THE QUEST FOR THE DRIVING FORCE IN ACTIVE GALACTIC NUCLEI

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ABSTRACT

Multiwavelength monitoring of continuum variability in active galactic nuclei (AGNs) can strongly constrain the emission mechanisms at work in these sources. For example, the first intensive monitoring program on the Seyfert 1 galaxy NGC 5548, undertaken by the International AGN Watch with *IUE* in 1989, showed that whatever mechanism was causing the continuum variations must propagate at close to the speed of light if the continuum was produced by a standard accretion disk. Further monitoring programs provided similar constraints. However, a continuous 49-day monitoring program on NGC 7469 carried out with *IUE* in its final year provides for the first time statistically significant evidence for time delays between continuum variations in different parts of the spectrum. Optical data confirm the trend seen in the UV. These interband time delays can be attributed to a temperature gradient in the continuum source combined with radiatively driven continuum variations. The time delays are consistent with the $\tau \propto \lambda^{4/3}$ dependence expected for a thin accretion disk. We will discuss the implications of this detection, as well as the absence of previous detections of wavelength-dependent continuum lags in AGNs.

Key words: active galactic nuclei; accretion disks; black holes; variability.

1. INTRODUCTION

It has been generally assumed for over three decades that active galactic nuclei are powered by gravity, specifically by accretion onto supermassive black holes. The simple arguments remain compelling: rapid variability implies a small size of the source that must be extremely massive on the basis of Eddington limit arguments. The deep gravitational potential of a supermassive black hole leads to an accretion disk that can produce thermal radiation over a broad range of the electromagnetic spectrum, as is observed in AGNs. Moreover, the accretion disk ought to produce prominent continuum emission through the optical and ultraviolet spectrum (Shields 1978), and Malkan & Sargent (1982) suggested that the feature now known as the "big blue bump" is in fact thermal continuum emission from the accretion disk. Finally, the intense magnetic fields expected in the inner parts of an accretion disk provide a plausible mechanism for collimating high speed outflows such as the jets seen in blazars. However, direct proof of the black-hole model is still lacking.

2. EVIDENCE OF BLACK HOLES FROM EMISSION LINES

What would constitute definitive proof that supermassive black holes reside in AGNs? Most astronomers would be convinced that AGNs are powered by supermassive black holes if orbiting material (stars or gas) demonstrated the presence of a large mass in a region too small to contain anything other than a collapsed object. This has provided much of the motivation for study of the structure and kinematics of the broad-line region (BLR). Even the first paper on the physics of Seyfert galaxies (Woltjer 1959) recognized the importance of the broad emission lines in this context; however, at that time the only limit on the BLR size was that it must be smaller than the angular resolution limit of about $1''$, which translates to a linear scale of order 100 pc for the nearest AGNs. The upper limits on the central masses thus imposed by the virial theorem were too large to be interesting. Twenty years ago, photoionization equilibrium arguments provided better estimates of the BLR sizes, of order 1 pc for nearby luminous Seyferts. However, the virial masses implied were still uncomfortably large, of order $10^9\,M_\odot$. Central masses this large would affect stellar kinematics on scales resolvable from the ground. The absence of evidence from stellar kinematics in AGNs led many to believe that the BLR cloud motions were not dominated by the gravity of the central source, but by radiation pressure (e.g., Blumenthal & Mathews 1975). However, a further significant reduction in the estimate of the BLR size was provided by emission-line lags (Peterson et al. 1985). Reverberation mapping (Blandford & McKee 1982; see Netzer & Peterson 1997 for a recent review) of the BLR now yields masses only an order of magnitude larger than required by the Eddington limit. The kinematics of the BLR are still not understood, although variability studies seem to indicate that the velocity field is not primarily radial (e.g., Korista et al. 1995); if this is indeed true, then the central-source masses are good to at least an order of magnitude. Thus the kinematic evidence that supermassive black holes reside in the central regions of AGNs is now quite good, but not irrefutable.

3. CONTINUUM VARIABILITY

Ten years ago, the International AGN Watch[1] (Alloin et al. 1994), an informal consortium that has involved more than 200 astronomers world-wide, carried out the first massive multiwavelength monitoring program on NGC 5548 (Clavel et al. 1991; Peterson et al. 1991), thus providing the first accurate measurements of emission-line lags for a wide range of ionization levels. The first AGN Watch program on NGC 5548 covered only the ultraviolet (with *IUE*) and optical (with ground-based telescopes), but later programs on the same galaxy as well as NGC 3783, NGC 4151, Fairall 9, and 3C 390.3 also involved at various times X-ray (*ROSAT* and *ASCA*), γ-ray (*CGRO*), and extreme UV (*EUVE*) observations.

At least as important as the emission line results has been the clear demonstration from the reverberation-mapping data that the UV (typically $\sim$ 1200–1400 Å) and optical (typically $\sim$ 5100 Å) continua vary with little if any time delay between them. This has important consequences for models of the continuum source (Krolik et al. 1991; Courvoisier & Clavel 1991; Collin-Soufrin 1991; Molendi et al. 1991). In Galactic accretion disks, some types of variations are attributable to instabilities that propagate through the disk at the sound speed. On account of the radial temperature gradient in the accretion disk, the variations appear in different wavebands at different times as the disturbances move radially through the disk. The lack of detectable time delays between different continuum wavebands in AGNs implies that whatever mechanism causes the continuum variations in Seyfert galaxies must propagate at close to the speed of light, if the standard thin-disk structure ($T(r) \propto r^{-3/4}$) is appropriate; in other words, if a thin disk is responsible for the continuum emission, the variations must be driven by radiation.

Until recently, only upper limits on possible time delays between variations in different continuum bands in AGNs were available. The situation has recently changed, however, as a consequence of of a very intensive UV/optical monitoring program that was undertaken in 1996 June – July, in the last few months of the long and productive *IUE* mission. The target of this investigation was the Seyfert 1 galaxy NGC 7469[2], and for the first time, statistically significant wavelength-dependent continuum lags were detected. The UV observations (Wanders et al. 1997) consist of somewhat more than 200 SWP spectra that were obtained over a 49-day period. Concurrent optical observations (Collier et al. 1998) were obtained at a number of observatories, but the low amplitude of source variability during the campaign required restricting the time-series analysis to only the three spectroscopic data sets (Wise Observatory, Ohio State/Lowell Observatory, and Crimean Astrophysical Observatory) that were sufficiently homogeneous and well-sampled that they could be intercalibrated to very high accuracy.

In Fig. 1, we show the UV and optical continuum light curves for NGC 7469. Cross-correlation of each of the longer wavelength light curves with the 1315 Å light curve reveals that the variations at longer wavelengths follow those at shorter wavelengths. The cross-correlation lags are given in Table 1, which gives in each case the centroid of the cross-correlation function, computed using all points within 80% of the peak value. As outlined in the next section, the uncertainties are small enough that both the lags and the trend with increasing wavelength are statistically significant. We thus believe that this presents the first reliable identification of UV and optical continuum variations arising in an accretion disk in an AGN.

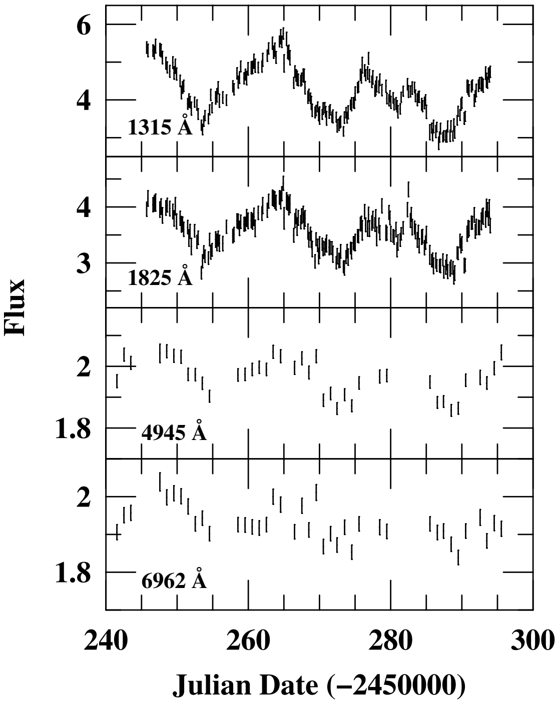

Figure 1. Continuum light curves for NGC 7469 during 1996 June–July. The first and second panels show the UV continuum at 1315 Å and 1825 Å, respectively (Wanders et al. 1997), and the third and fourth panels show the optical continuum at 4945 Å and 6962 Å, respectively (Collier et al. 1998).

4. HOW SIGNIFICANT ARE THE INTERBAND CONTINUUM LAGS?

The real question, of course, is how significant are the lag detections? Determining the uncertainties in AGN cross-correlation lags is non-trivial, and has

[1] See http://www.astronomy.ohio-state.edu/~agnwatch/.

[2] The original target for this investigation was to be Mrk 335. However, loss of a gyro in 1996 March rendered much of the sky no longer accessible to *IUE*, and NGC 7469 was one of a handful of observable Seyfert 1s that were suitably bright for this experiment.

Table 1. NGC 7469 Continuum Lags Relative to 1315 Å.

Wavelength (Å)	$\tau_{\rm cent}$ (days)	$P(\tau_{\rm cent} \leq 0\,{\rm days})$
1825 Å	$0.22^{+0.12}_{-0.13}$	0.024
4845 Å	$1.25^{+0.48}_{-0.35}$	0.004
6962 Å	$1.84^{+0.93}_{-0.94}$	0.030

been discussed in the literature in a variety of papers with no real consensus emerging. There seems to be general agreement that Monte-Carlo methods give the most reliable estimates of lag uncertainties; indeed, if one performs the cross-correlation without making use of the uncertainties in the light curve fluxes (such as in the Gaskell & Sparke 1986 interpolation method), one is automatically driven to Monte-Carlo methods to estimate lag uncertainties. The principal difficulty with the usual implementation of Monte-Carlo methods is that the results are highly model dependent, and are valid only to the degree that the adopted model approximates reality.

In order to overcome this model-dependence, we have developed a Monte-Carlo method of error estimation that is *model independent* (Peterson et al. 1998). This method has two independent components:

1. **Flux randomization (FR):** One of the sources of uncertainty in cross-correlations is the flux uncertainties in the light curves. In the best data sets, these errors are small (1-5%), but often the variations detected are not very large (only $\sim 10\%$, in some cases). To test the effects of flux uncertainties, for each Monte-Carlo realization, we perturb each flux value by a random Gaussian deviate with σ given by the quoted uncertainty for each point. This effectively randomizes the fluxes in the light curve in a way that is consistent with their measured values and error estimates.

2. **Random subset selection (RSS):** Individual data points or small clusters of data points in a light curve can have a very strong effect on cross-correlation results. In order to test the effects of different individual points in a light curve consisting of N measurements, for each Monte-Carlo realization we select at random N points from the light curve one at a time *without regard to whether or not the points have been previously selected*. On average, this means that $1/e$ of the points will be selected more than once. The redundant selections have no effect on the cross-correlation analysis since the order of the points is not changed, and thus is equivalent to selection of a subset of $\sim N(1-1/e)$ points. Random subset selection is similar to a "bootstrap" process (which indeed provided the original motivation for this method), except that the ordering of the points matters in a time series.

Of course what is *not* tested by this method is sensitivity to what might have happened between the actual observations, but attempting to account for this brings in a level of model-dependence. Fundamentally, this limitation is the same as the limitation on the reliability of the Gaskell–Sparke interpolation method itself, namely that linear interpolation between measurements is a reasonable approximation to the true behavior of the light curve. All this says is that the light curves must be well sampled.

We have carried out extensive simulations and comparisons with model-dependent Monte Carlo simulations, and we find that the uncertainties we obtain by combining FR and RSS (hereafter called FR/RSS) appear to be quite reliable for a wide range of plausible behaviors. If anything, the uncertainties are conservative in the sense that the errors are slightly *overestimated*, which is as expected since the original light curve has N points and each simulation has on average only $N(1-1/e)$ points. In order to test the statistical significance of interband continuum lags, this is the preferred direction to err, since the uncertainties we derive are thus likely to be overestimates rather than underestimates.

To evaluate the interband continuum lags in NGC 7469, we used Gaskell–Sparke interpolation method as described in detail by White & Peterson (1994). To estimate the uncertainties in the measurements, we carried out a large number of realizations using the FR/RSS method. Each realization yields two light curves, which are cross-correlated as though they were real data. Each lag measurement so derived is used to build up a "cross-correlation peak distribution" (CCPD; see Maoz & Netzer 1989), which can then be integrated to determine the probability that the actual lag lies in a given range. A sample CCPD from the NGC 7469 FR/RSS analysis is shown in Fig. 2. Unless stated otherwise, the lag uncertainties we quote are such that approximately 16% of the realizations give lags larger than the upper limit quoted, and 16% of the realizations give lags smaller than the lower limit quoted; the uncertainties would thus correspond to 1σ limits for a Gaussian error distribution. In Table 1, the uncertainties in the cross-correlation function centroid $\tau_{\rm cent}$ are the values computed by the FR/RSS method. We also quote in Table 1 the probability of obtaining a lag of zero or less $P(\tau \leq 0)$, which is obtained by integrating the CCPD over this range. This can be interpreted as the probability that the actual lag is less than or equal to zero, which demonstrates that the interband lags given in Table 1 are all statistically significant at $\gtrsim 97\%$ confidence.

Further demonstration of the wavelength dependence of interband continuum lags is shown in Fig. 3. Here we have simply averaged the fluxes in each spectrum into broad wavelength bins and cross-correlated the resulting light curves with the 1315 Å light curve, thus producing a "lag spectrum." The peaks in the spectrum correspond to contamination by emission lines, which lag behind the continuum variations by a few to several days (see Wanders et al. 1997 and Collier et al. 1998). This clearly shows that the continuum lag increases with wavelength. We also plot for comparison the expected wavelength dependence for a thin accretion disk, $\tau \propto \lambda^{4/3}$.

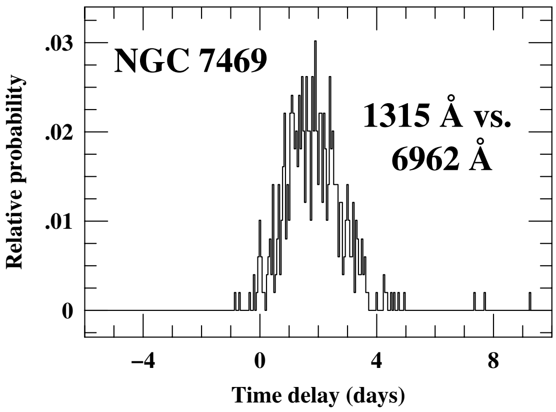

Figure 2. A sample CCPD for NGC 7469. This figure shows the distribution of cross-correlation centroids computed in the FR/RSS simulations for the 1315 Å and 6925 Å light curves. The probability of obtaining a cross-correlation centroid $\tau \leq 0$, obtained by integrating the CCPD from 0 to $-\infty$, is only 0.030, as noted in Table 1.

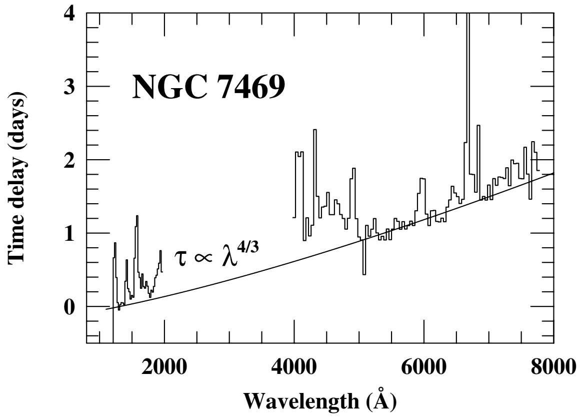

Figure 3. Lag vs. wavelength for NGC 7469. The average flux in each wavelength band is cross-correlated with the 1315 Å continuum, yielding a "lag spectrum". The expected dependence for a thin accretion disk, $\tau \propto \lambda^{4/3}$, is also shown.

5. WHY HAVEN'T INTERBAND CONTINUUM LAGS BEEN DETECTED IN OTHER SOURCES?

It is natural at this point to ask why these wavelength-dependent continuum lags have been detected in NGC 7469, but not in any of the other well-studied variable Seyfert galaxies. The short answer to this is only NGC 7469 has been sampled at sufficiently high frequency and for long enough duration for the cross-correlation results to be significant.

Aside from NGC 7469, the best-sampled non-blazar AGN light curves are for NGC 4151, obtained in 1993 December (Crenshaw et al. 1996; Kaspi et al. 1996; Warwick et al. 1996; Edelson et al. 1996). The time resolution of the NGC 4151 experiment was superior to that of the NGC 7469 experiment in both the UV (0.05 days for NGC 4151, versus 0.23 days for NGC 7469) and the optical (0.5 days for NGC 4151 versus 1.4 days for NGC 7469); however, the duration of the NGC 4151 experiment (10 days) was far shorter than that of the NGC 7469 program (49 days).

In Table 2, we show the results of cross-correlating the UV and optical continuum light curves with the shortest wavelength UV light curve (1275 Å) for NGC 4151. As in Table 1, we give both the cross-correlation centroid with its FR/RSS uncertainties and the probability $P(\tau \leq 0)$ of obtaining a lag of zero or less. It is notable that the 1820 Å variations *lead* the 1275 Å variations at a high level of significance; we suspect that this is in fact due to weak contamination of the 1275 Å band by emission-line flux, which is delayed relative to the continuum. In Fig. 4, we show a lag spectrum for NGC 4151, similar to that shown for NGC 7469 in Fig. 3. The UV data alone provide no evidence for wavelength-dependent continuum lags, and the optical data show convincing wavelength dependence only longward of about 5000 Å. But even for the longest wavelength bin in Table 2 (6925 Å), we find that the relatively short duration of the experiment limits the level of significance that can be ascribed to the results; we find $P(\tau \leq 0) \approx 0.1$, which means that the detection of a positive lag between the 1275 Å and 6925 Å variations is significant only at the 90% level.

Table 2. NGC 4151 Continuum Lags Relative to 1275 Å.

Wavelength (Å)	$\tau_{\rm cent}$ (days)	$P(\tau_{\rm cent} \leq 0\,{\rm days})$
1820 Å	$-0.049^{+0.028}_{-0.023}$	0.991
2688 Å	$0.086^{+0.039}_{-0.048}$	0.042
5125 Å	$0.296^{+0.307}_{-0.938}$	0.423
6925 Å	$2.462^{+0.315}_{-0.308}$	0.102

Of course, it is not just the sampling characteristics of the light curves that matter — the continuum lags depend on the physics of the accretion disk as well. Specifically, the interband continuum time delays for a thin accretion disk are expected to scale like

$$\tau \propto \left(M\dot{M}\right)^{1/3} \lambda^{4/3}, \tag{1}$$

where M is the mass of the central source and $\dot{M}$ is the rate at which it is accreting matter (Collier et al. 1998). For NGC 7469, a fit to thex data yields an estimate $M\dot{M} \approx 10^6\, M_\odot^2\, {\rm year}^{-1}$. If we assume that the wavelength-dependent continuum lags detected in NGC 7469 are indeed the signature of an accretion disk in this source, we can scale the properties of other well-studied sources to NGC 7469 and ask what lags might be expected. We will assume that BLR is virialized, and that the mass of the central source is thus proportional to $v_{\rm FWHM}^2 c\tau$, where $v_{\rm FWHM}$ and τ

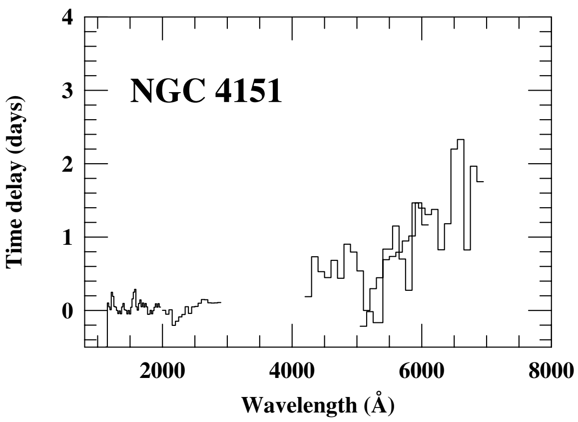

Figure 4. Lag vs. wavelength for NGC 4151. The average flux in each wavelength band is cross-correlated with the 1275 Å continuum, as was done for NGC 7469 in Fig. 3.

are the emission-line full-width at half maximum and lag, respectively[3]. We also assume that the mass accretion rate $\dot{M}$ is proportional to the luminosity L, and in each case we scaled by the mean luminosity in the shortest-wavelength UV band measured. In Table 3, we list four galaxies that have been extensively monitored by the International AGN Watch consortium, along with their masses and mass accretion rates, scaled relative to NGC 7469 as just described[4]. Column (4) of Table 3 gives the predicted time delay between the UV and optical continuum bands expected by this scaling, and in the final column we give the 90% confidence interval (using the FR/RSS method) for the lags actually measured.

In each case, we see that the expected lag lies comfortably within the 90% confidence interval, as does $\tau = 0$. In other words, the observations are in each case consistent with the existence of wavelength-dependent continuum lags as observed in NGC 7469, but also consistent with no wavelength dependence at all.

Table 3. Comparison with Theoretical Scaling.

Galaxy	M^a	$\dot{M}^a$	$\Delta\tau_{\rm pred}$ (days)	90% Confidence Interval (days)
NGC 4151	0.53	0.44	1.1	[−1.90, +3.00]
NGC 3783	2.92	0.38	1.1	[−4.69, +4.62]
NGC 5548	2.00	1.04	1.4	[−0.54, +2.49]
Fairall 9	4.74	8.42	4.0	[−9.83, +10.28]

[a] Relative to NGC 7469.

[3]In each case, we used the Lyα emission line, and measured the line width in the root-mean-square spectrum, so that only the variable part of the line was included.

[4]See Peterson et al. (1998) for a more complete version of Table 3.

6. DISCUSSION

6.1. Where Are We Now?

As noted earlier, it has not been proven definitively that AGNs are powered by supermassive black holes. Nevertheless, the evidence for black holes is gradually improving. The detection of wavelength-dependent continuum lags in NGC 7469 seems to be consistent with an accretion-disk origin for the UV/optical continuum; the wavelength dependence of the lags seems to be about right, and the product $M\dot{M}$ is consistent with other estimates.

The origin of the X-ray continuum remains a problem, particularly on account of the surprising results of the *RXTE* program on NGC 7469 that was carried out in connection with the UV/optical experiment described here (Nandra et al. 1998; Edelson, these proceedings), which does not indicate a strong connection between the X-rays and other bands. This result is quite surprising compared to the results on NGC 4151 (Edelson et al. 1996) and 3C 390.3 (O'Brien, these proceedings), which suggests that the variations in the X-ray and UV/optical are connected, and provided strong arguments for reprocessing models.

6.2. What Comes Next?

The primary goal AGN research ought to be to prove definitively whether or not the fundamental energy source is accretion by supermassive black holes, which requires determination of the velocity field of stars or gas as close as possible to the Schwarzschild radius $R_{\rm S} = 2GM/c^2$. Reverberation mapping of the BLR provides a tremendously powerful tool in this regard since at $\sim 10^3 R_{\rm S}$, it is more than an order of magnitude closer to the central source than megamaser sources and several orders of magnitude smaller than the scales that can be spatially resolved (Peterson 1997). While reverberation-mapping experiments have been successful in measuring the size of the BLR, the kinematics of the BLR are still only constrained and not determined; indeed, three entirely different kinematic models have been used to explain the *HST-IUE* results on NGC 5548, one of the best-studied sources (Wanders et al. 1995; Done & Krolik 1996; Bottorff et al. 1997). Reverberation-mapping experiments that will resolve the velocity-field problem will require very well sampled, high signal-to-noise observations over a long time baseline. This is not a suitable task for *HST*, but it would be for an Explorer-class spacecraft.

More (and longer) simultaneous X-ray/UV/optical monitoring programs are necessary to determine if and how the variations at different energies are causally connected and whether reprocessing models are viable. Do the NGC 7469 results mean we need to pay renewed attention to non-thermal models, and if so, what are the possible consequences in other wavebands?

The consistency of measured wavelength-dependence continuum lags such as those seen in NGC 7469 and thin-accretion disk models is heartening. Maybe we *do* understand the basic processes at work in AGNs. As pointed out by Horne et al. (1998), this affords an intriguing possibility: with the assumption of a thin-disk model and measurement of the time delay, the luminosity of the accretion disk is determined to within a factor of $(\cos i)^{1/2}$, where i is the inclination of the disk. A statistical sample of time delays thus could be used to determine cosmological parameters (H_0 and q_0).

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