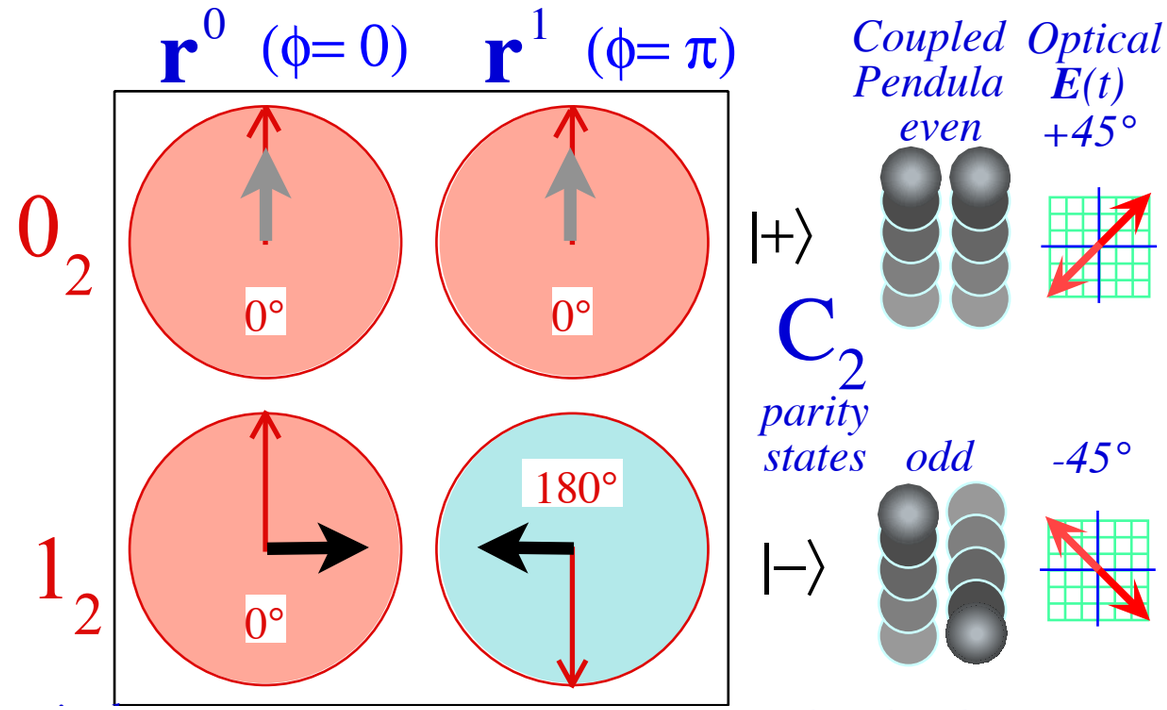
symmetric A_1

vs.

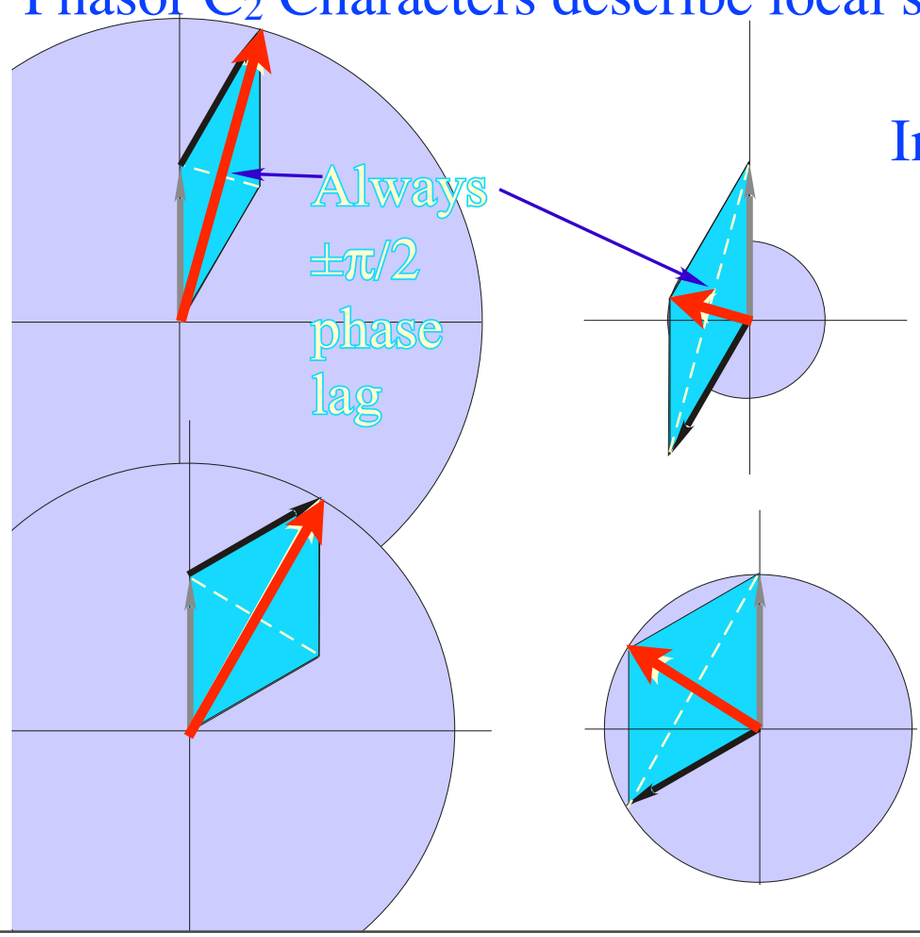
antisymmetric A_2

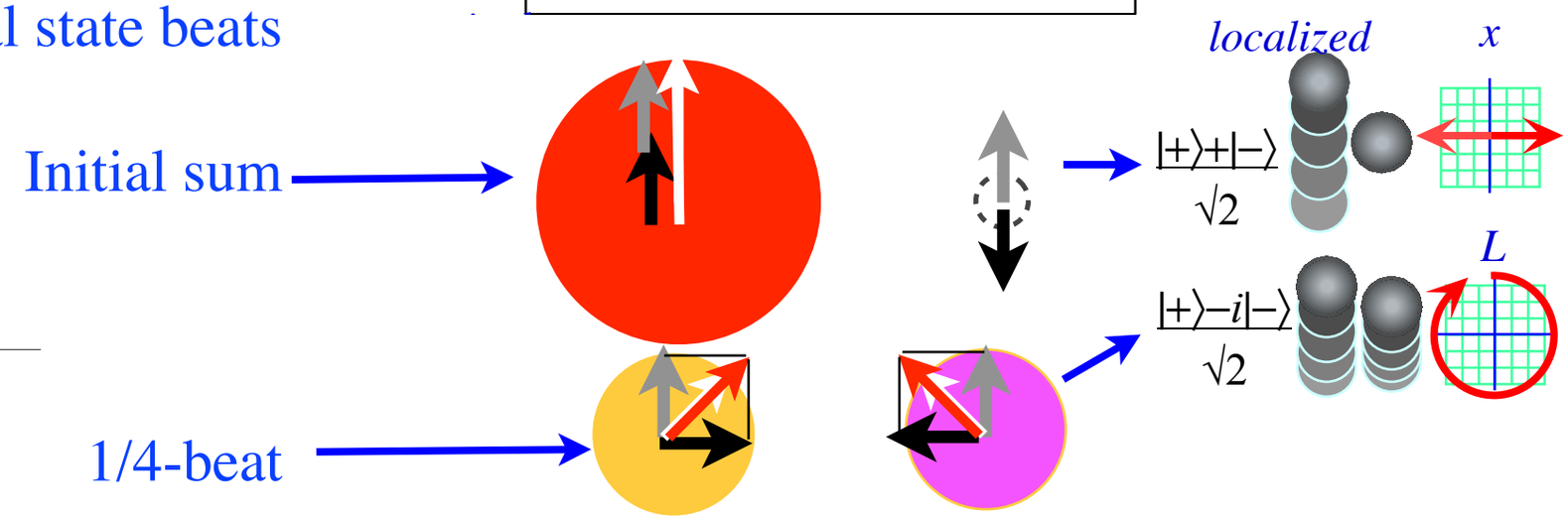
	$1 = r^0$	$r = r^1$
$0 \bmod 2$	1	1
$\pm 1 \bmod 2$	1	-1

C_2 *Phasor*-Character Table



Phasor C_2 Characters describe local state beats





2-level-system and C_2 symmetry phase dynamics

C_2 Character Table describes eigenstates

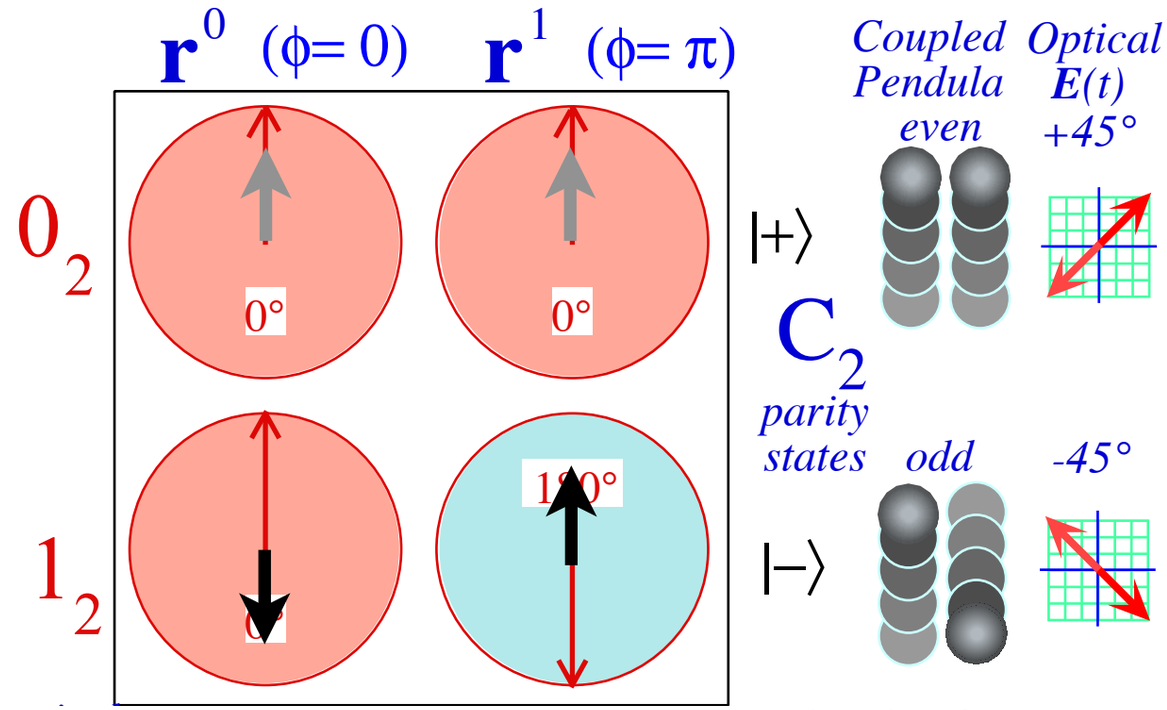
symmetric A_1

vs.

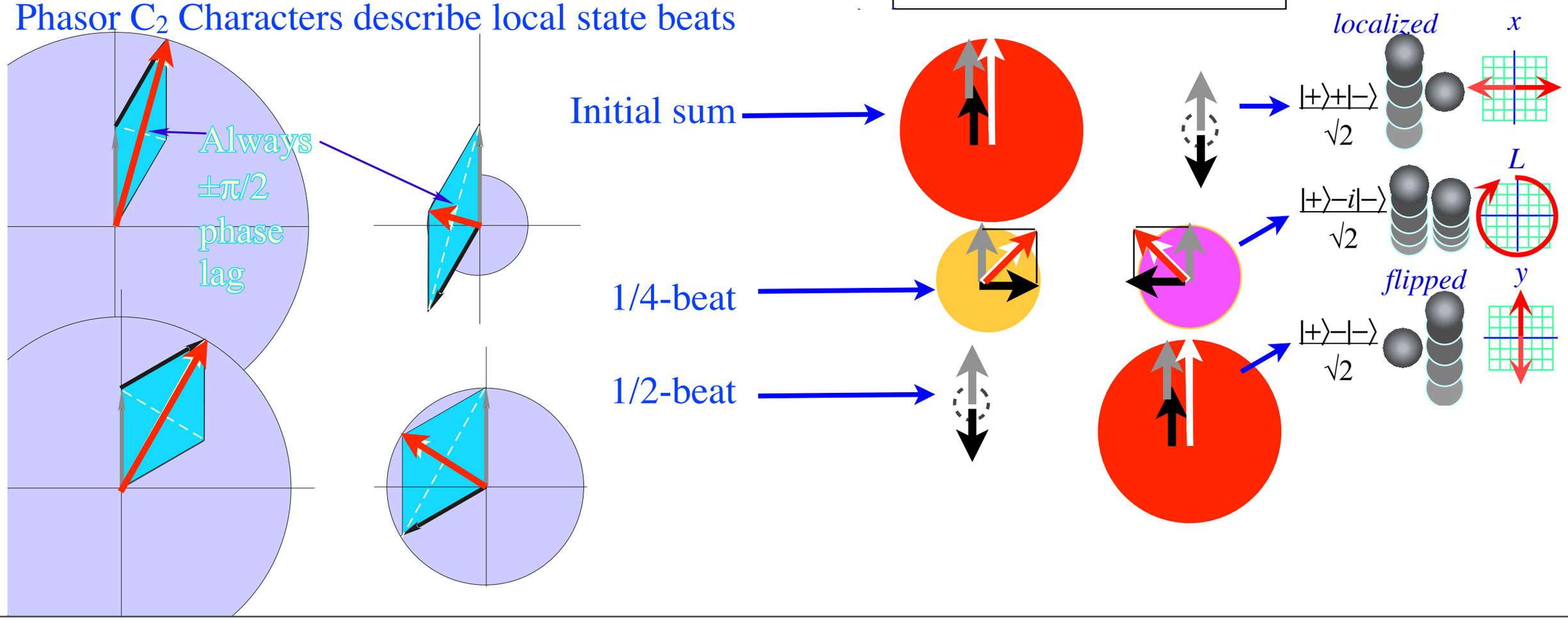
antisymmetric A_2

	$1 = r^0$	$r = r^1$
$0 \bmod 2$	1	1
$\pm 1 \bmod 2$	1	-1

C_2 *Phasor*-Character Table



Phasor C_2 Characters describe local state beats



2-level-system and C_2 symmetry beat dynamics

C_2 Character Table describes eigenstates

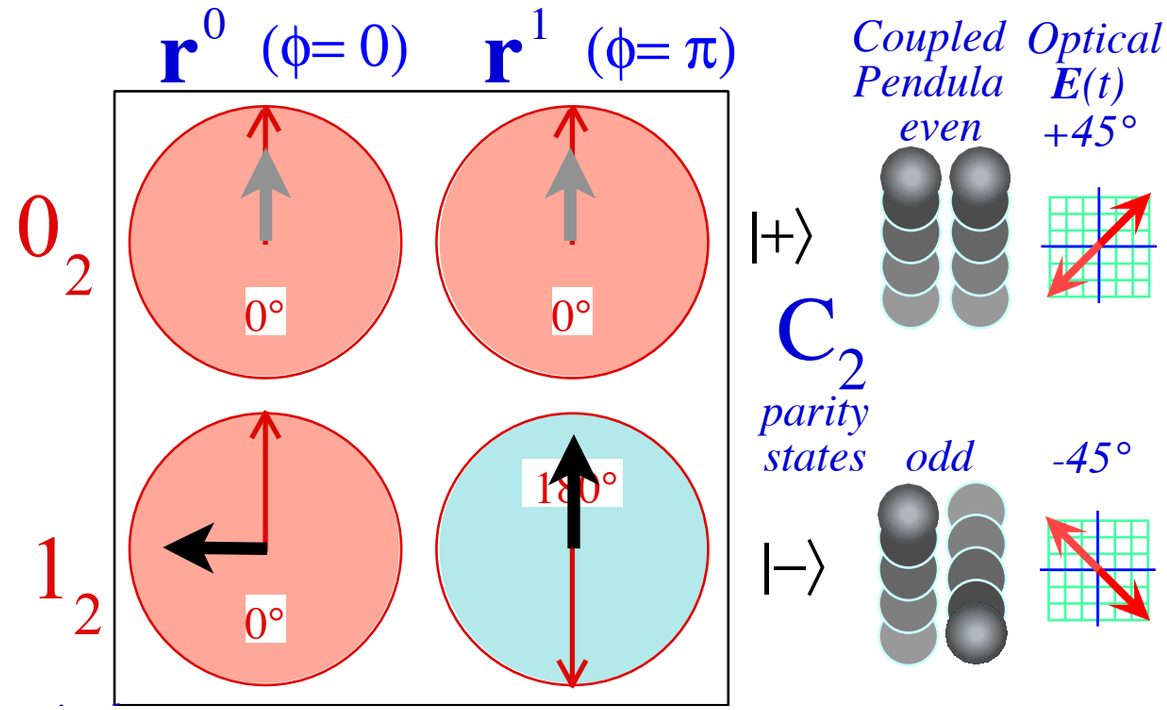
symmetric A_1

vs.

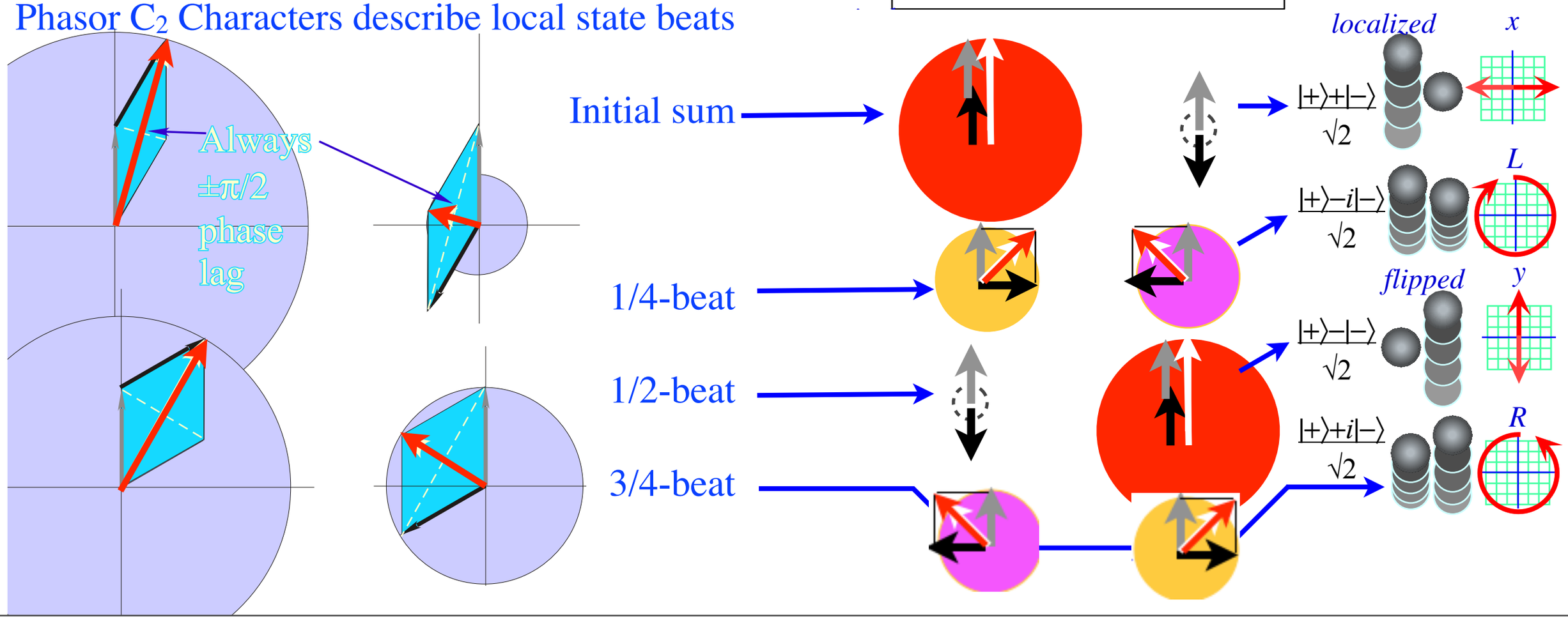
antisymmetric A_2

	$1 = r^0$	$r = r^1$
$0 \bmod 2$	1	1
$\pm 1 \bmod 2$	1	-1

C_2 *Phasor*-Character Table



Phasor C_2 Characters describe local state beats



2-level-system and C_2 symmetry phase dynamics

C_2 Character Table describes eigenstates

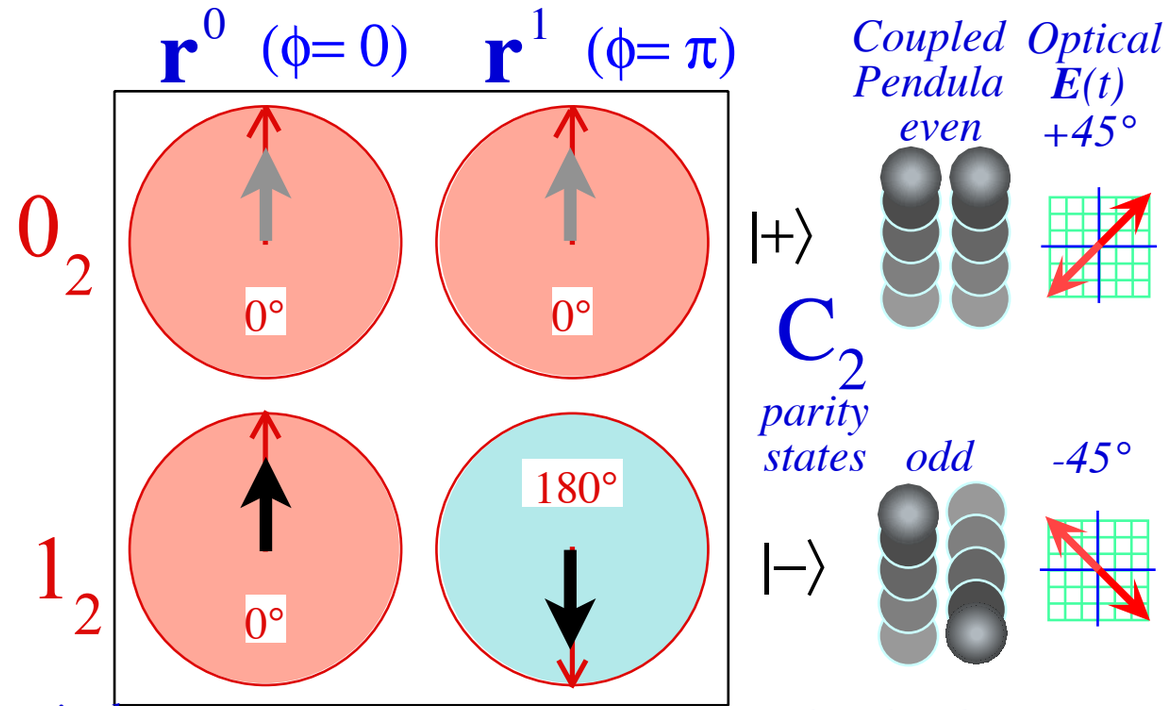
symmetric A_1

vs.

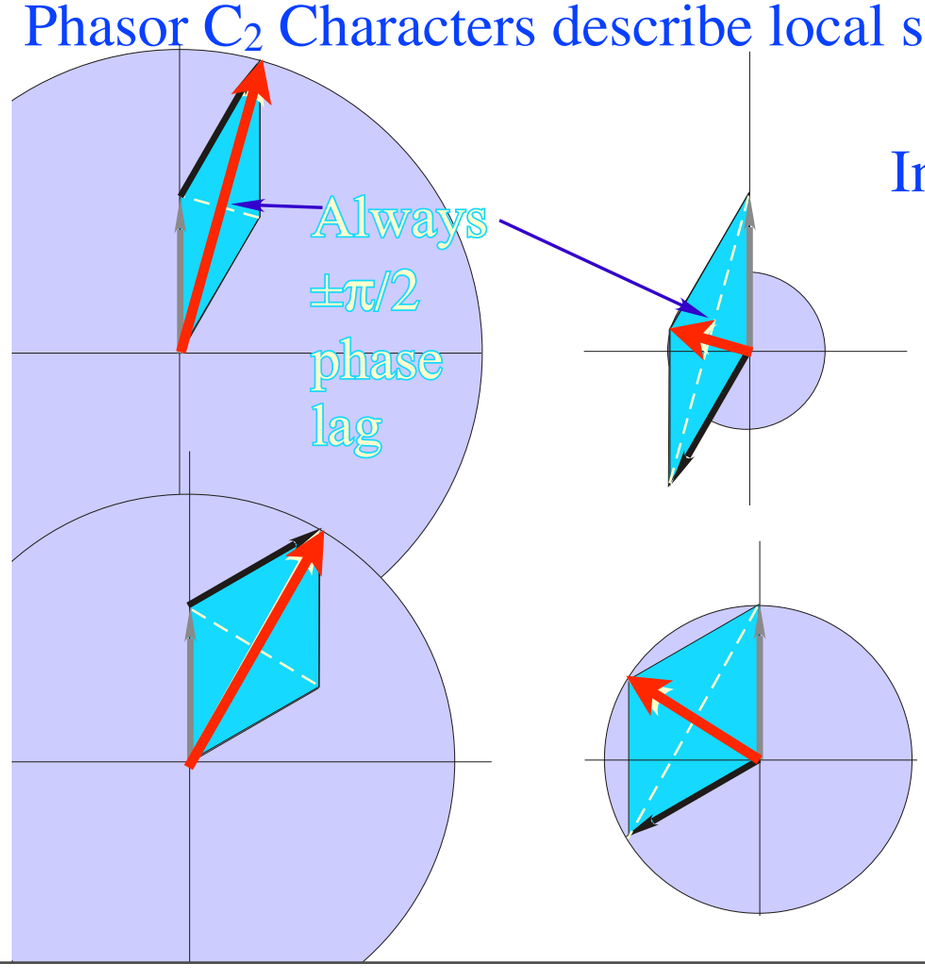
antisymmetric A_2

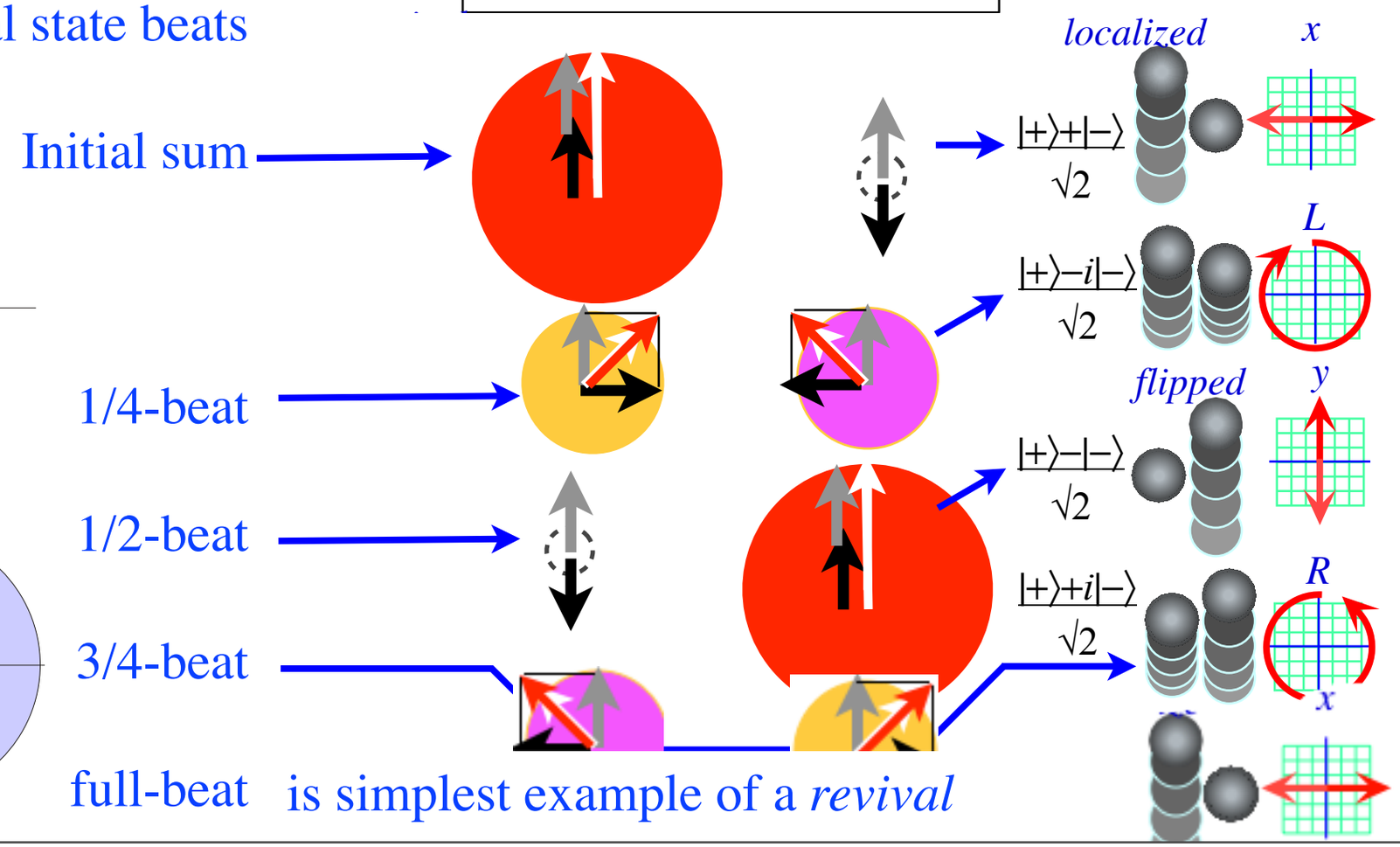
	$1 = r^0$	$r = r^1$
$0 \bmod 2$	1	1
$\pm 1 \bmod 2$	1	-1

C_2 *Phasor*-Character Table



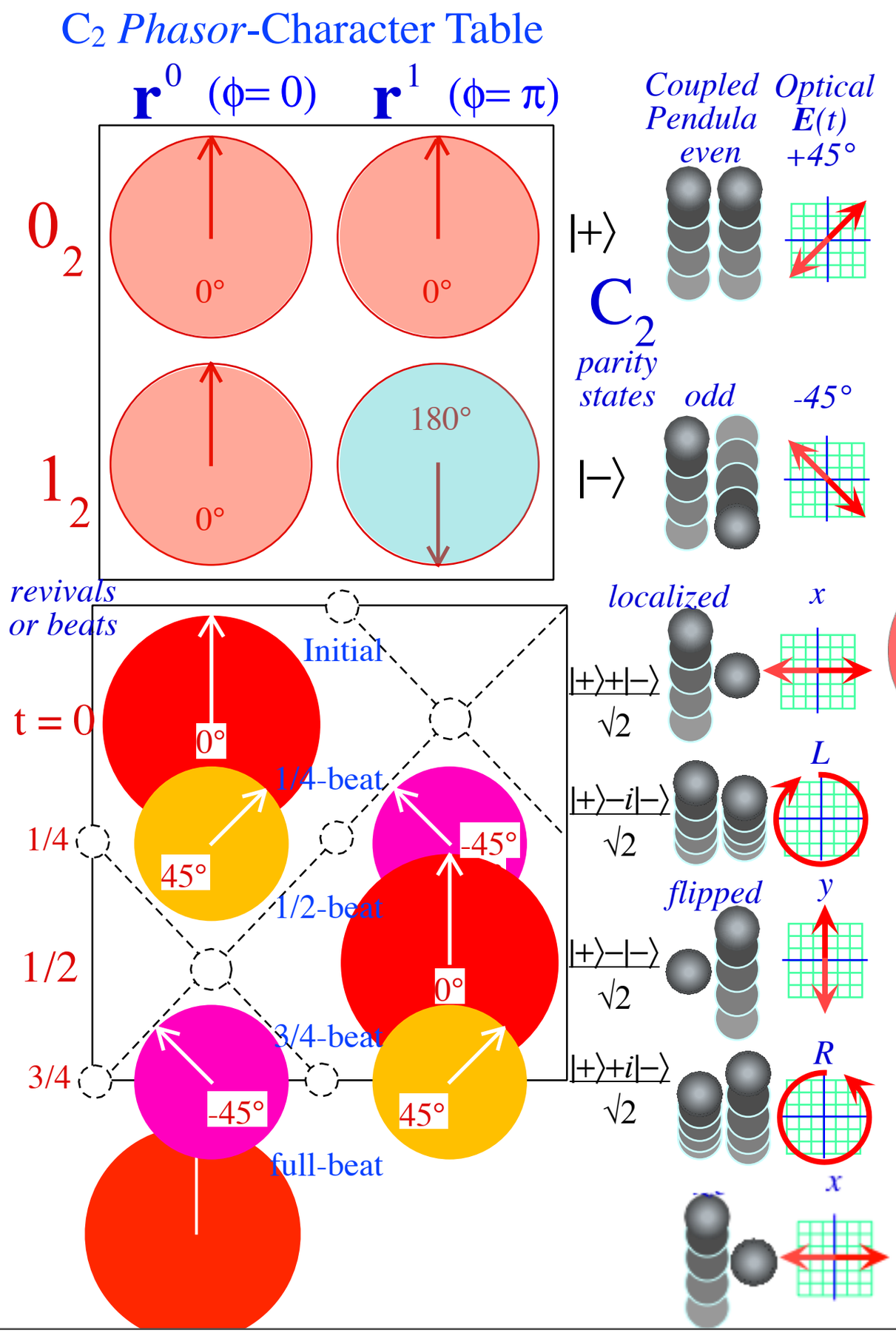
Phasor C_2 Characters describe local state beats

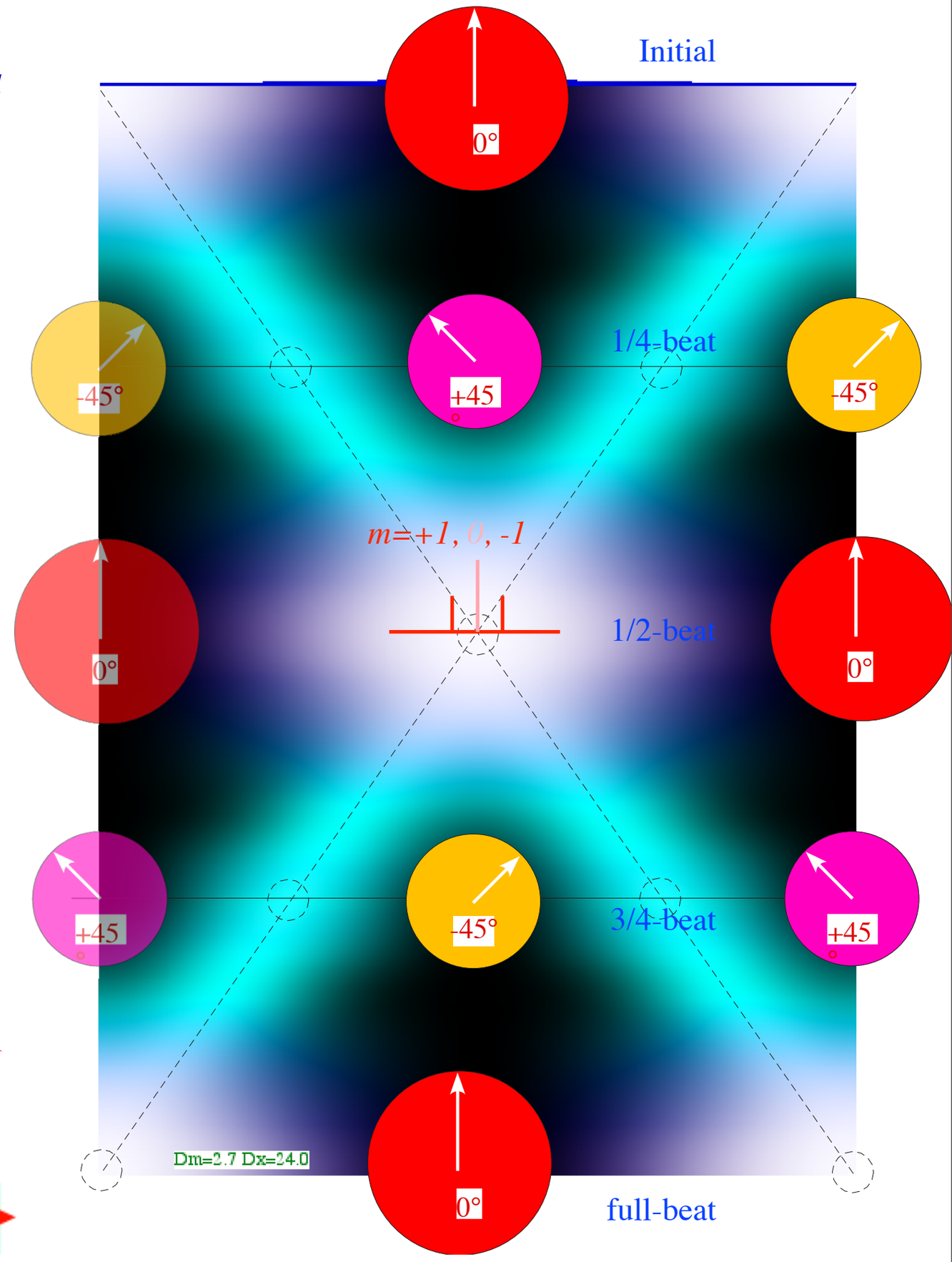




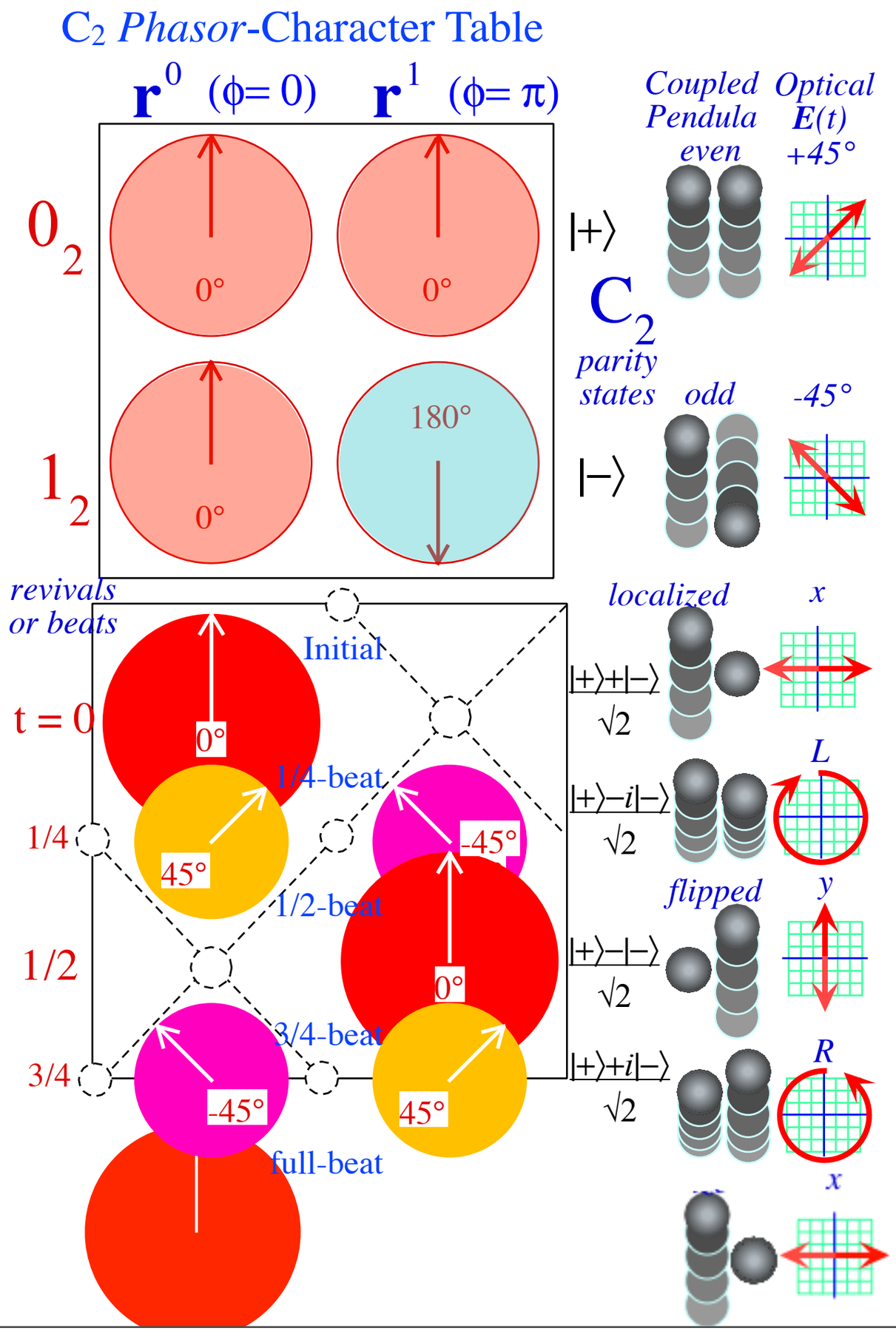
is simplest example of a *revival*

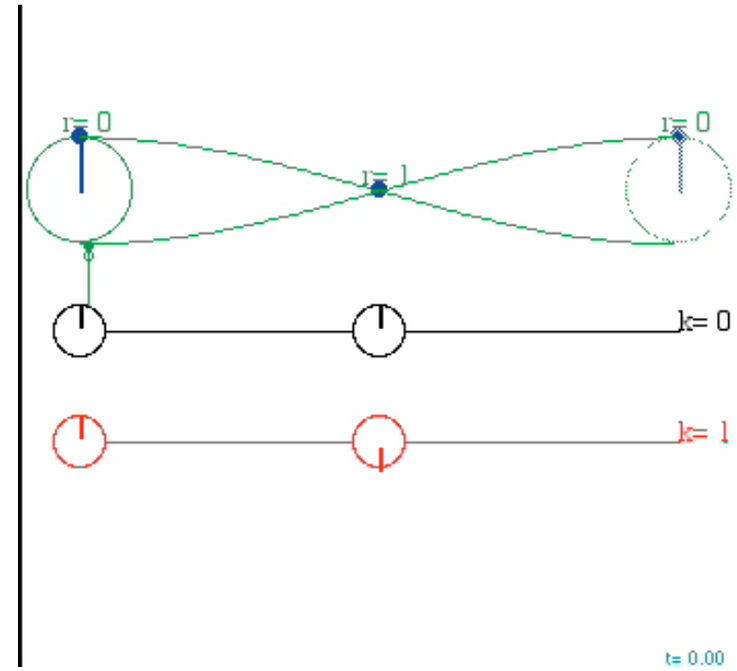
2-level-system and C_2 symmetry phase dynamics

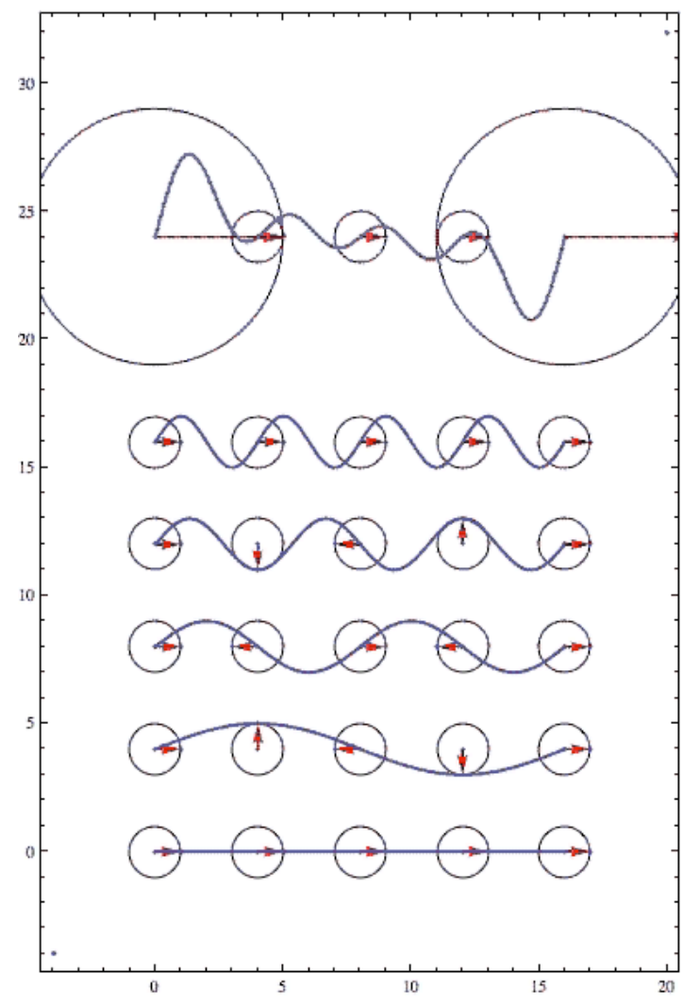




2-level-system and C_2 symmetry phase dynamics





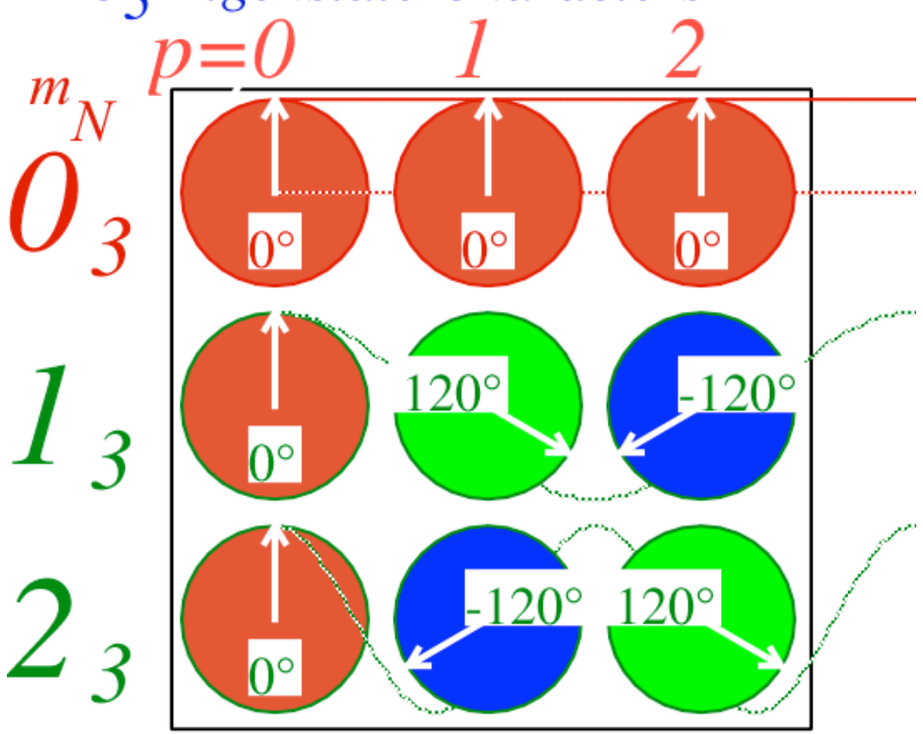


C_3 symmetry phase in *1*, *2*, *or* *3*-level-systems

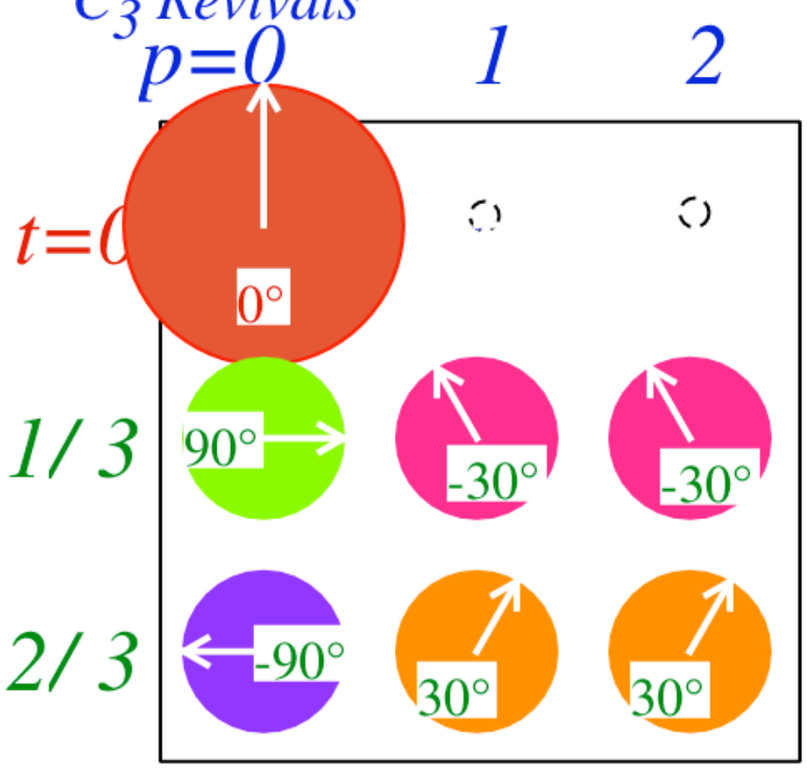
Non - chiral
C_{3v} system

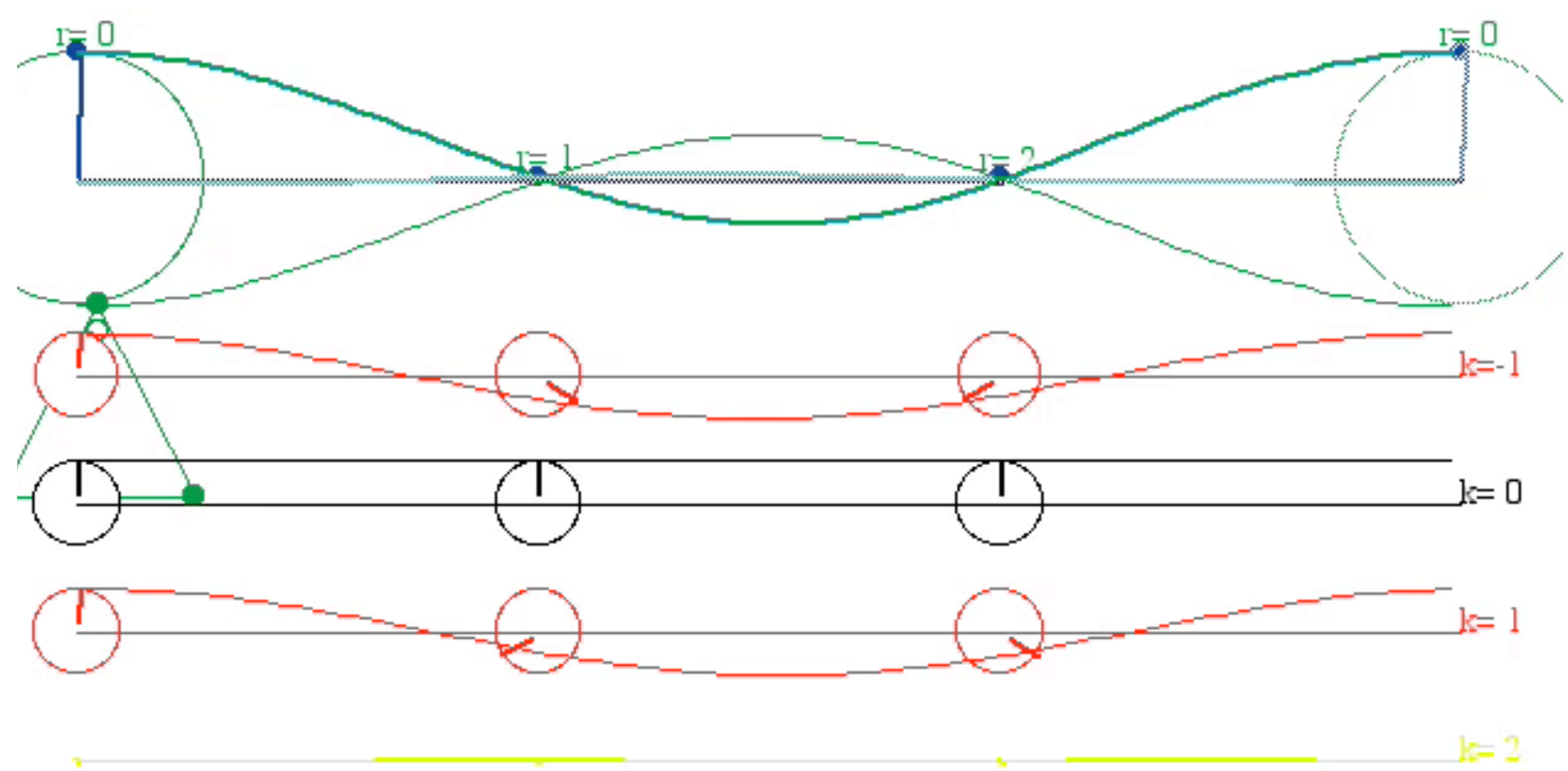
Chiral
"quantum-Hall-like"
systems
deserve special treatment

C_3 Eigenstate Characters



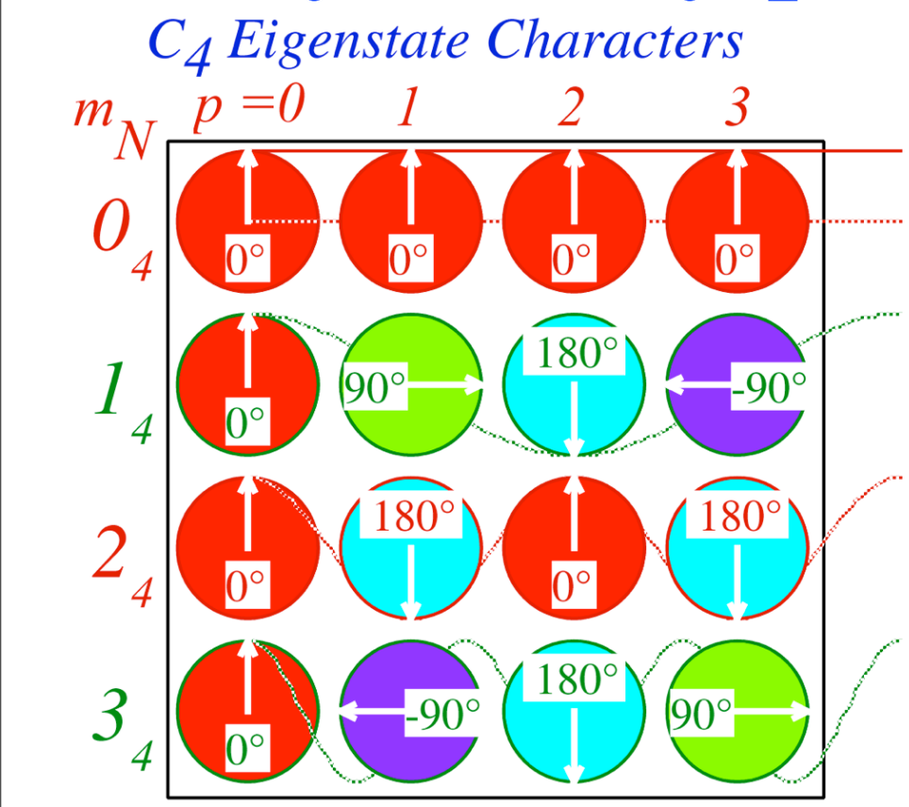
C_3 Revivals

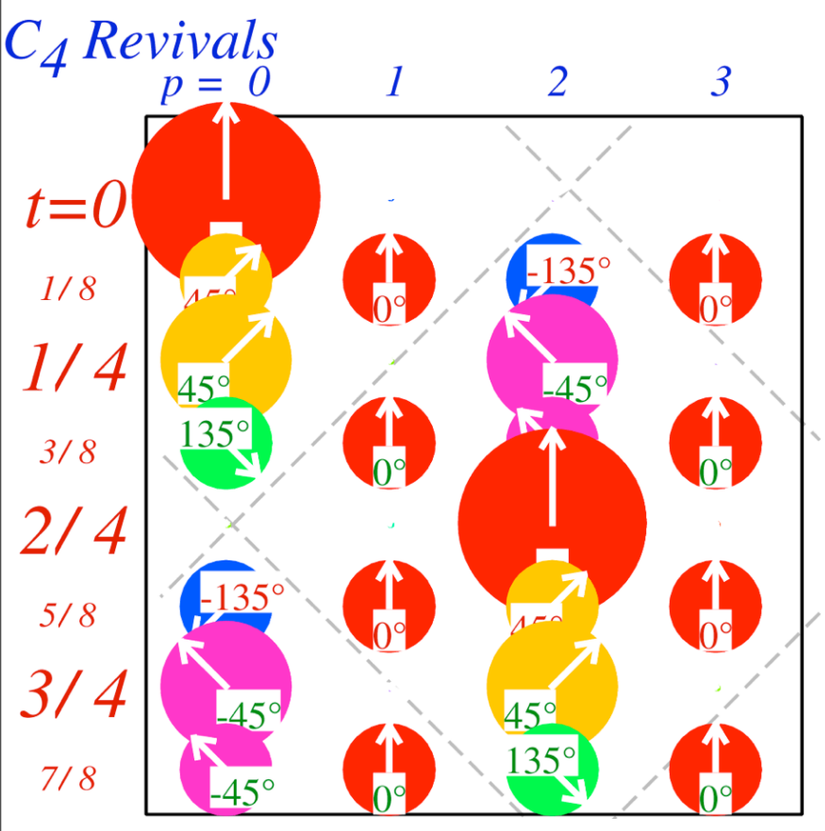


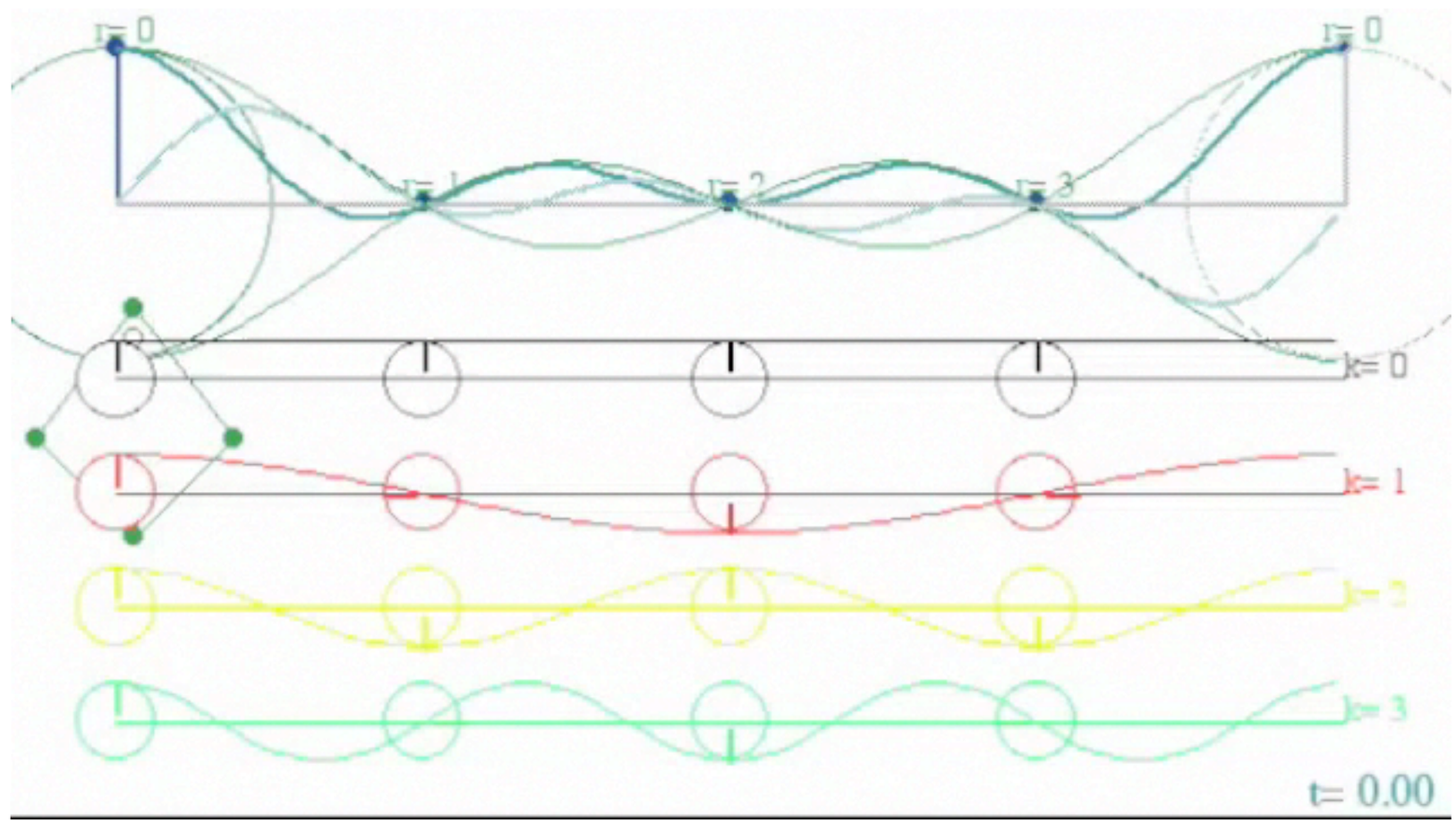


C_4 symmetry phase in *1*, *2*, *3* , *or 4* level-systems

Non - chiral
C_{4v} system



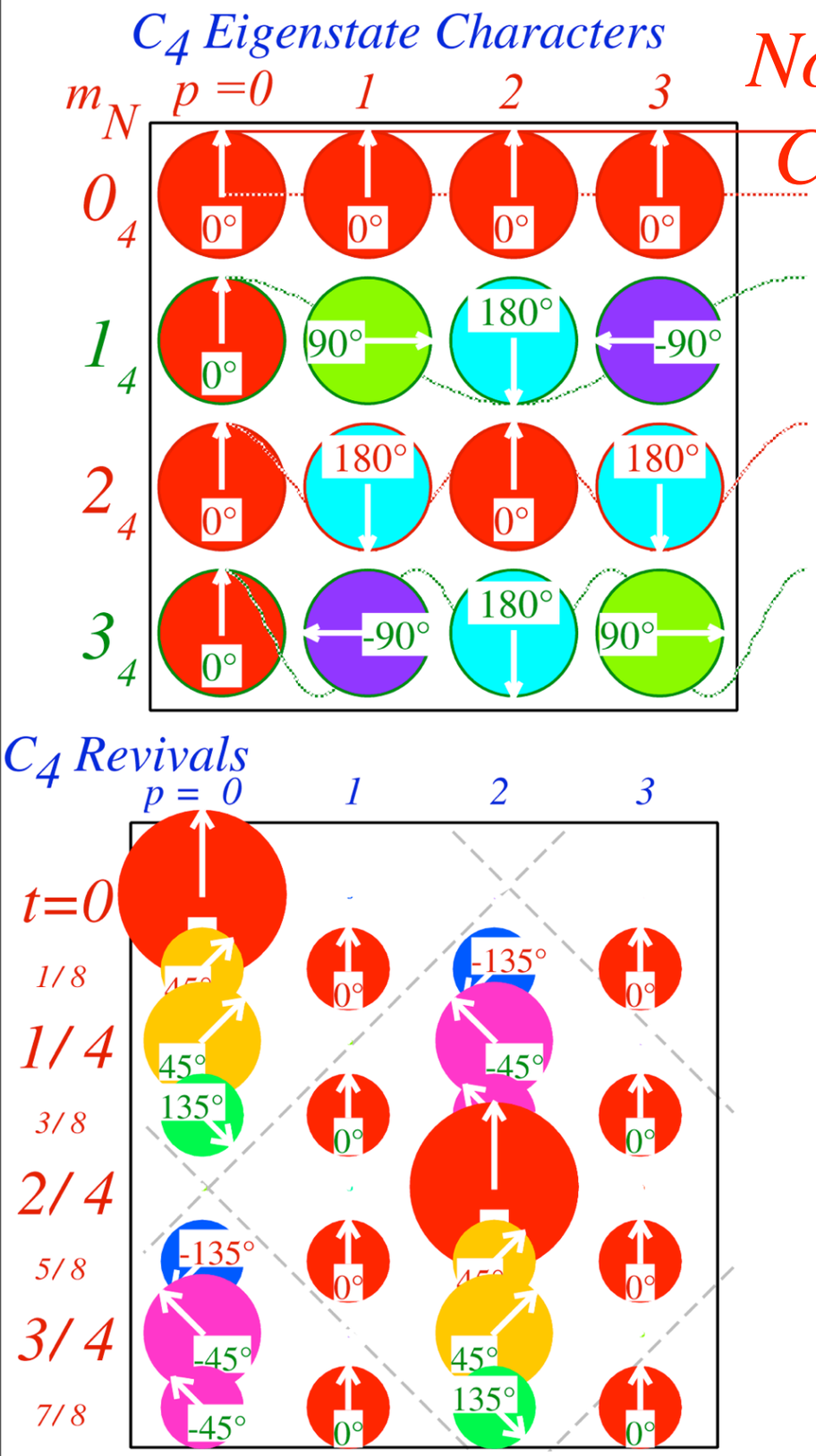


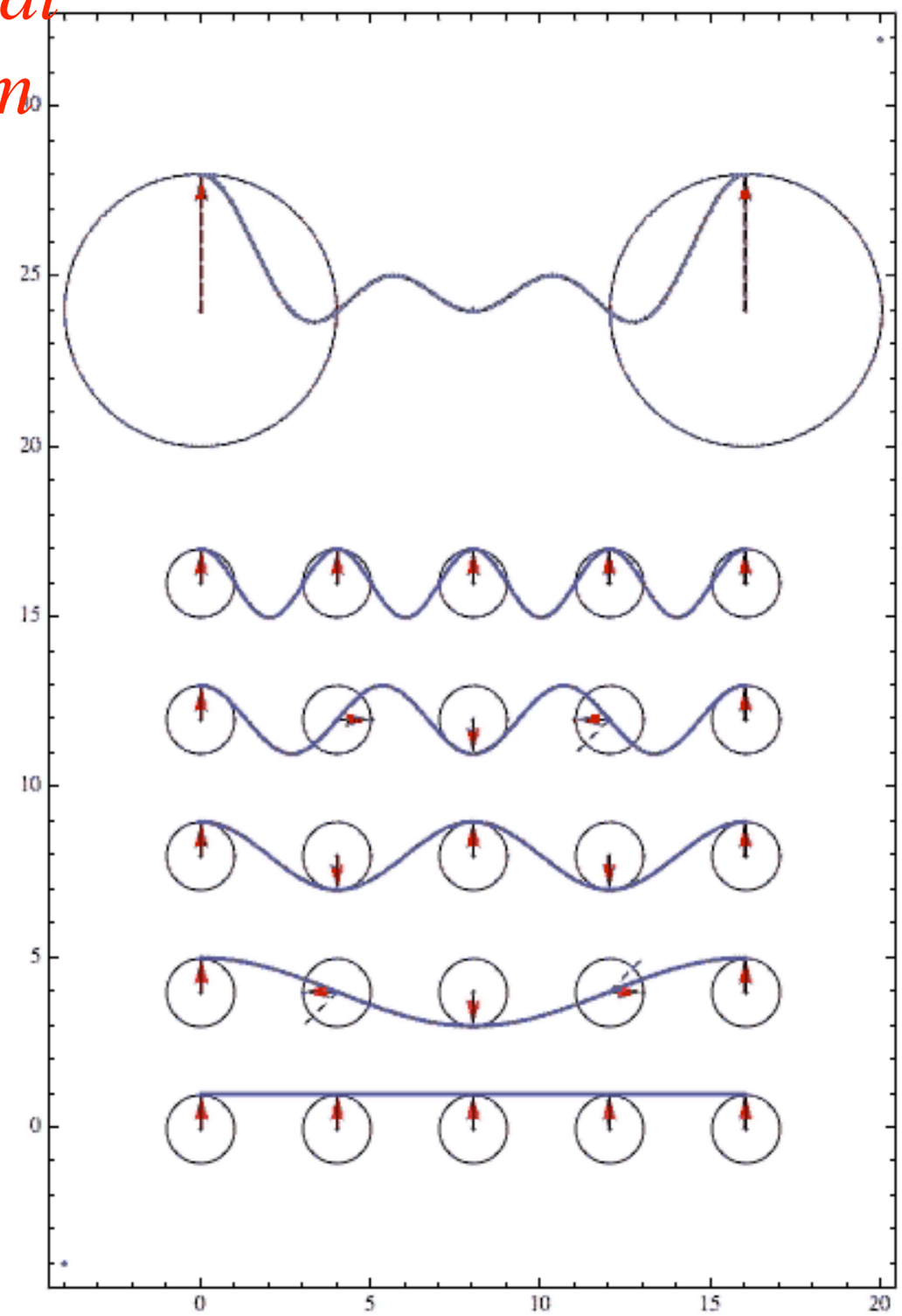


C_4 symmetry phase in *1*, *2*, *3* , *or 4* level-systems

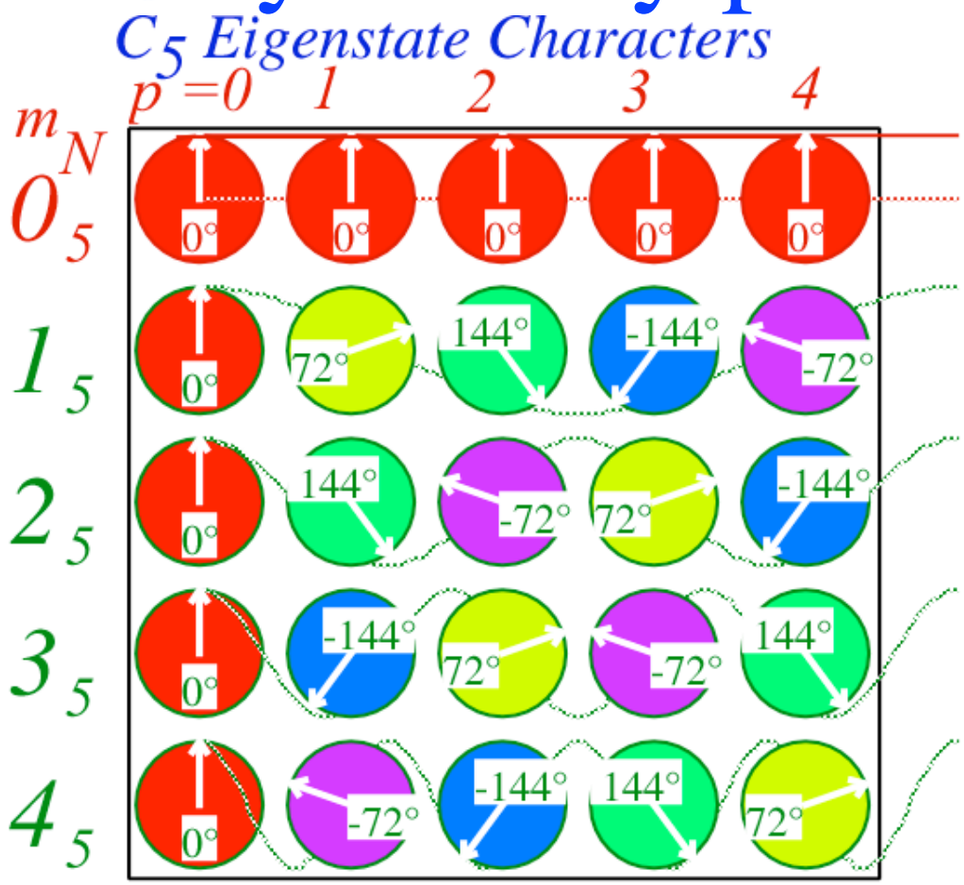
Non - chiral

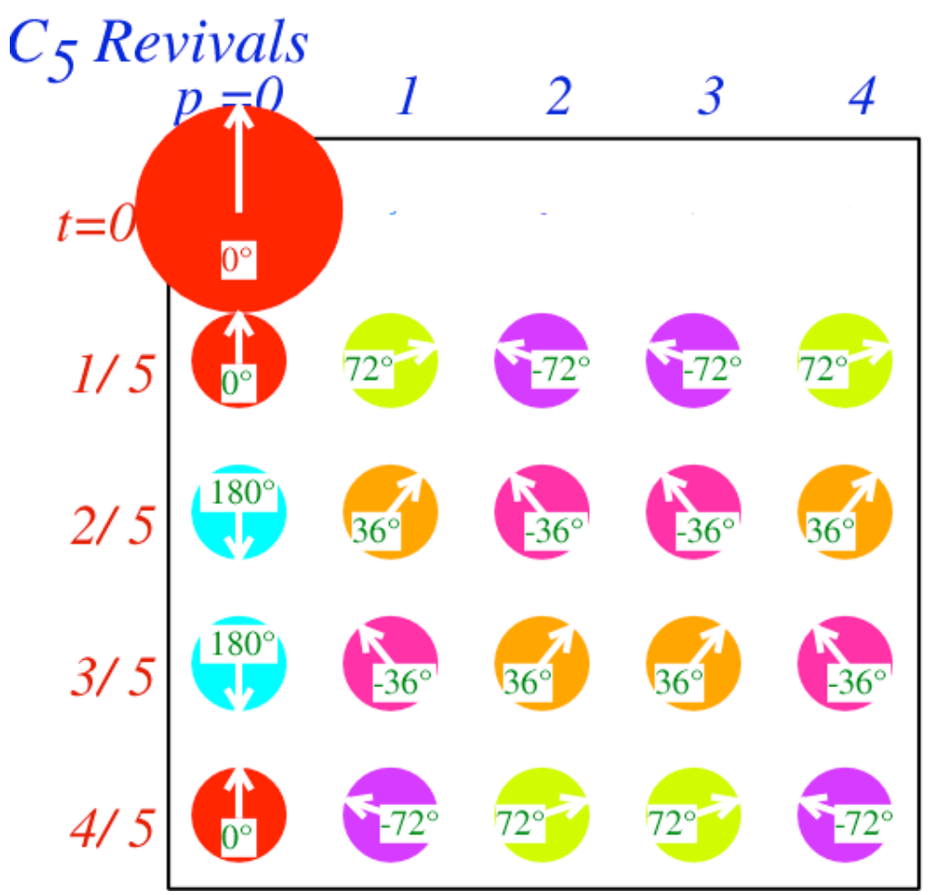
C_{4v} system

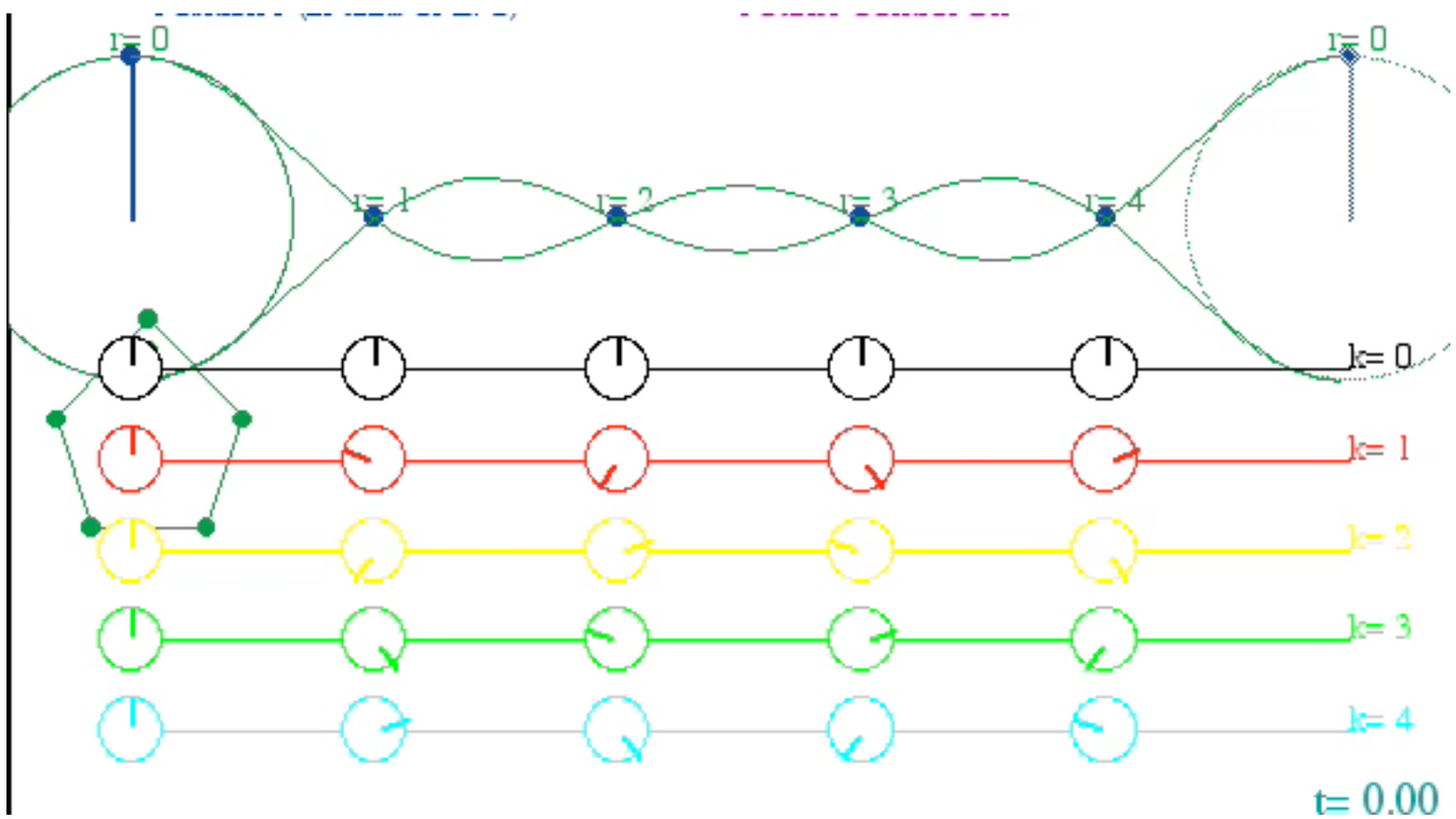




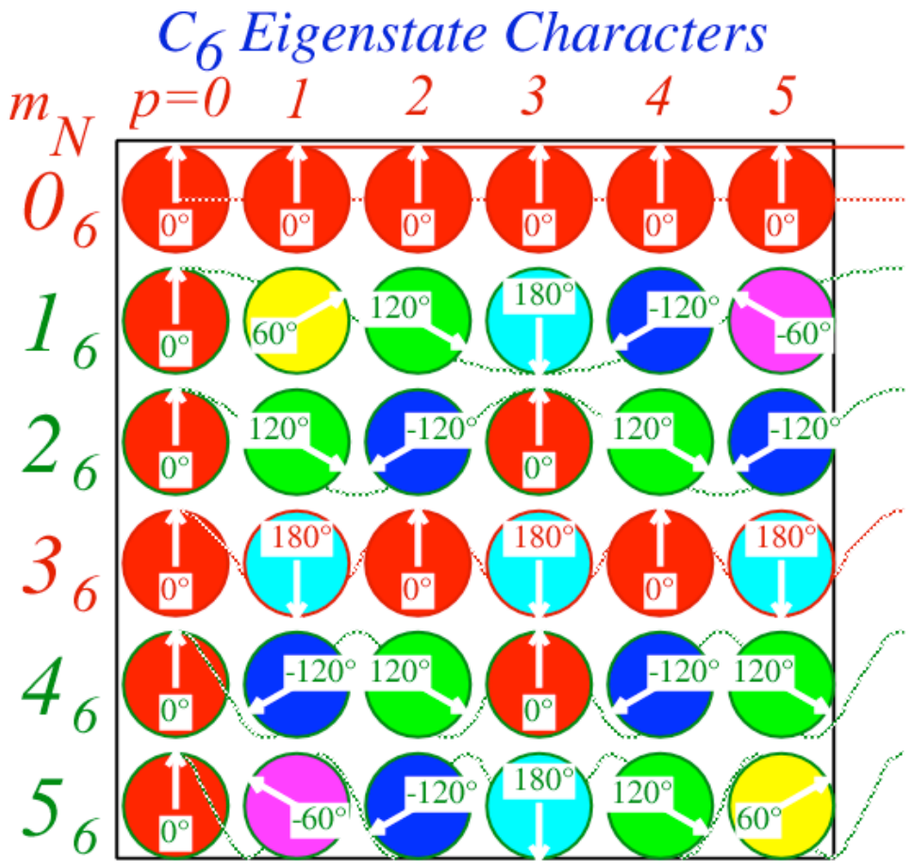
C_5 symmetry phase in *1*, *2*,...*5* level-systems

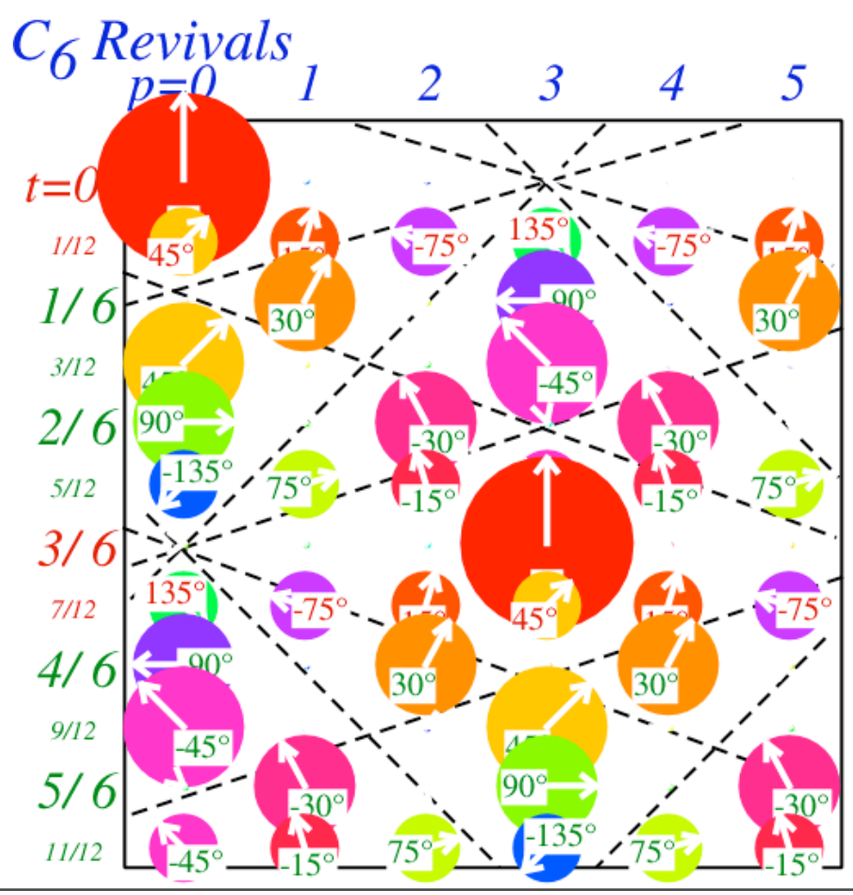


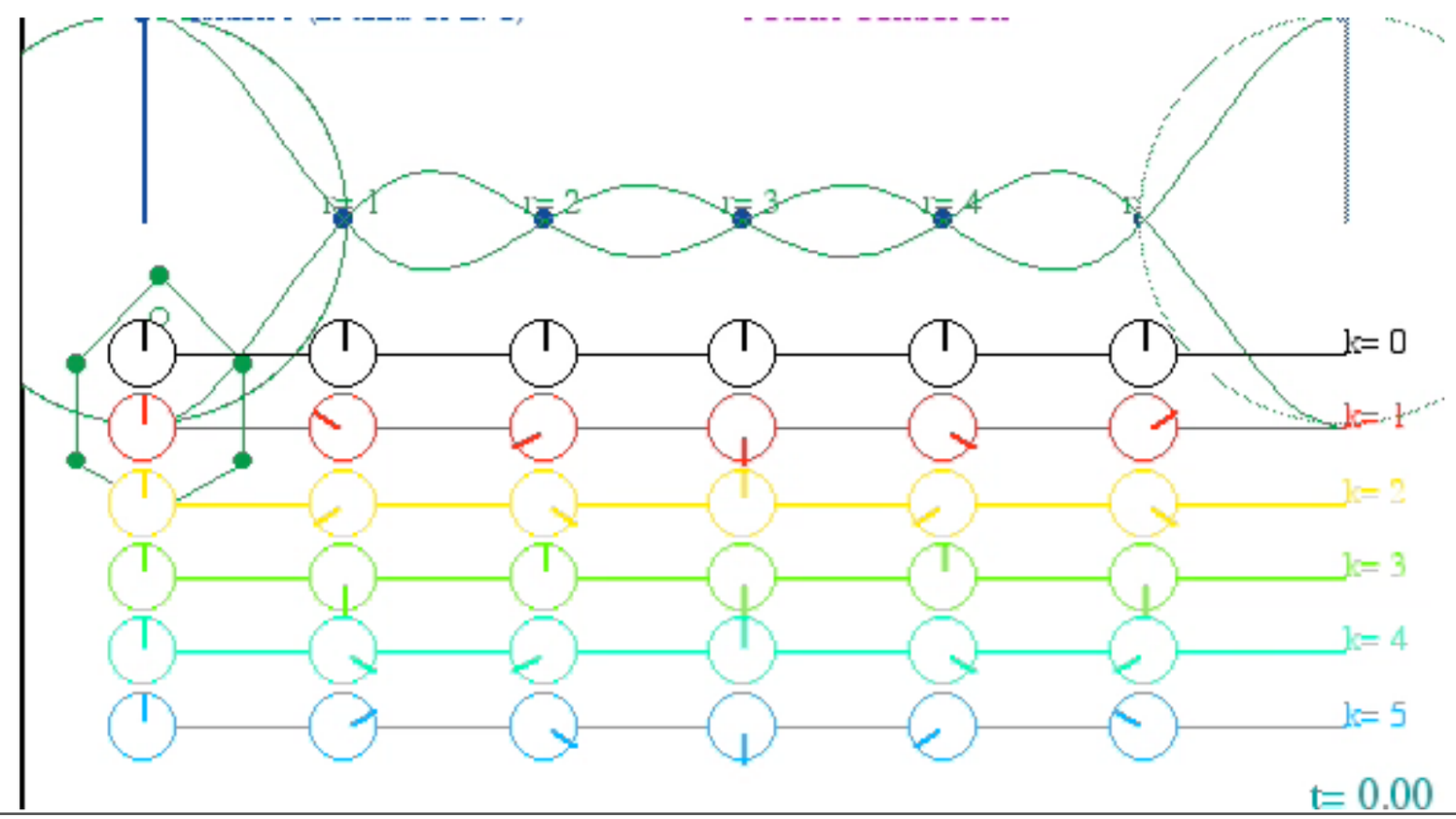




C_6 symmetry phase in *1*, ...*6* level-systems



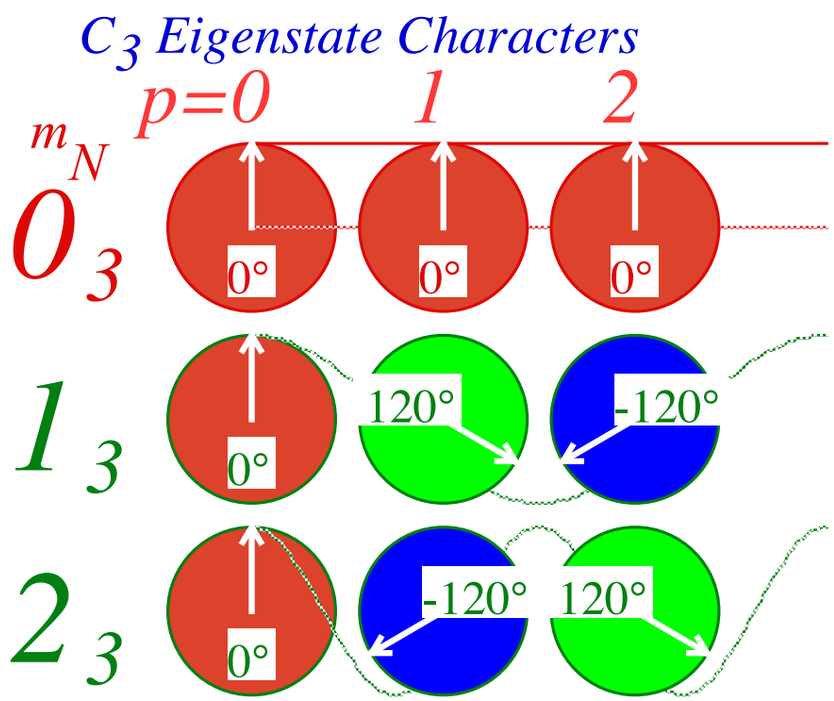


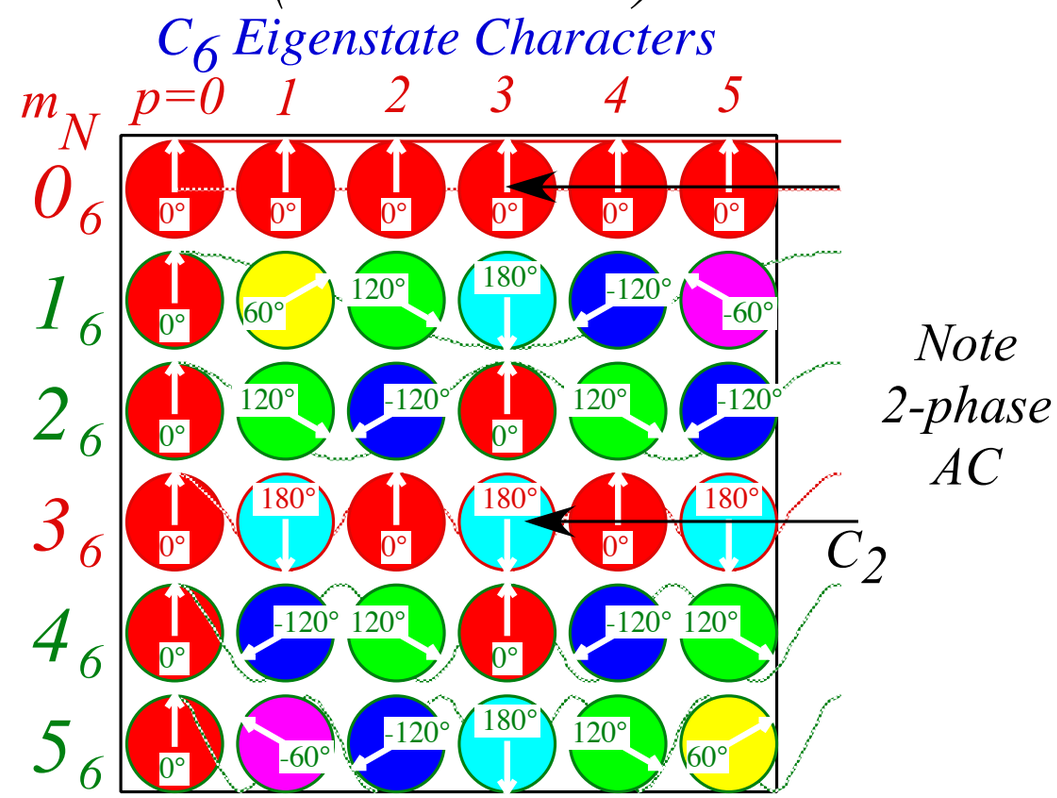


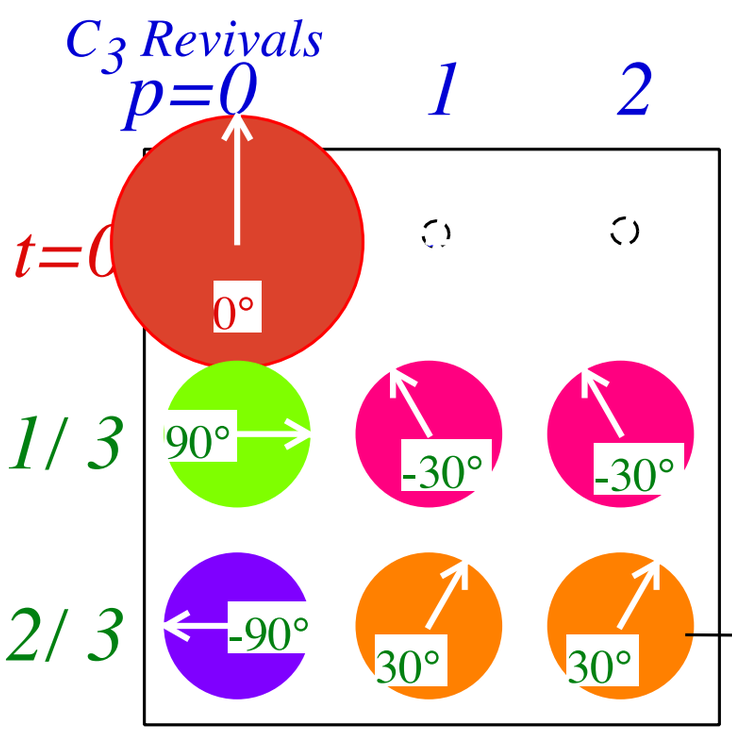
C_m algebra of revival-phase dynamics

Discrete 3-State or Trigonal System (Tesla's 3-Phase AC)

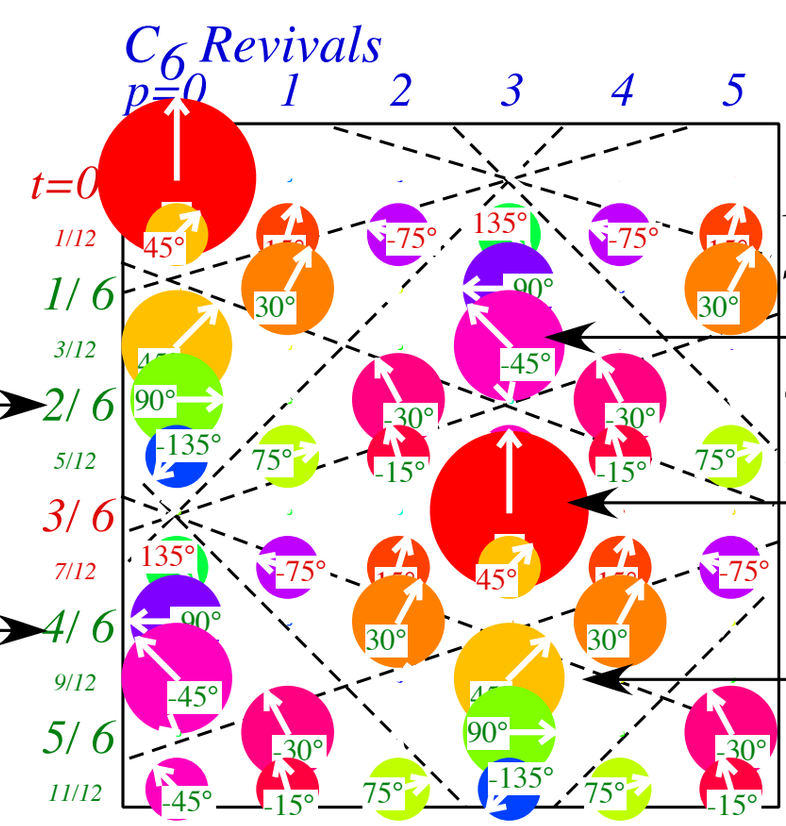
Discrete 6-State or Hexagonal System (6-Phase AC)







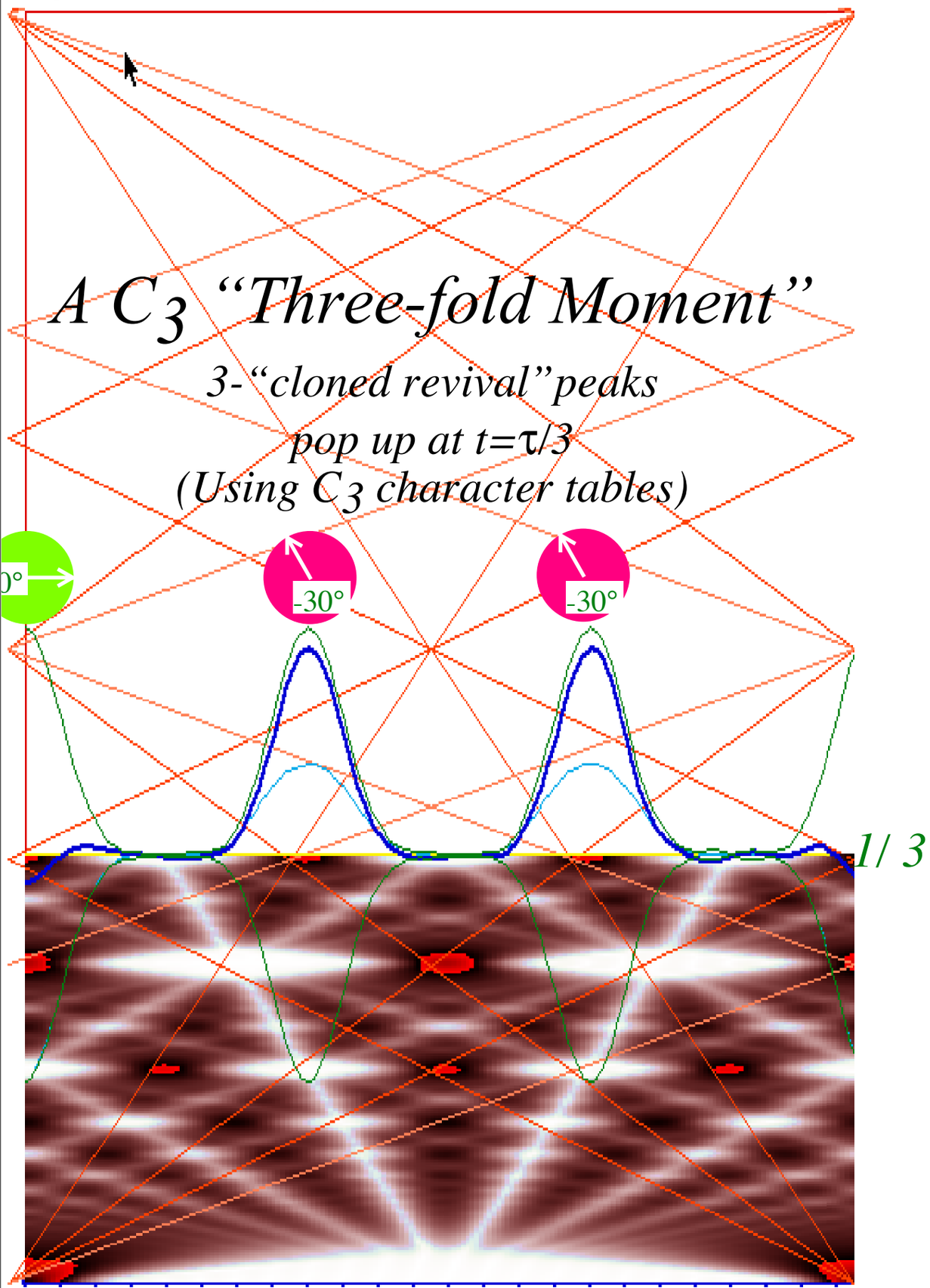
Note 3-phase sub-symmetry

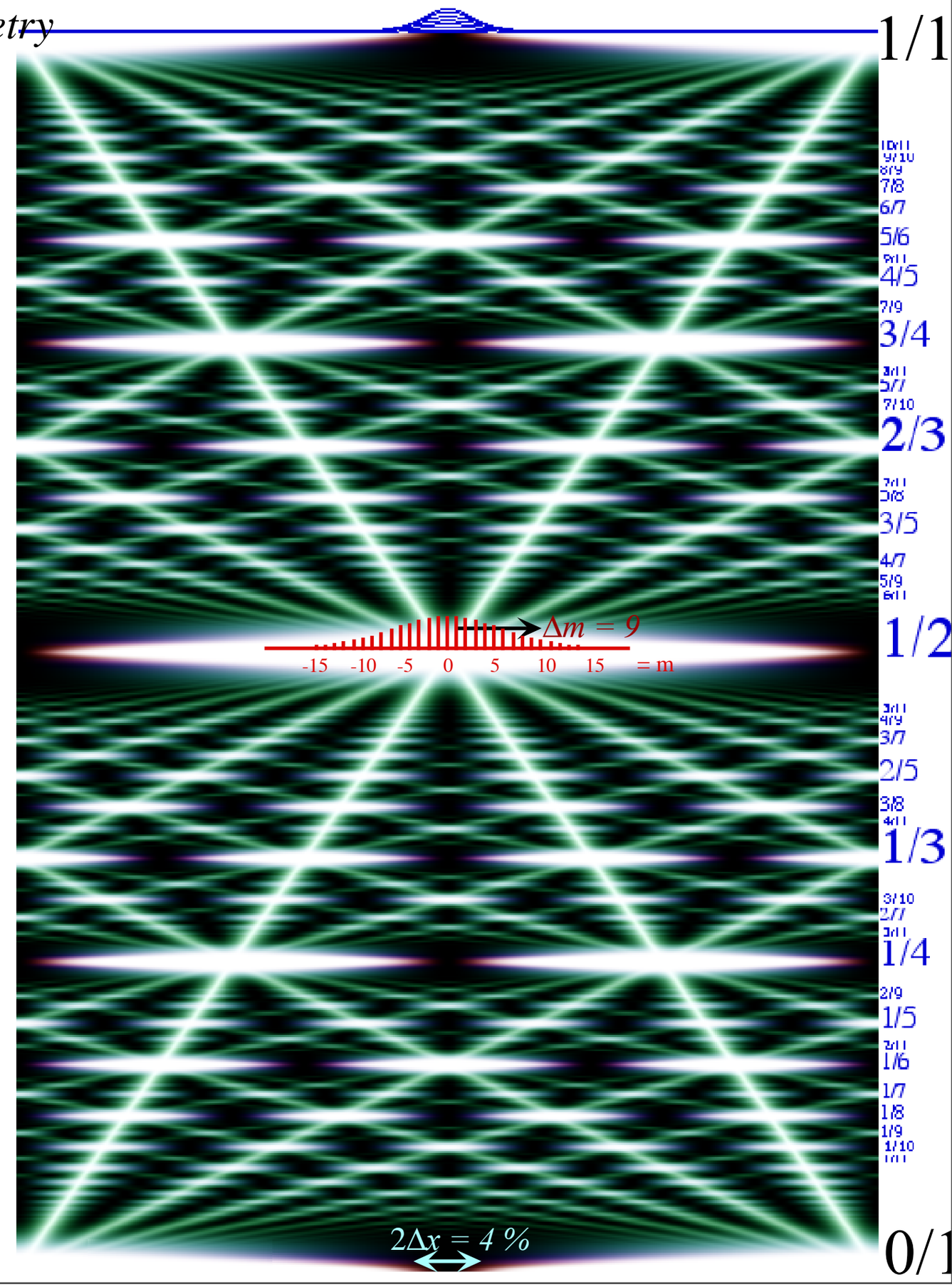


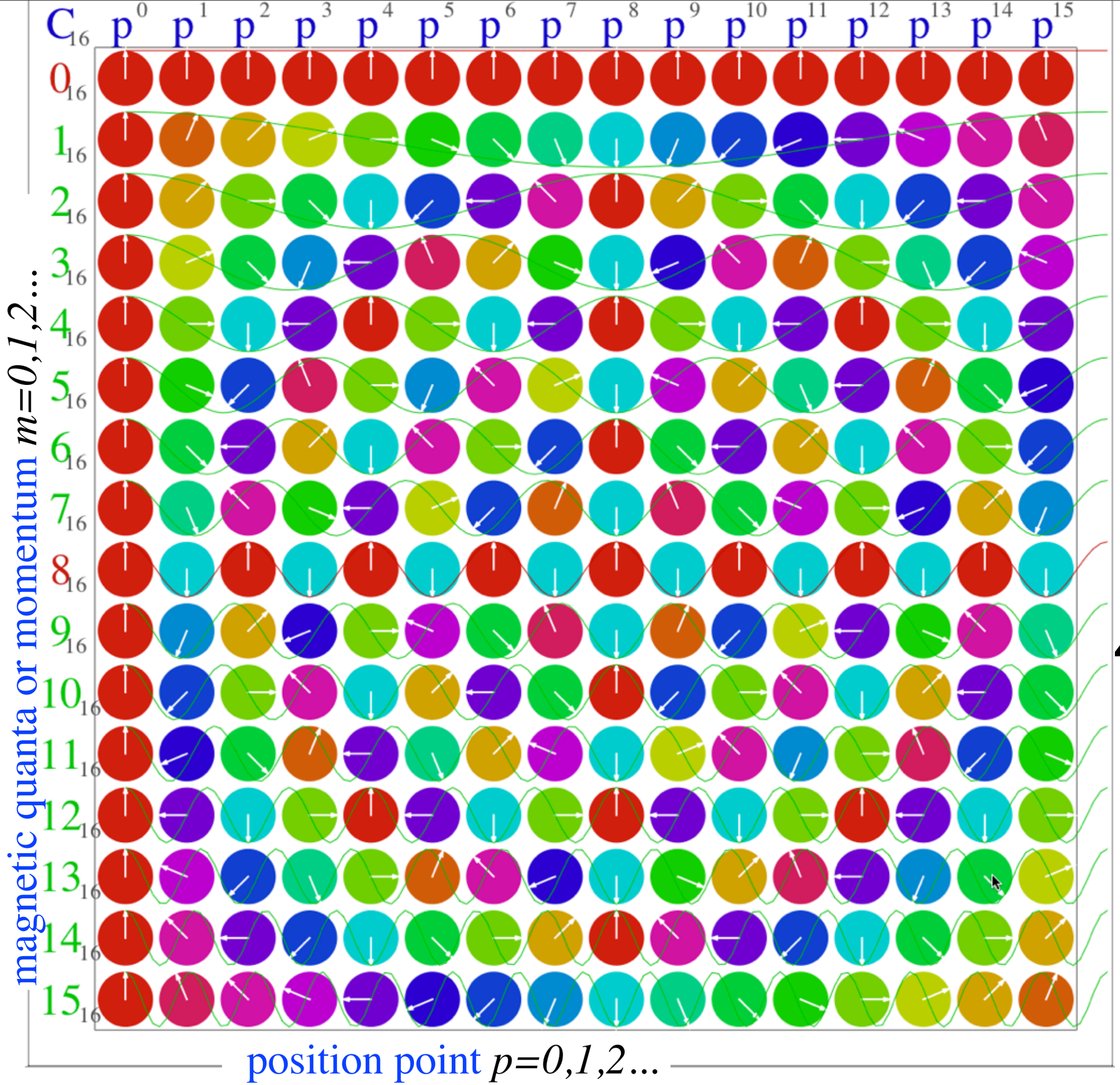
Note 2-phase sub-symmetry (The "Mother of all symmetry" is C_2)

C_m algebra of revival-phase dynamics

Quantum rotor fractional take turns at Cn symmetry





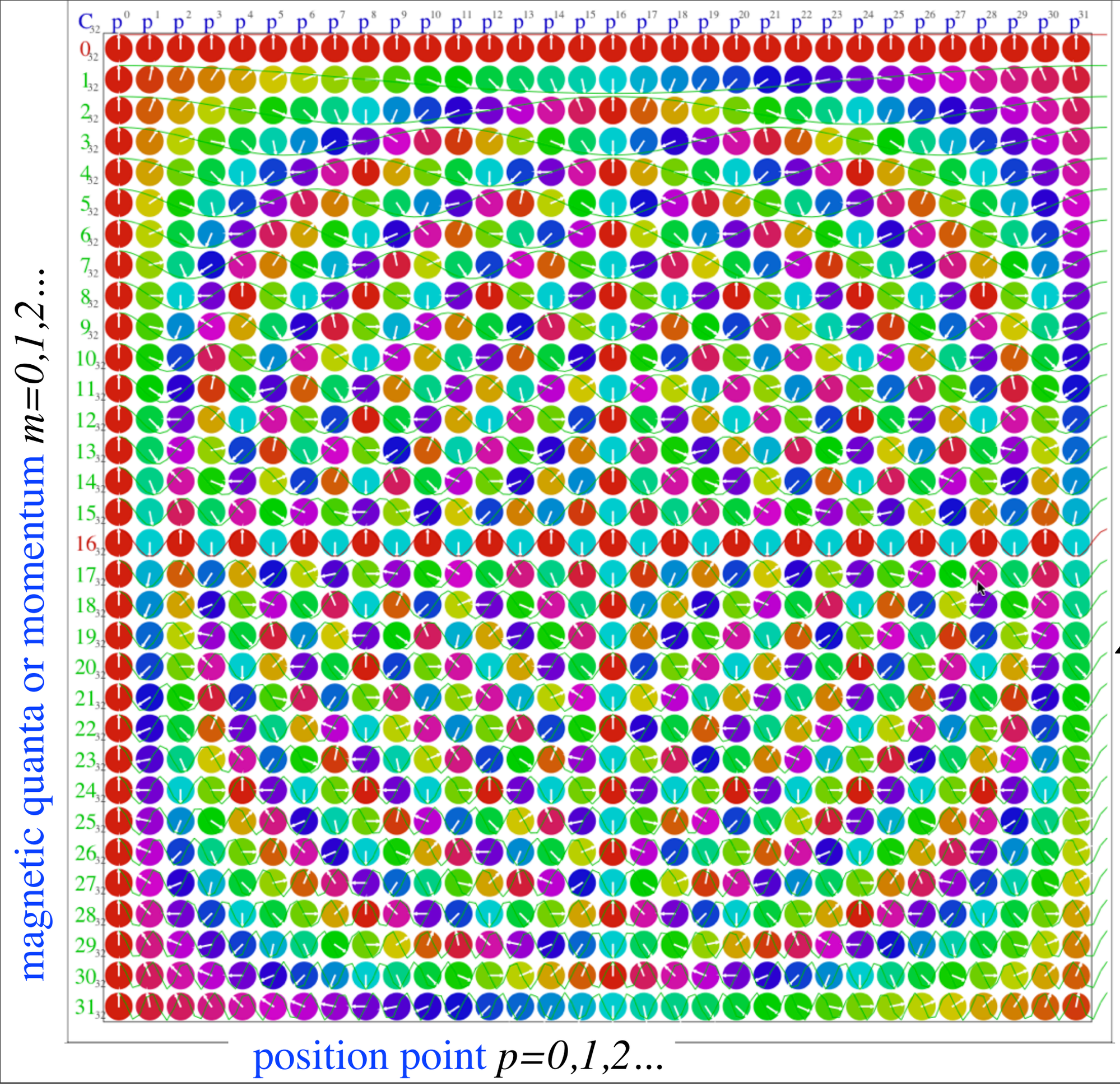


C_{16}

phasor character table

$$\chi_p^m = e^{ik_m r^p}$$

$$= e^{\frac{2\pi imp}{16}}$$

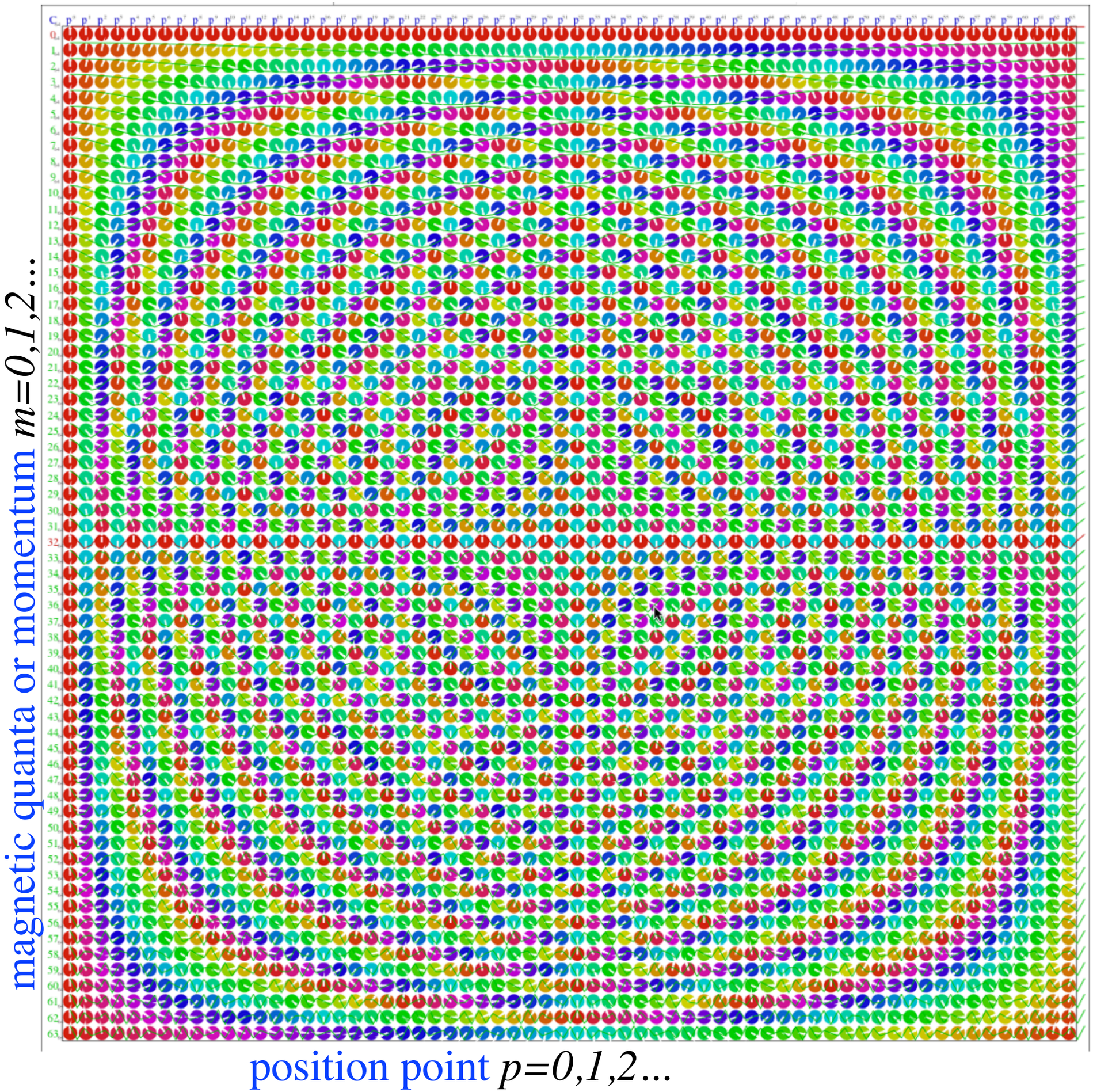


C_{32}

phasor character table

$$\chi_p^m = e^{ik_m r^p}$$

$$= e^{\frac{2\pi imp}{32}}$$



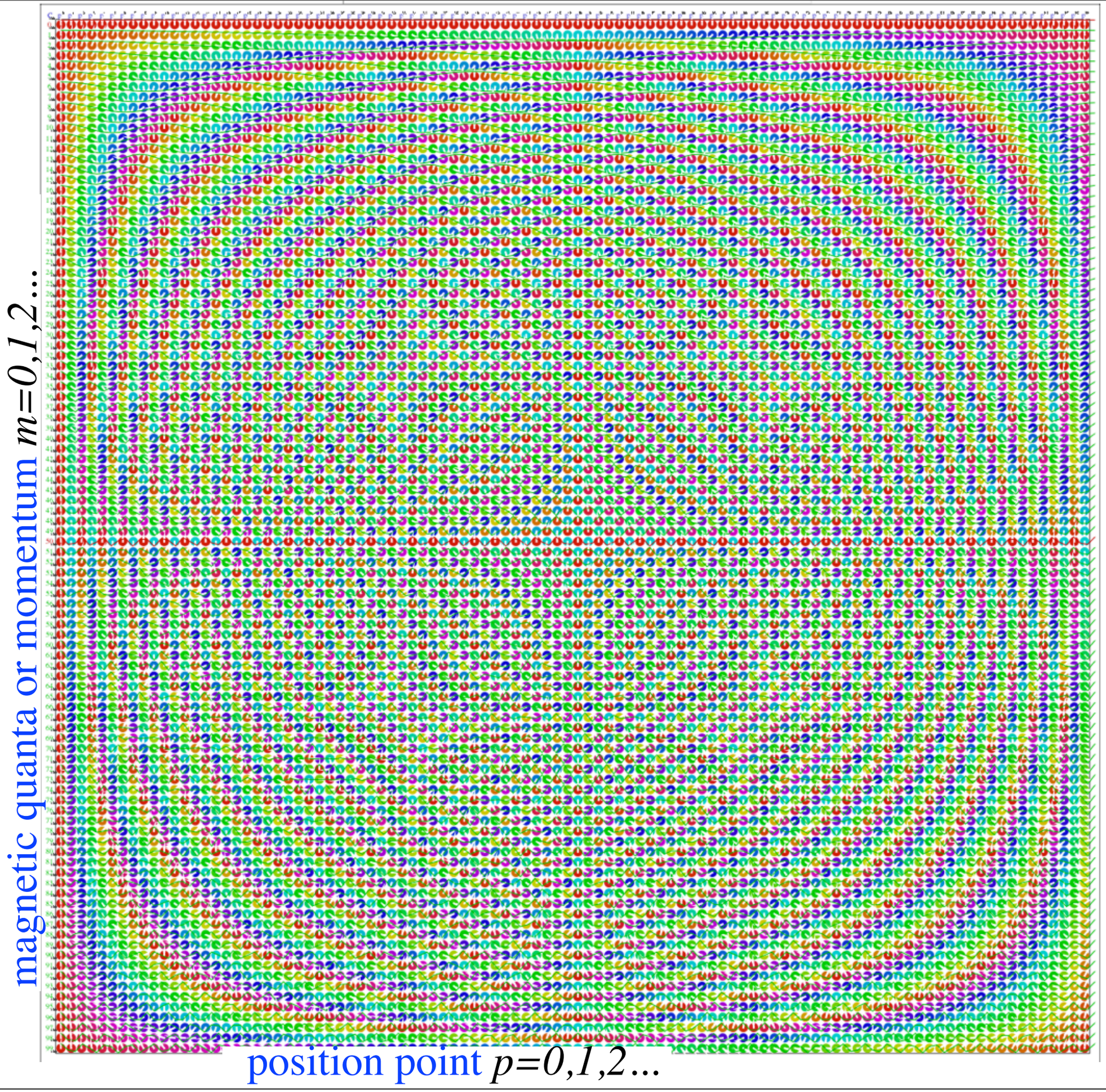
C_{64}

phasor character table

$$\chi_p^m = e^{ik_m r^p}$$

$$= e^{\frac{2\pi imp}{64}}$$

Invariant phase "Uncertainty" hyperbolas: $m \cdot p = const.$



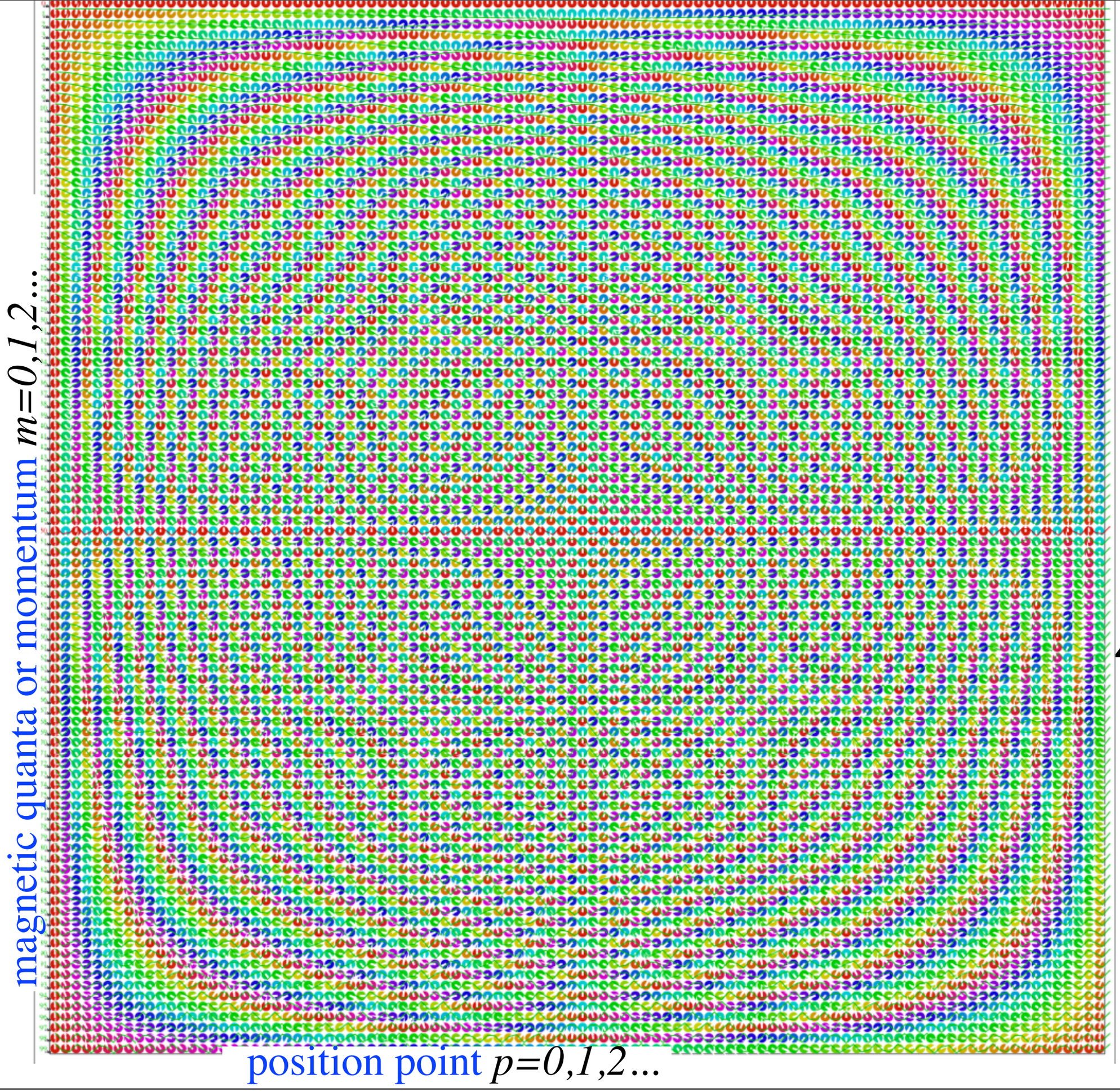
C_{100}

phasor character table

$$\chi_p^m = e^{ik_m r^p}$$

$$= e^{\frac{2\pi imp}{100}}$$

Invariant phase "Uncertainty" hyperbolas: $m \cdot p = const.$



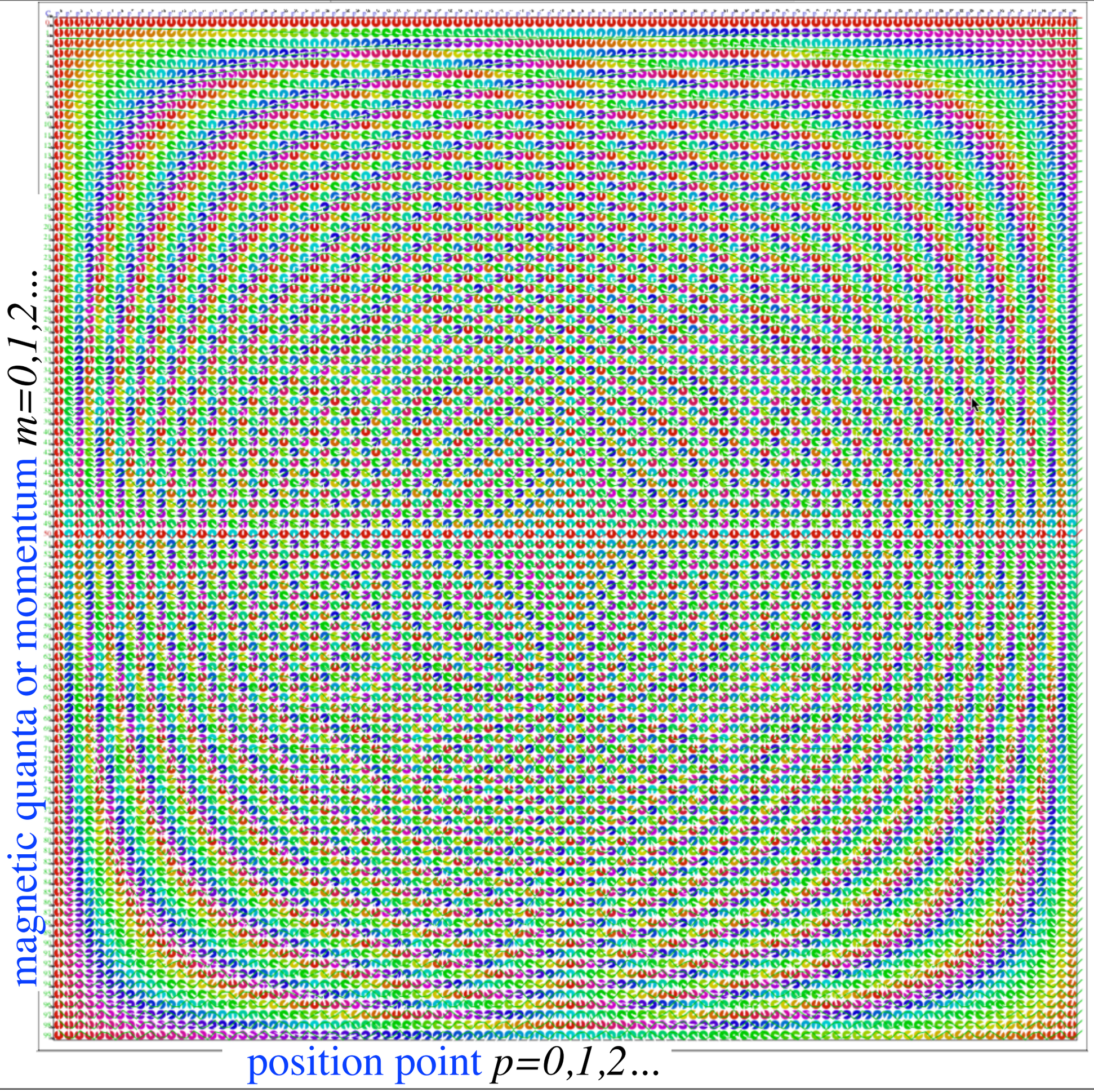
C_{128}

phasor character table

$$\chi_p^m = e^{ik_m r^p}$$

$$= e^{\frac{2\pi imp}{128}}$$

Invariant phase "Uncertainty" hyperbolas: $m \cdot p = const.$



C_{256}

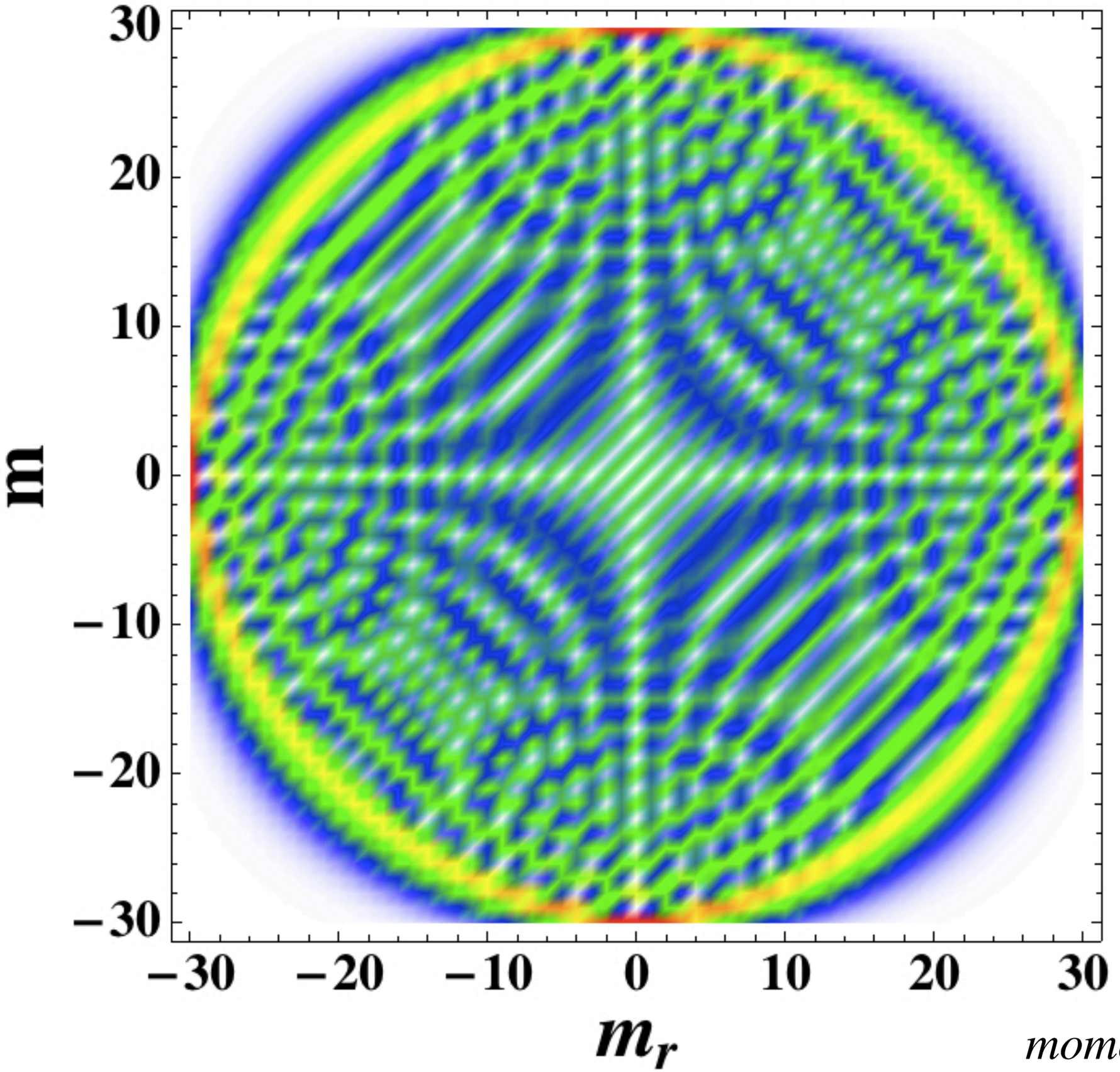
phasor character table

$$\chi_p^m = e^{ik_m r^p}$$

$$= e^{\frac{2\pi imp}{256}}$$

Invariant phase "Uncertainty" hyperbolas: $m \cdot p = const.$

$$\left(|D^{j}_{m_r\, m}| \text{ at } \beta = \frac{\pi}{2} \right)$$

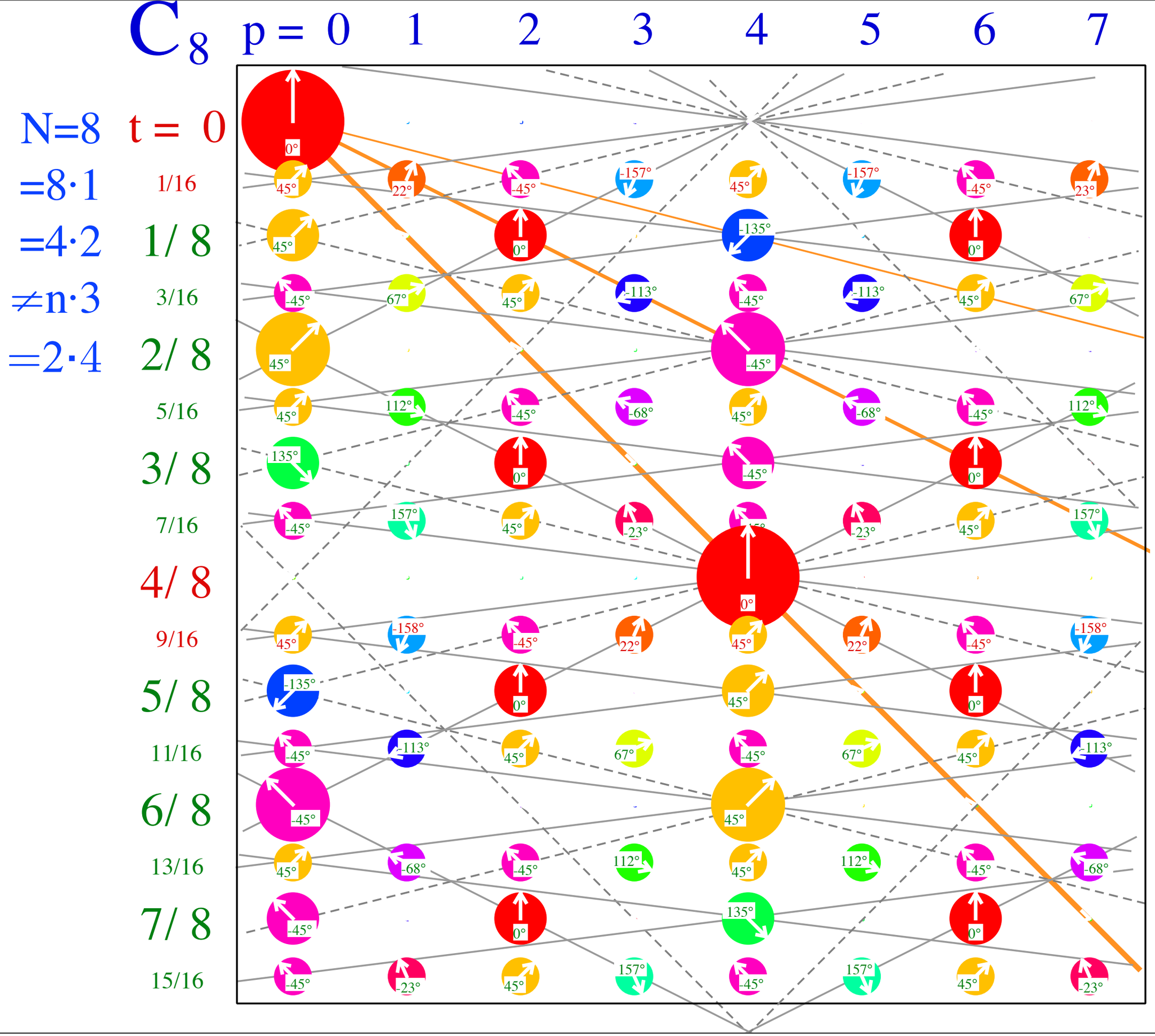


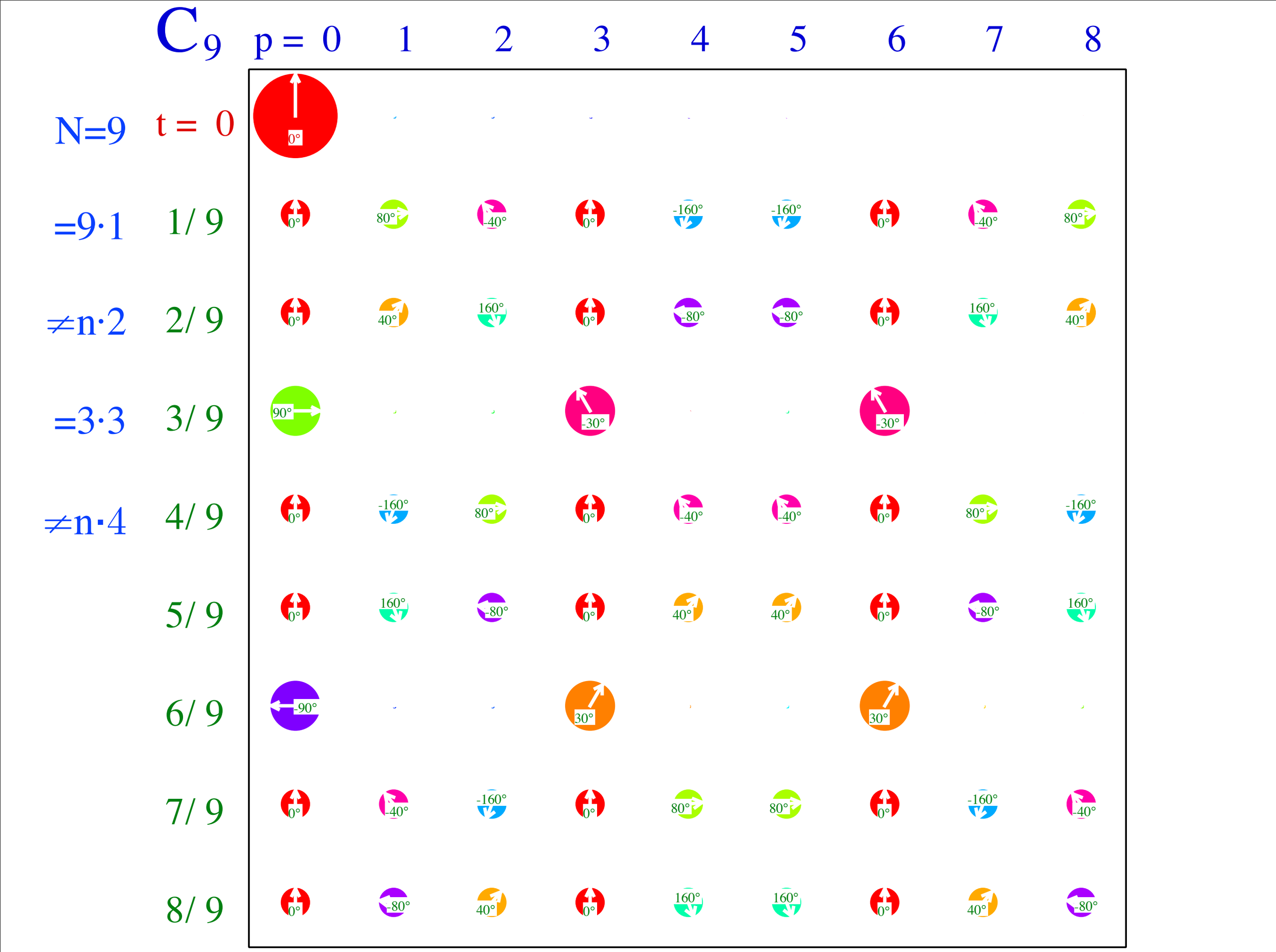
Compare to Alvason Li's plot of Wigner R(3) $D^{j=30}$-matrix representation

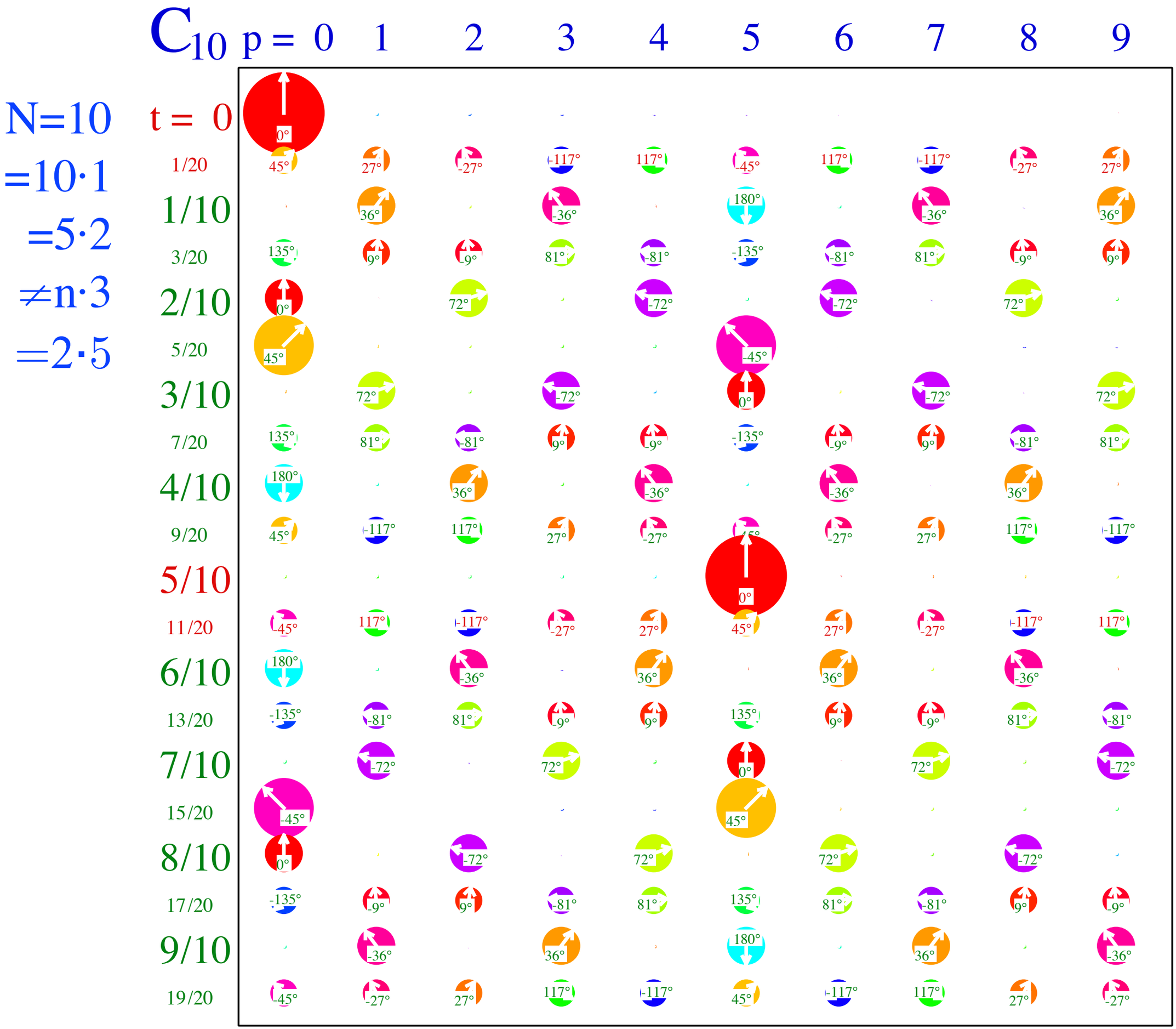
Invariant phase "Uncertainty" hyperbolas:

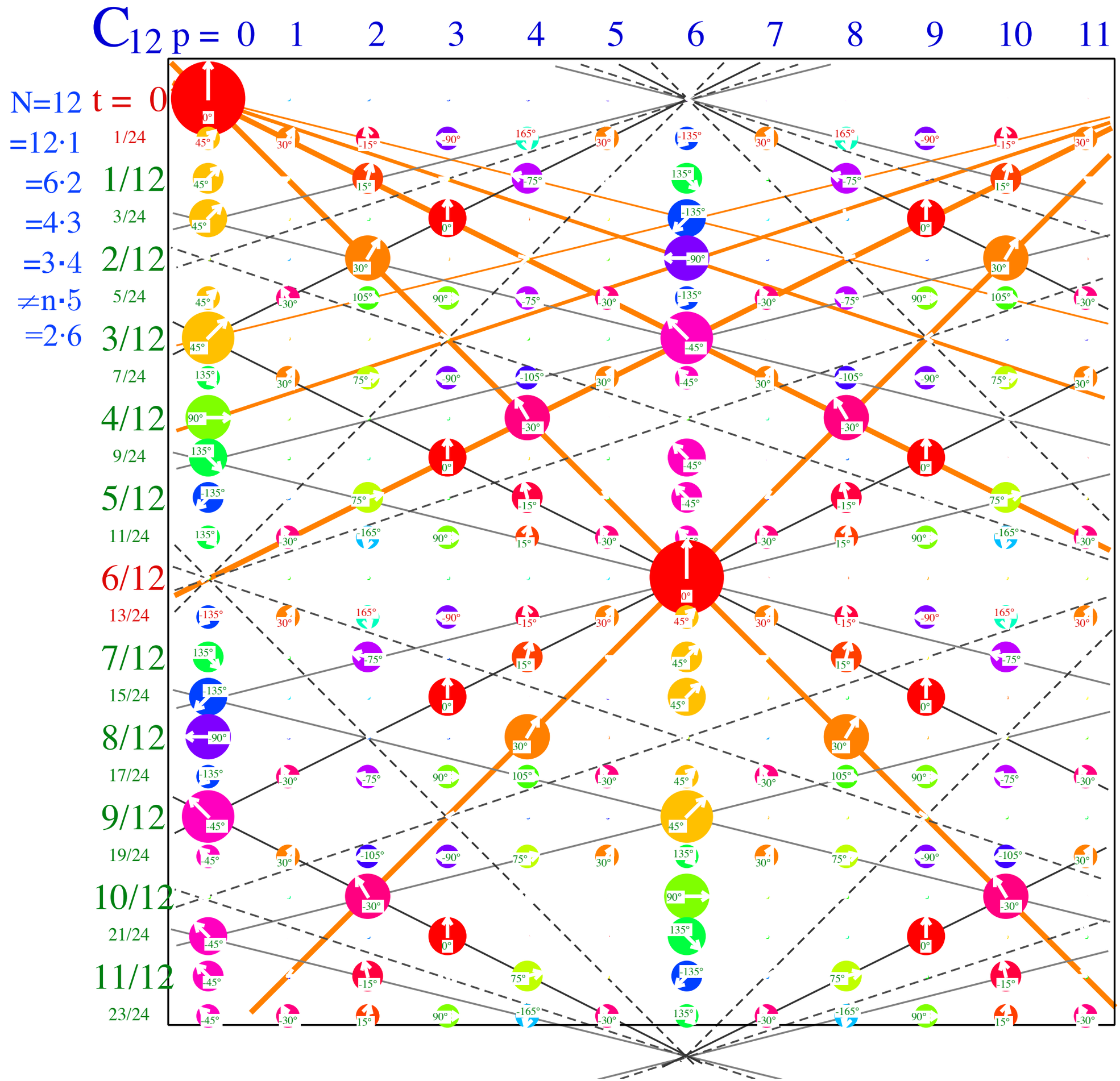
momentum·position=const.

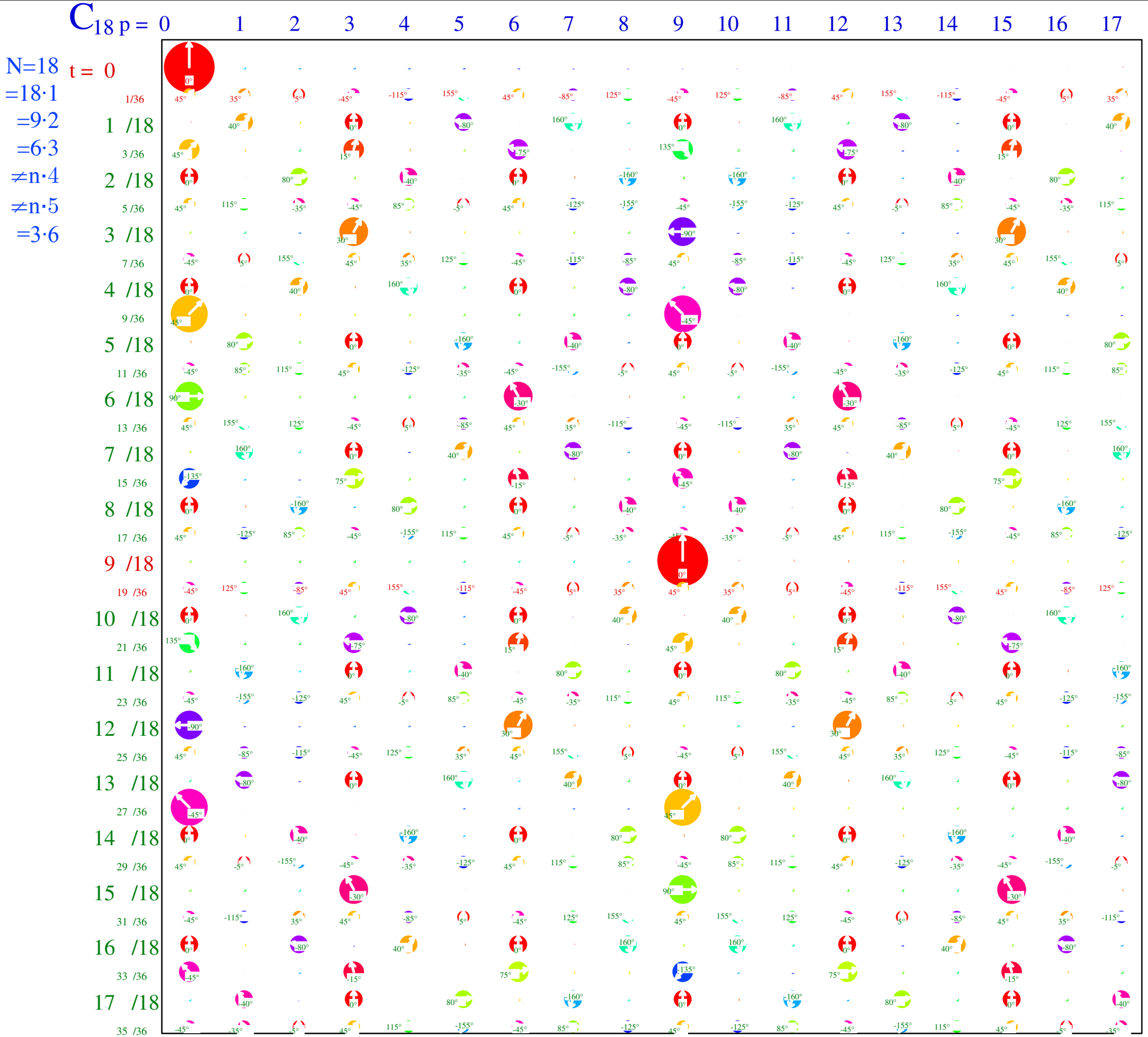
Finite C_N-revivals display all factors of N

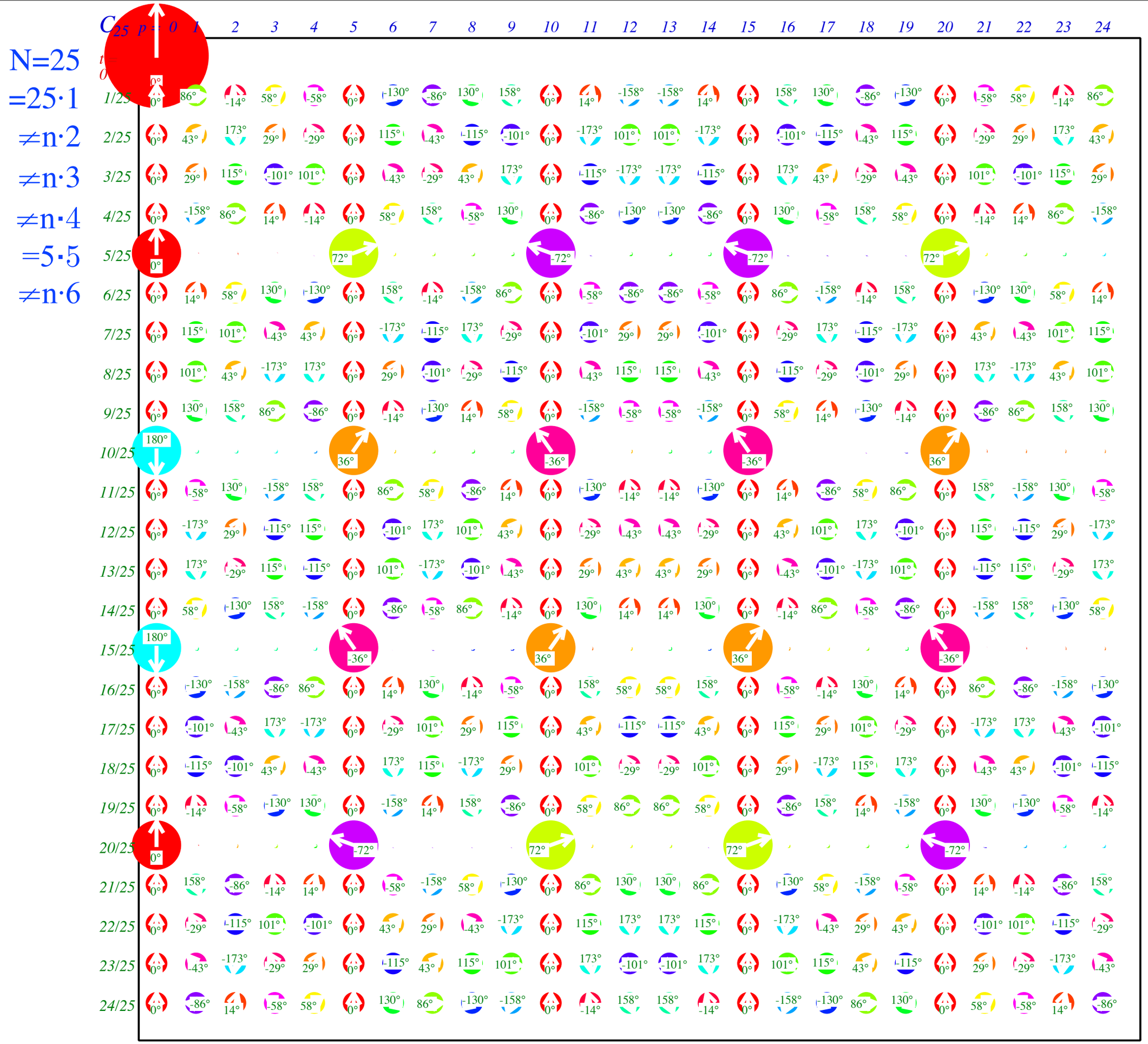

C8
p = 0 1 2 3 4 5 6 7
N=8 t = 0
=8·1 1/16
=4·2 1/ 8
≠n·3 3/16
=2·4 2/ 8
5/16
3/ 8
7/16
4/ 8
9/16
5/ 8
11/16
6/ 8
13/16
7/ 8
15/16


C9
p = 0 1 2 3 4 5 6 7 8
N=9 t = 0
0°
=9·1 1/ 9
0° 80° -40° 0° -160° -160° 0° -40° 80°
≠n·2 2/ 9
0° 40° 160° 0° -80° -80° 0° 160° 40°
=3·3 3/ 9
90° -30° -30°
≠n·4 4/ 9
0° -160° 80° 0° -40° -40° 0° 80° -160°
5/ 9
0° 160° -80° 0° 40° 40° 0° -80° 160°
6/ 9
-90° 30° 30°
7/ 9
0° -40° -160° 0° 80° 80° 0° -160° -40°
8/ 9
0° -80° 40° 0° 160° 160° 0° 40° -80°


C10 p = 0 1 2 3 4 5 6 7 8 9
N=10
=10·1
=5·2
≠n·3
=2·5
t = 0
1/20
1/10
3/20
2/10
5/20
3/10
7/20
4/10
9/20
5/10
11/20
6/10
13/20
7/10
15/20
8/10
17/20
9/10
19/20


C12 p = 0 1 2 3 4 5 6 7 8 9 10 11
N=12 t = 0
=12·1 1/24
=6·2 1/12
=4·3 3/24
=3·4 2/12
≠n·5 5/24
=2·6 3/12
7/24
4/12
9/24
5/12
11/24
6/12
13/24
7/12
15/24
8/12
17/24
9/12
19/24
10/12
21/24
11/12
23/24


C18 p = 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
N=18 t = 0
=18·1
=9·2
=6·3
≠n·4
≠n·5
=3·6
1/36
1 /18
3 /36
2 /18
5 /36
3 /18
7 /36
4 /18
9 /36
5 /18
11 /36
6 /18
13 /36
7 /18
15 /36
8 /18
17 /36
9 /18
19 /36
10 /18
21 /36
11 /18
23 /36
12 /18
25 /36
13 /18
27 /36
14 /18
29 /36
15 /18
31 /36
16 /18
33 /36
17 /18
35 /36

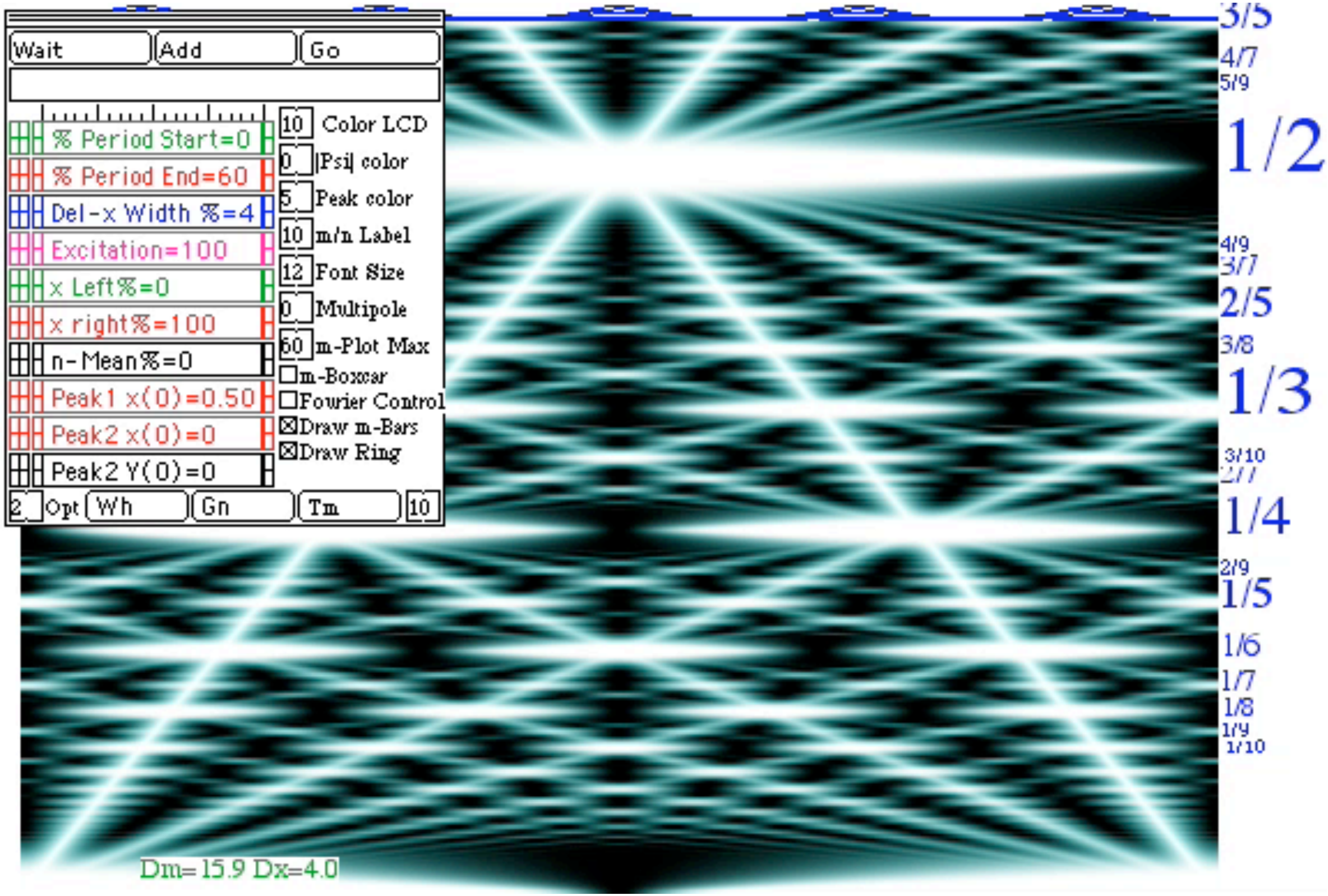

C_{25} p = 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
N=25
=25·1
≠n·2
≠n·3
≠n·4
=5·5
≠n·6
1/25
2/25
3/25
4/25
5/25
6/25
7/25
8/25
9/25
10/25
11/25
12/25
13/25
14/25
15/25
16/25
17/25
18/25
19/25
20/25
21/25
22/25
23/25
24/25

Summary

Quantum rotor revivals have wonderfully simple geometry, number, and group theoretical analyses

...But, how they do factoring is still an issue..

Simulation of revival-intensity dynamics



Observable dynamics of N-level-system state $|\Psi\rangle$

Depends on Fourier spectrum of probability distribution $\langle\Psi|\Psi\rangle$

$$|\Psi\rangle = \sum_{n=0}^{N} e^{-i\omega_n t}\psi_n$$

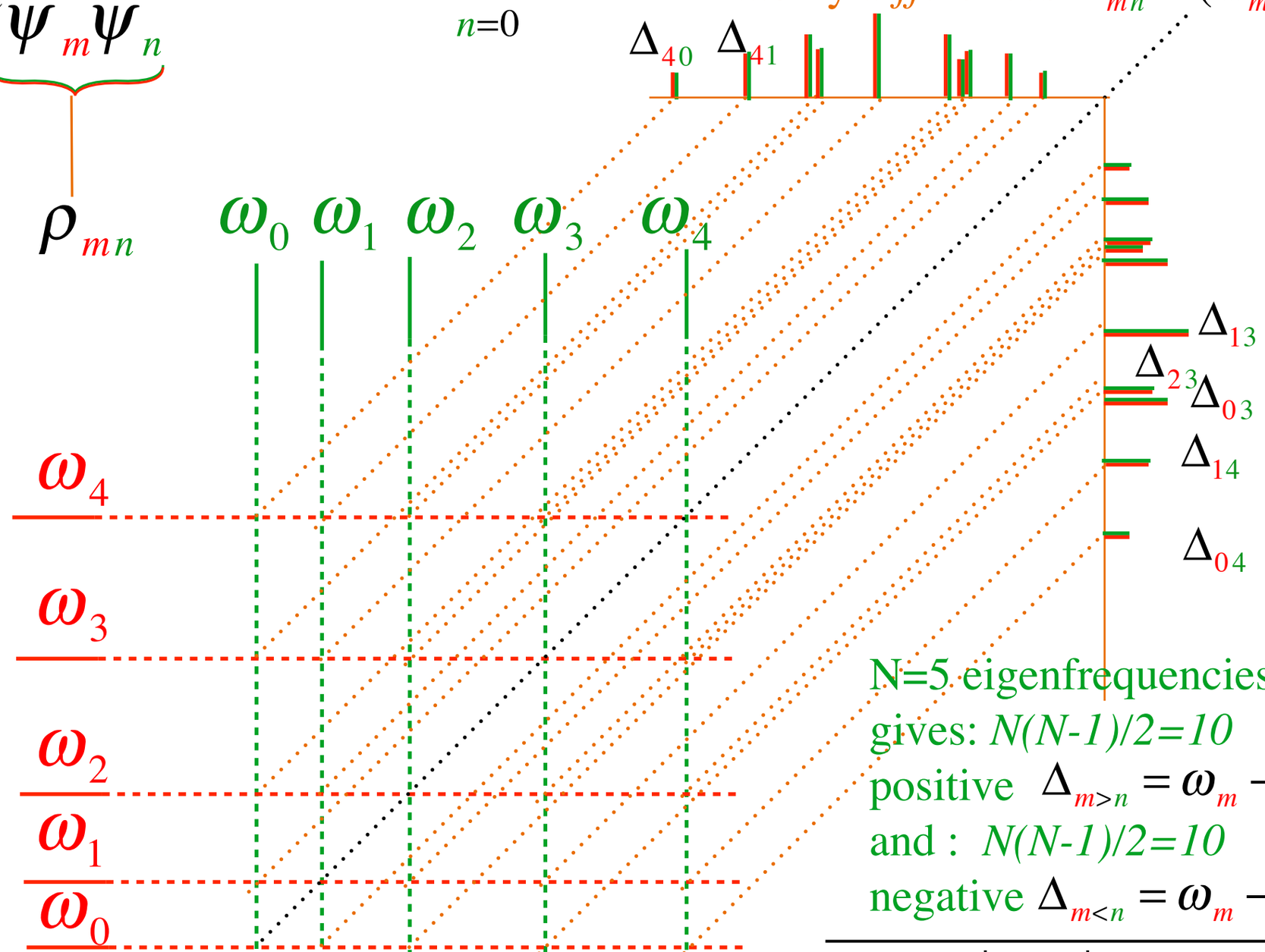
...But individual eigenfrequencies are not directly observable…

...only *differences* $\Delta_{mn} = (\omega_m - \omega_n)$

$$\langle\Psi|\Psi\rangle = \sum_{n=0}^{N} e^{i(\omega_m-\omega_n)t}\psi_m^*\psi_n$$

$$= \sum_{m,n=0}^{N} e^{i\Delta_{mn}t} \quad \rho_{mn}$$

$$\langle\Psi| = \sum_{m=0}^{N} e^{+i\omega_m t}\psi_m^*$$



N=5 eigenfrequencies gives: *N(N-1)/2=10* positive $\Delta_{m>n} = \omega_m - \omega_n$ and : *N(N-1)/2=10* negative $\Delta_{m<n} = \omega_m - \omega_n$

...that is, *N(N-1)/2=10* observable beats $|\Delta_{m<n}| = |\omega_m - \omega_n|$

Observable dynamics of 2-level-system state $|\Psi\rangle$

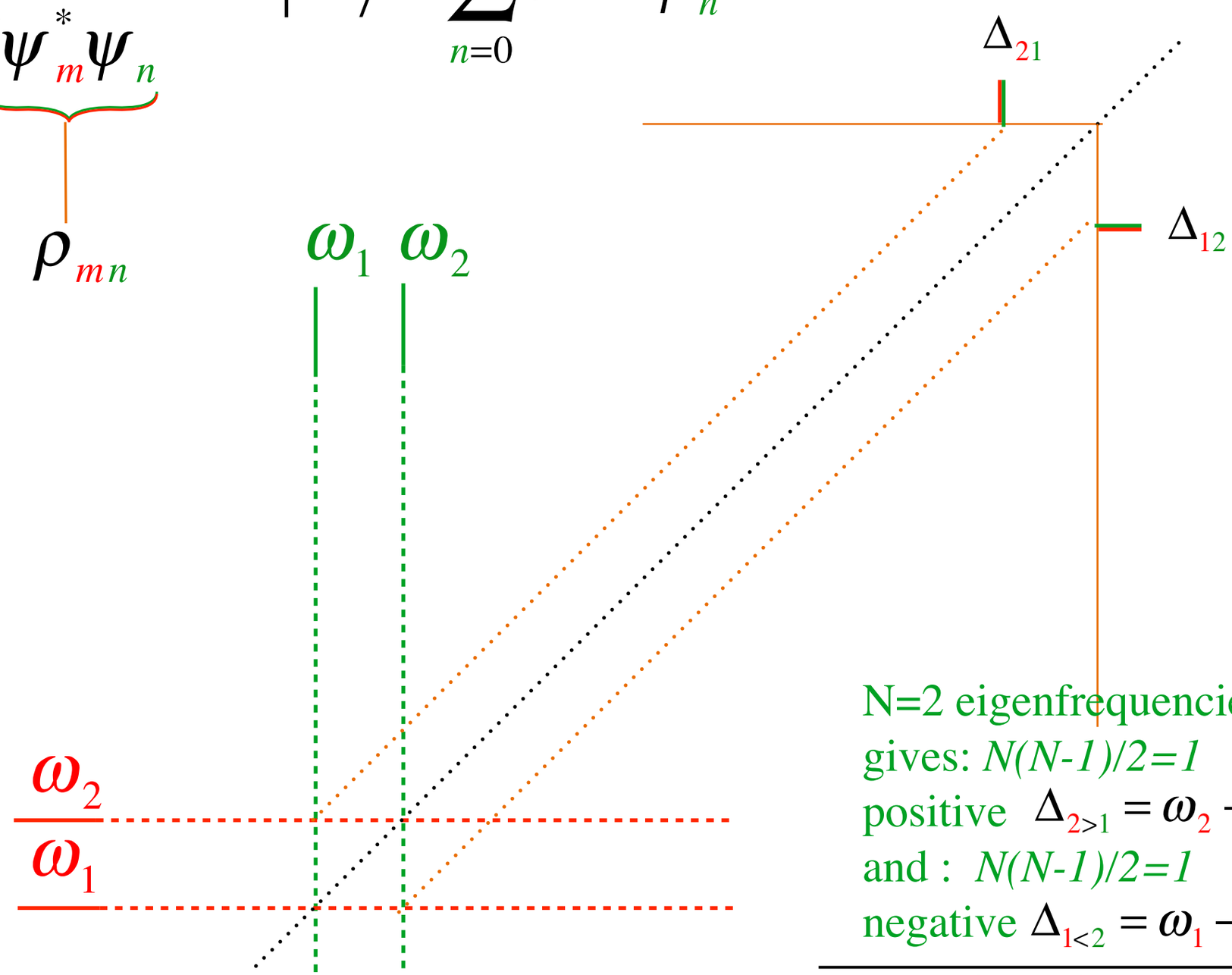
Fourier spectrum of $\langle\Psi|\Psi\rangle$ has *ONE* beat frequency $\Delta_{21} = -\Delta_{12}$

$$|\Psi\rangle = \sum_{n=0}^{N} e^{-i\omega_n t}\psi_n$$

$$\langle\Psi|\Psi\rangle = \sum_{n=0}^{N} e^{i(\omega_m-\omega_n)t}\psi_m^*\psi_n$$

$$= \sum_{m,n=0}^{N} e^{i\Delta_{mn}t} \quad \rho_{mn}$$

$$\langle\Psi| = \sum_{m=0}^{N} e^{+i\omega_m t}\psi_m^*$$



N=2 eigenfrequencies
gives: *N(N-1)/2=1*
positive $\Delta_{2>1} = \omega_2 - \omega_1$
and : *N(N-1)/2=1*
negative $\Delta_{1<2} = \omega_1 - \omega_2$

...that is, *N(N-1)/2=1* observable beat $|\Delta_{m<n}| = |\omega_m - \omega_n|$

What do revivals look like?

...in *per*-space-time…

(… that is:

frequency ω_m radian/sec.

vs

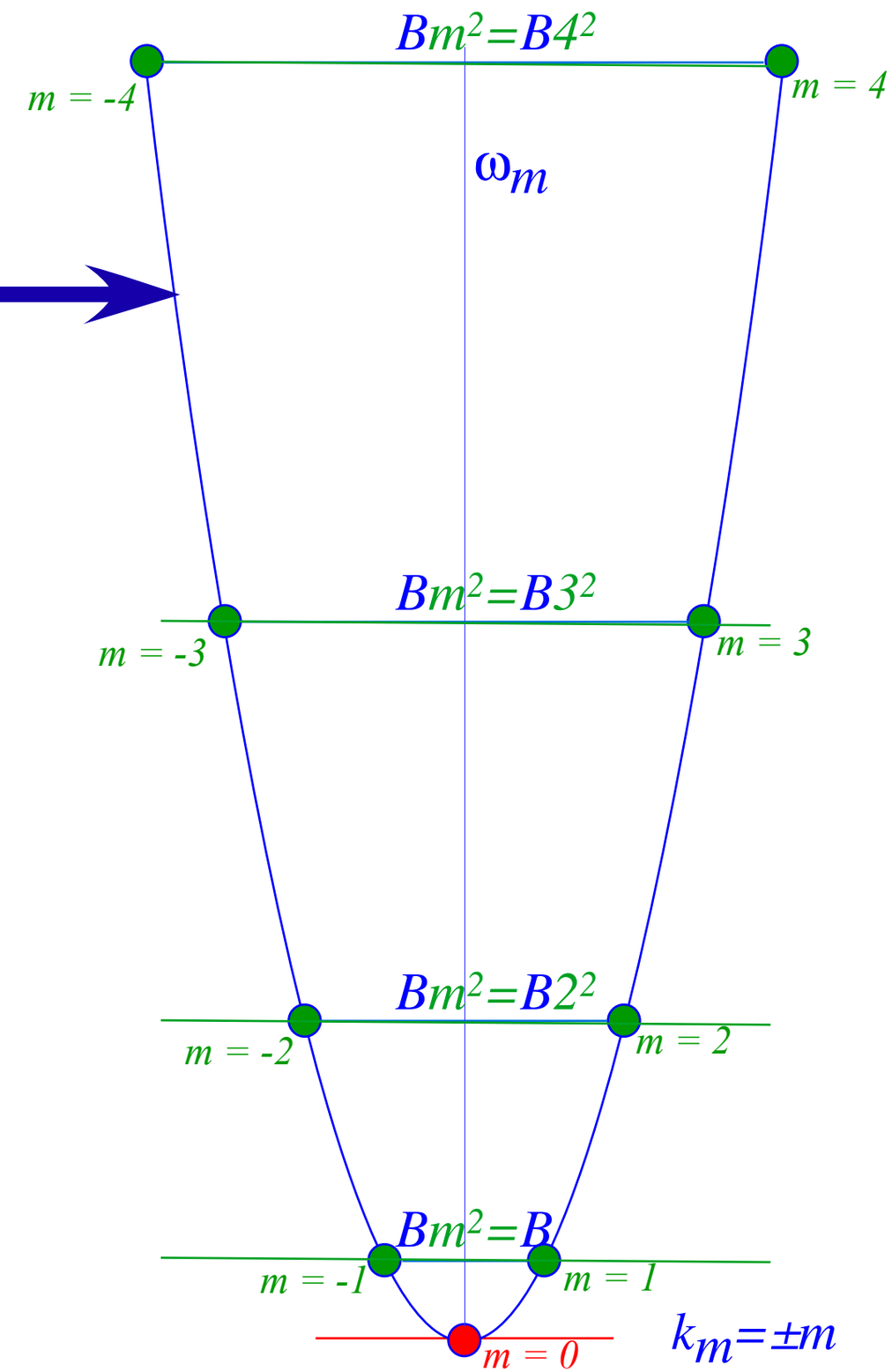
k-vector k_m radian/cm)

N-level-system and revival-beat wave dynamics

Levels
for
Quadratic (Bohr-Rotor) Spectrum

$\omega_m = Bm^2$

$k_m = \pm m$



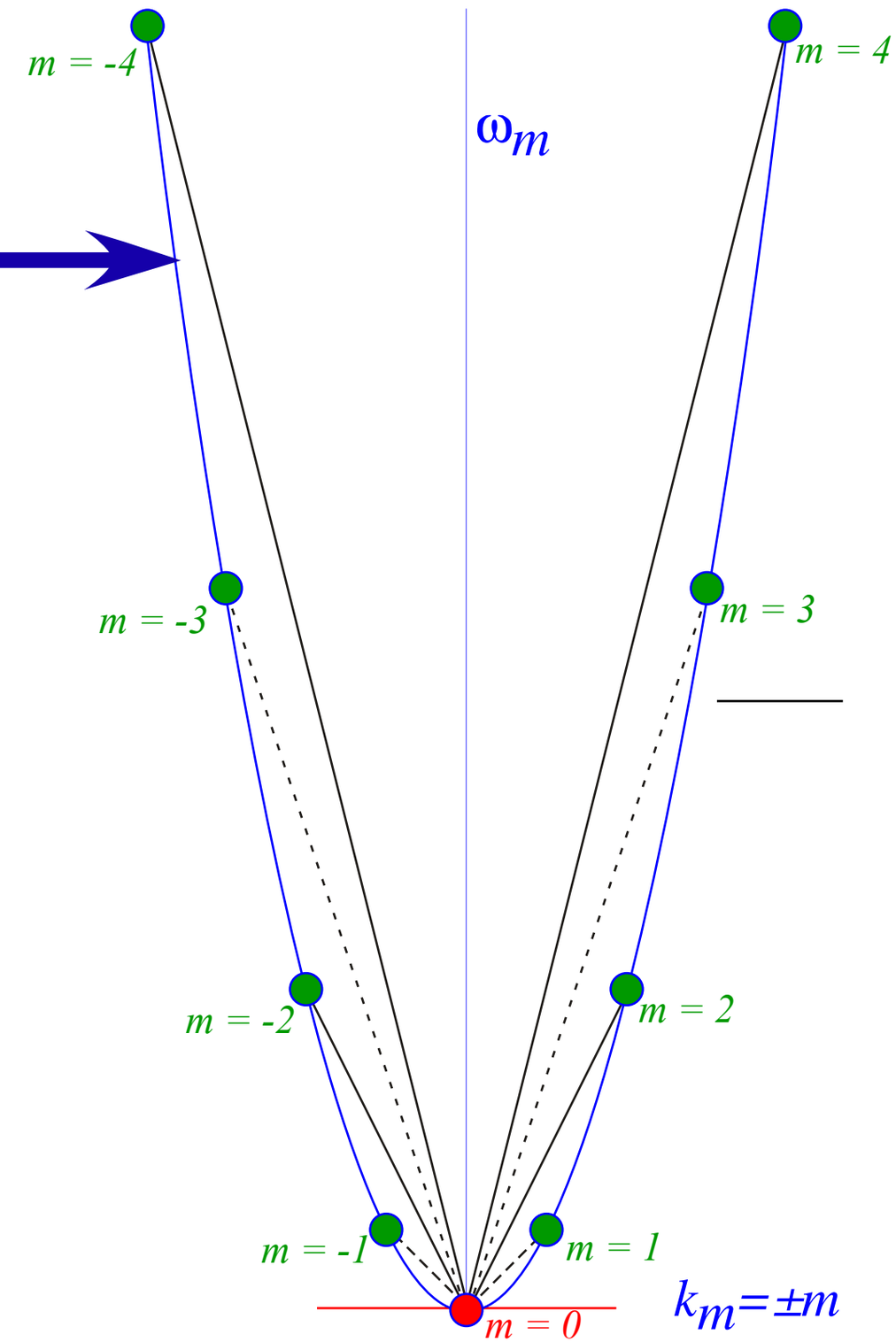
N-level-system and revival-beat wave dynamics

Possible wave velocities
for
Quadratic (Bohr-Rotor) Spectrum

$$\omega_m = Bm^2$$

$$k_m = \pm m$$

$$V_{phase} = \frac{\omega_m}{k_m} = \frac{Bm^2}{m} = mB$$



N-level-system and revival-beat wave dynamics

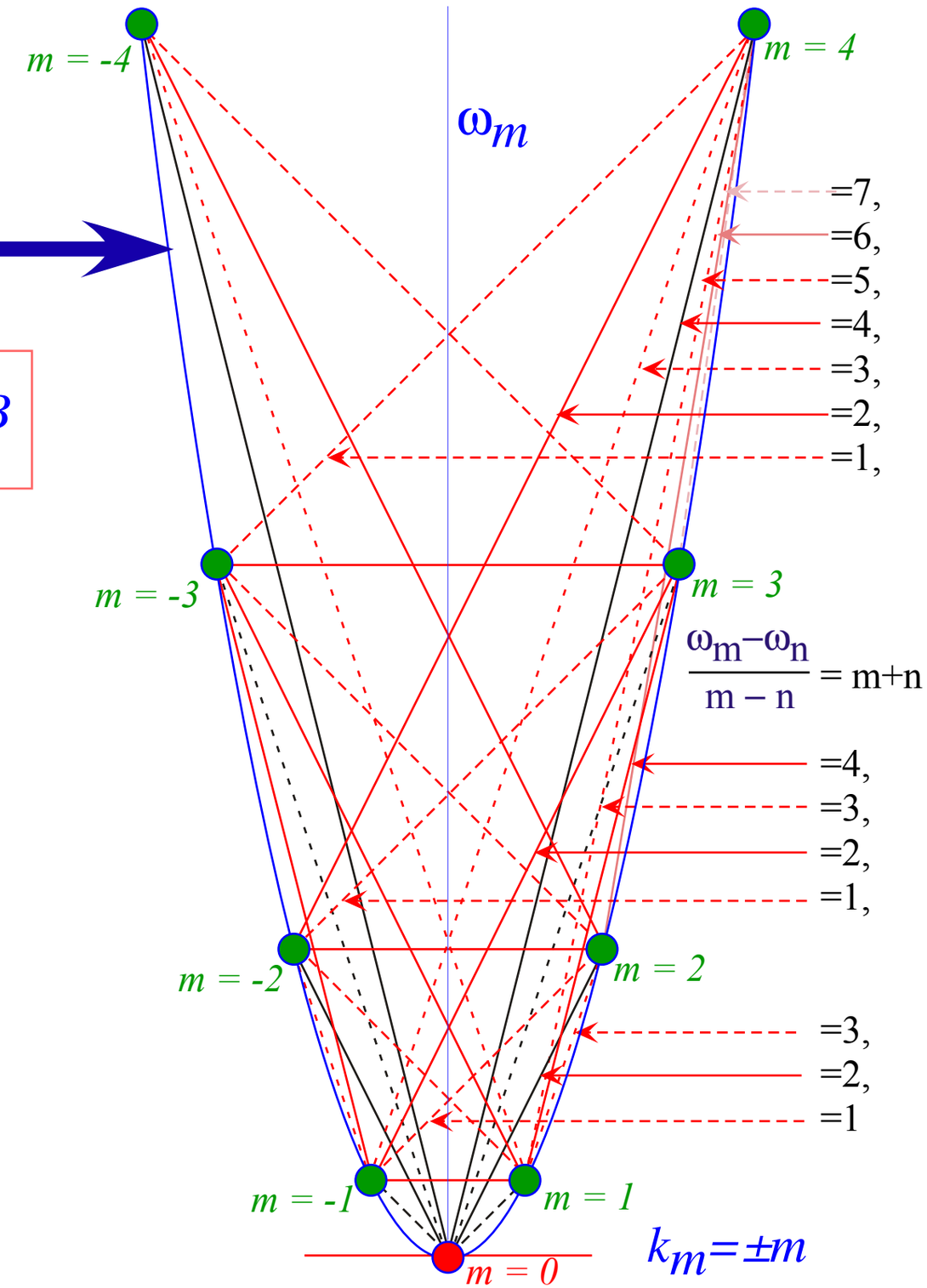
Possible wave velocities
for
Quadratic (Bohr-Rotor) Spectrum

$$\omega_m = Bm^2$$

$$k_m = \pm m$$

$$V_{phase} = \frac{\omega_m}{k_m} = \frac{Bm^2}{m} = mB$$

$$V_{group} = \frac{\omega_m - \omega_n}{k_m - k_n} = \frac{m^2 - n^2}{m \pm n} B = (m \pm n)B$$



N-level-system and revival-beat wave dynamics

Possible wave velocities
for
Quadratic (Bohr-Rotor) Spectrum

$\omega_m = Bm^2$

$k_m = \pm m$

$$V_{phase} = \frac{\omega_m}{k_m} = \frac{Bm^2}{m} = mB$$

$$V_{group} = \frac{\omega_m - \omega_n}{k_m - k_n} = \frac{m^2 - n^2}{m \pm n} B = (m \pm n)B$$

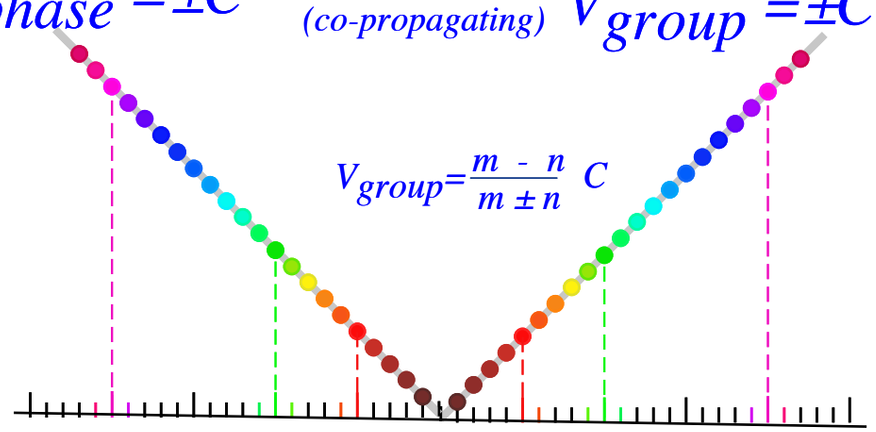
Possible wave velocities
for
Linear (Optical) Spectrum

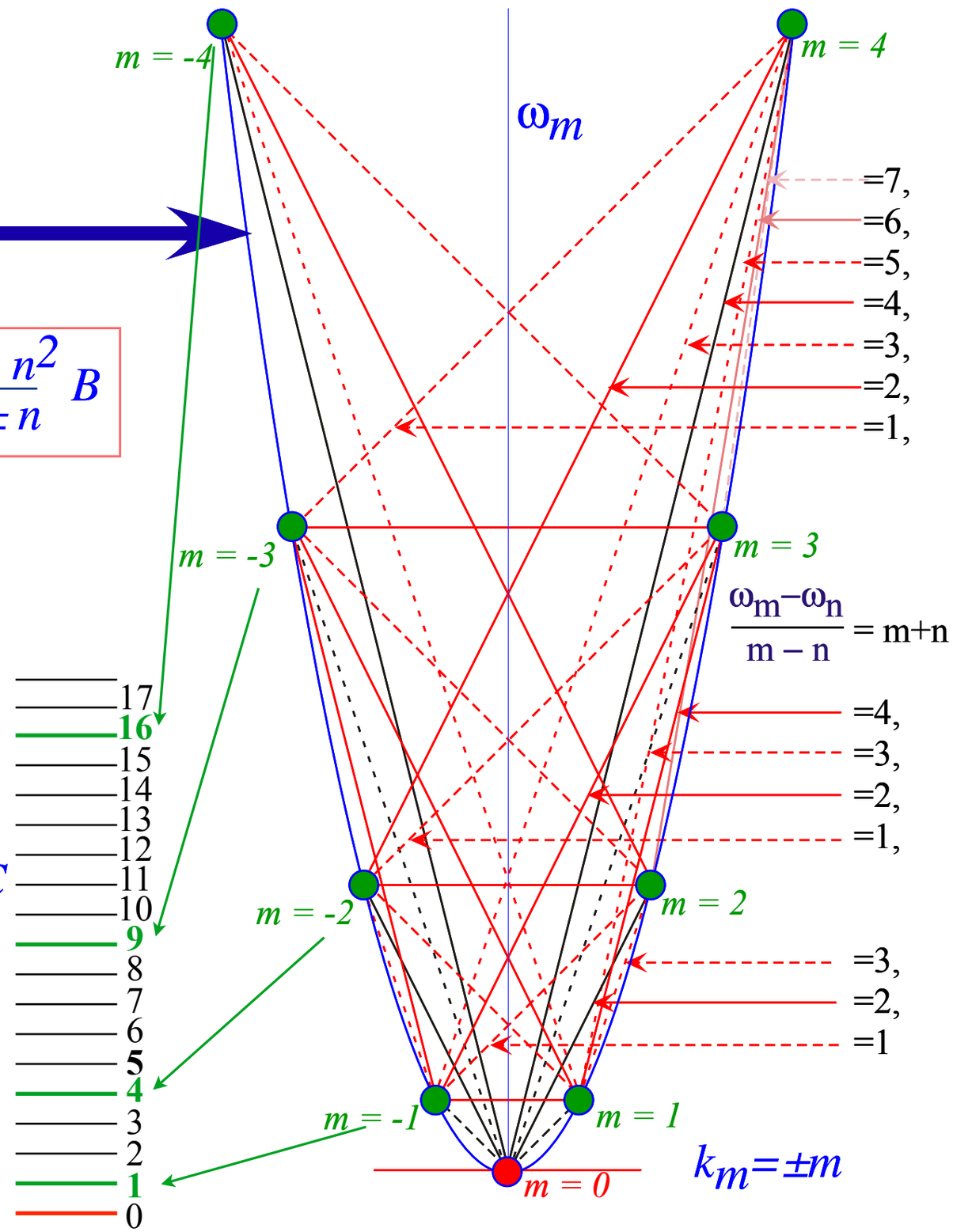
$\omega_m = C|m|^1$

$k_m = m$

$V_{phase} = \pm C$ (co-propagating) $V_{group} = \pm C$

$$V_{group} = \frac{m - n}{m \pm n} C$$

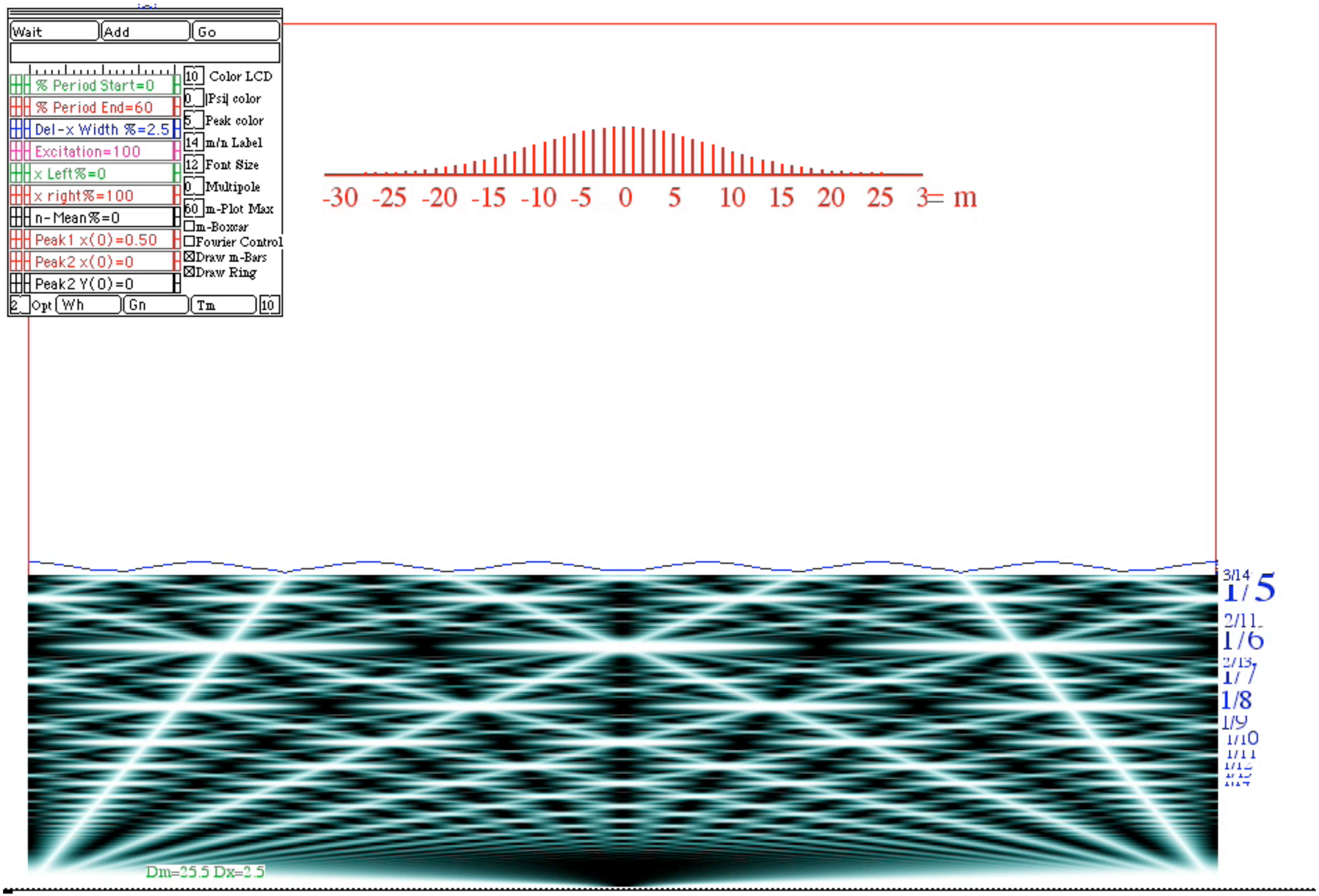




Harmonic Oscillator level spectrum contains the **Rotor Levels** as a <u>subset</u>

Summary

Quantum rotor revivals obey wonderfully simple geometry, number, and group theoretical analysis


Wait
Add
Go
% Period Start=0
% Period End=60
Del-x Width %=2.5
Excitation=100
x Left%=0
x right%=100
n-Mean%=0
Peak1 x(0)=0.50
Peak2 x(0)=0
Peak2 Y(0)=0
2
Opt
Wh
Gn
Tm
10
10 Color LCD
0 |Psi| color
5 Peak color
14 m/n Label
12 Font Size
0 Multipole
60 m-Plot Max
m-Boxcar
Fourier Control
Draw m-Bars
Draw Ring
-30 -25 -20 -15 -10 -5 0 5 10 15 20 25 30 m
3/14
1/5
2/11
1/6
2/13
1/7
1/8
1/9
1/10
Dm=25.5 Dx=2.5