

Theoretical comparisons of average normalized gain calculations

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Since its introduction, the normalized gain or the **g**-factor has been widely used in assessing students' performance in pre- and post-tests. The average **g**-factor can be calculated using either the average scores of the class or individual student's scores. In general, these two calculations produce different results. The nature of these two results is explored for several idealized situations. The results suggest that we may be able to utilize the difference between the two results to extract information on how the population may have changed as a result of instruction. © 2006 American Association of Physics Teachers.

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I. INTRODUCTION

Pre- and post-test analyses have been widely used as a method of assessment in education and social science. Researchers have developed a variety of tools to perform such analysis.^{1,2} Some 30 years ago, Frank Gery³ proposed a gap-closing measure as the dependent variable for studies of educational methods,

$$\mathbf{g} = \frac{\text{Posttest Score} - \text{Pretest Score}}{\text{Maximum Score} - \text{Pretest Score}}. \quad (1)$$

In the physics education community, this gap-closing measure is most commonly associated with the work of Richard Hake and is known as the normalized gain.^{4,5}

The three test scores (maximum, post-test, and pre-test) could be defined for an individual student or as an average measure for a population. The average **g** assigned to a group of students can be determined by averaging the **g** for each student in the group. Alternatively, the average **g** for a group can be obtained by applying Eq. (1) to the average scores for the group. In practice, the two methods usually give very similar results for classes of 50 students or more, but the results from these two methods can sometimes yield different outcomes.

There has been little discussion of how the results of these two methods of calculating the average value of **g** can differ and if any useful information can be inferred from differences when they arise. There are also more fundamental issues such as the underlying cognitive models and the measurement uncertainties of the gap-closing measure.^{6,7} Although researchers still question the models underlying the normalized gain (often called the **g**-factor), it is useful to consider the mathematical characteristics of this measure. The focus of this paper is the theoretical relation between the two types of average **g** calculations for several idealized situations.

II. MATHEMATICAL FEATURES OF A MODIFIED **g**-FACTOR

We denote the ratio of the number of correct answers to the total number of questions on the pre-test by x and call this ratio the "pre-test score." We denote this ratio on the post-test by y and call it the "post-test score." The scores can be the scores of an individual student or the average scores of a group of students. In theory, we can treat x and y as two independent variables. The definition of **g** in Eq. (1) is the ratio of a student's or a class' score change to the maximum possible score change. Typically, such changes are positive, but students and/or classes sometimes have negative score changes. To have a consistent definition of **g** for both positive and negative score changes, we will use a definition first introduced by Marx and Cummings,⁸

$$\mathbf{g}(x,y) = \begin{cases} \frac{y-x}{1-x} > 0 & (y \geq x) \\ \frac{y-x}{x} < 0 & (y < x) \end{cases}. \quad (2)$$

The values at the two undetermined (singular) points at $x=0$ and $x=1$ are defined by forcing y to equal x .

$$\mathbf{g}(x,y) = \begin{cases} 1 & (y = x = 1) \\ 0 & (y = x = 0) \end{cases}. \quad (3)$$

In the three-dimensional space spanned by x , y , and **g**, **g** represents a surface (see Fig. 1). For a given x , the **g**-factor increases linearly with y except for the discontinuity at $x=y$. In the positive branch, the slope becomes larger as x increases, while in the negative branch the slope becomes larger as x decreases. Figure 2 shows the y -**g** relation at different values of x . From Eq. (2), we see that **g** can be considered to be a scaled measure of the absolute score change ($y-x$), i.e., a fixed value of ($y-x$) will produce different values of **g** for different values of x .

The relation between **g** and x at constant values of y and ($y-x$) is shown in Figs. 3 and 4 along with the scatter plots

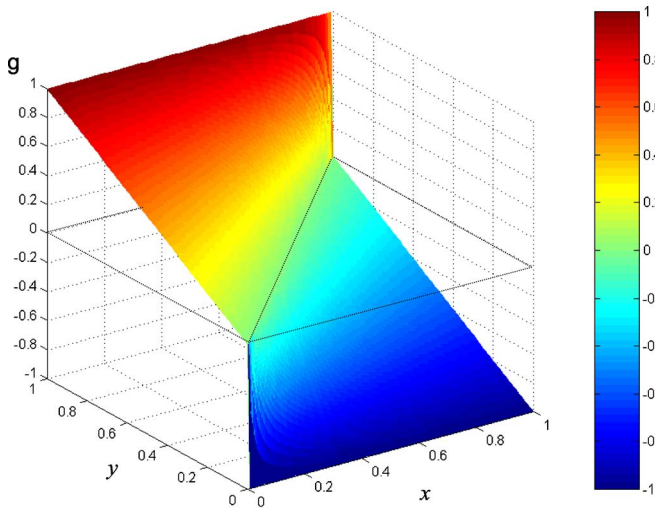


Fig. 1. $g(x, y)$ represents a 3D surface in the space spanned by x , y , and g .

of the data of a class. We can clearly see each student's pre-test score, post-test score, absolute score change, and normalized gain. This type of overlaid scatter plot can help researchers quickly see possible issues in the data such as "ceiling effects" and identify interesting clusters of students (for example, the students with negative gains). Depending on the emphasis of a particular analysis, we can use different types of scatter plots (for example, y versus x) and background curves to show relations among different variables.

III. TWO METHODS OF CALCULATING THE AVERAGE g -FACTOR

Consider a class of N students. We denote the average score of the class on the pre-test by $\bar{x}$, the post-test average score by $\bar{y}$, and the gain calculated using these average

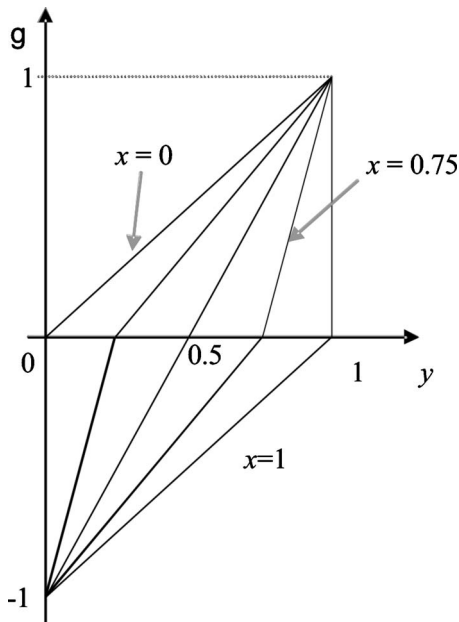


Fig. 2. With a fixed pre-test score, g changes linearly with respect to y and the slope is determined by the value of x .

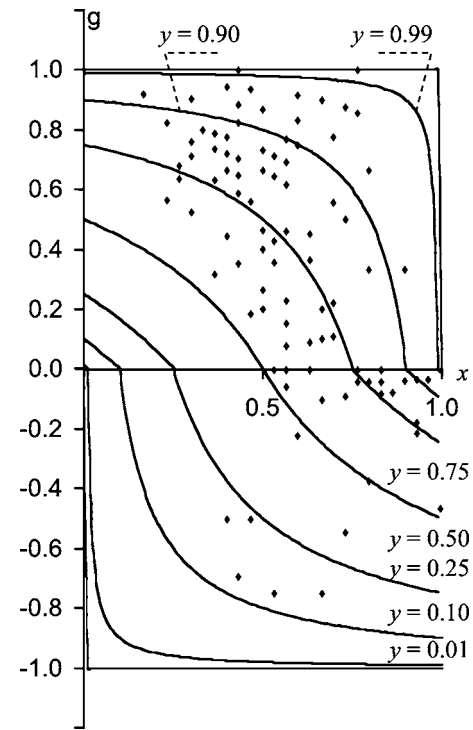


Fig. 3. The relation between g and x at constant values of post-test scores. The scatter plot represents data from a calculus-based introductory physics class at The Ohio State University ($N=105$).

scores by g . We represent the pre and post scores of the k th student in the class by x_k and y_k , and the student's individual gain by g_k . The average gain calculated using the average of

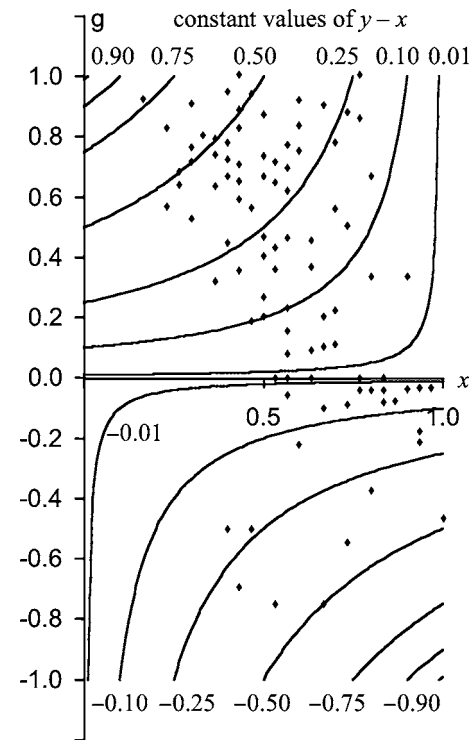


Fig. 4. The relation between g and x at constant values of absolute score changes. The scatter plot is from the same data shown in Fig. 3.

these individual gains is denoted by $\bar{\mathbf{g}}$. These two methods of calculating the average value of $\mathbf{g}$ are described by Eqs. (4) and (5).

$$\mathbf{g} = \frac{\bar{y} - \bar{x}}{1 - \bar{x}} = \frac{\frac{1}{N} \left(\sum_{k=1}^N y_k - \sum_{k=1}^N x_k \right)}{1 - \frac{1}{N} \sum_{k=1}^N x_k} = \frac{\sum_{k=1}^N (y_k - x_k)}{\sum_{k=1}^N (1 - x_k)}. \quad (4)$$

$$\bar{\mathbf{g}} = \frac{1}{N} \sum_{k=1}^N \mathbf{g}_k = \frac{1}{N} \sum_{k=1}^N \frac{(y_k - x_k)}{(1 - x_k)}. \quad (5)$$

Note that only the positive branch ($\bar{y} \geq \bar{x}$) of Eq. (2) is considered. That is, our discussion is valid only for the Hake definition of gain, Eq. (1), if each student scores higher on the post-test than he or she did on the pre-test.

It is easy to see from Eqs. (4) and (5) that when individual students have similar pre-test scores, that is, $x_k \approx \bar{x}$, we can write $\sum_{k=1}^N (1 - x_k) \approx N(1 - \bar{x})$, which leads to $\mathbf{g} \approx \bar{\mathbf{g}}$. This result is trivial, is unusual for real groups of students, and has no implications for reasoning in the reverse direction. That is, when the two methods give identical results, the cause of such an outcome is not necessarily that all students have similar pre-test scores.

Equations (4) and (5) show that $\bar{\mathbf{g}}$ and $\mathbf{g}$ are different in general, and the difference depends on the distribution of pre and post scores. It is interesting to see how this difference is related to certain features of the population and if such relations can be used in assessment.

IV. DIFFERENCES BETWEEN $\bar{\mathbf{g}}$ AND $\mathbf{g}$ FOR IDEALIZED SITUATIONS

To explore how differences between $\bar{\mathbf{g}}$ and $\mathbf{g}$ may arise from specific characteristics of the population, we consider a few idealized situations. The population is assumed to have a normal distribution and all gains are assumed to be positive. Although it is most unlikely for any of these idealized situations to occur in real classrooms, the results based on these conditions may help researchers interpret actual data.

Suppose that a class is given a pre-test and a post-test and that the data are “matched”—all students have taken both tests. Consider the case in which students with below average pre-test scores have below average post-test scores and students with above average pre-test scores have above average post-test scores; that is, all students remain on the same side of the class score distribution curve (with respect to the average class score) before and after instruction. We refer to this type of situation as a *normal shift*. Several cases may occur in a normal shift.

Constant shift. In this case it is assumed that all the students in the class have the same score change, which would equal the shift of the class average scores. The pre and post score distributions should also have identical shapes. Denote the change of the class average scores by β_0 ($\beta_0 = \bar{y} - \bar{x}$) and let $\bar{\mathbf{g}}_0$ represent the average of the individual student’s normalized gains. The class gain calculated by the class’ average scores is still represented by $\mathbf{g}$, which, by using the positive branch of Eq. (2), is $\mathbf{g} = \beta_0 / (1 - \bar{x})$.

Because the class score distribution is symmetric and each student has the same score change, we can identify students

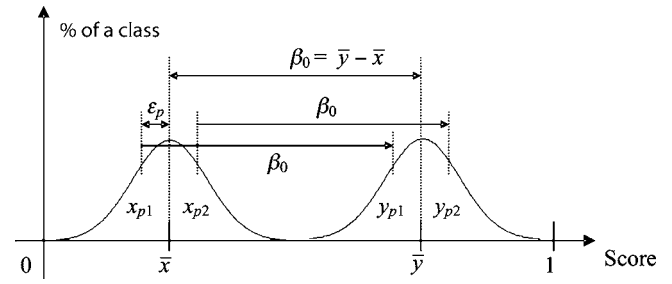


Fig. 5. A class going through a Normal Translation process—all students have the same absolute score change, which is denoted with β_0 .

from the class score distribution in symmetrical pairs with respect to the class average. That is, if one or several students has a score of $(\bar{x} - \epsilon_p)$, an equal number of students will have a score of $(\bar{x} + \epsilon_p)$. Here we let $\epsilon_p \equiv |x_p - \bar{x}|$, which represents the difference between the p th student pair’s score and the class average (see Fig. 5). Now consider the p th pair of students with pre-test scores of $(\bar{x} - \epsilon_p)$ and $(\bar{x} + \epsilon_p)$, respectively. The average gain of the two students, $\bar{\mathbf{g}}_{0,p}$, is given by

$$\begin{aligned} \bar{\mathbf{g}}_{0,p} &= \frac{1}{2} \left(\frac{\beta_0}{(1 - \bar{x} + \epsilon_p)} + \frac{\beta_0}{(1 - \bar{x} - \epsilon_p)} \right) \\ &= \frac{\beta_0}{(1 - \bar{x}) - \epsilon_p^2 / (1 - \bar{x})}, \end{aligned} \quad (6)$$

which gives

$$\bar{\mathbf{g}}_{0,p} = \frac{\beta_0}{(1 - \bar{x}) - \epsilon_p^2 / (1 - \bar{x})} > \frac{\beta_0}{(1 - \bar{x})} = \mathbf{g}. \quad (7)$$

Then the average gain of the class, $\bar{\mathbf{g}}_0$, is

$$\bar{\mathbf{g}}_0 = \frac{2}{N} \sum_{p=1}^{N/2} \frac{\beta_0}{(1 - \bar{x}) - \epsilon_p^2 / (1 - \bar{x})}. \quad (8)$$

Obviously, $\bar{\mathbf{g}}_0 > \mathbf{g}$. Equation (8) also indicates that if a constant shift occurs and many students have large values of ϵ_p (which implies a large standard deviation of the class score distribution σ_x) and/or if the class has a large $\bar{x}$, the difference between $\bar{\mathbf{g}}_0$ and $\mathbf{g}$ will increase. In this case if we calculate the correlation between the individual student’s normalized gains and the pre-test scores, a positive correlation is expected because the students with low pre-test scores will have low individual gains due to the scaling shown in Fig. 2. (Note that all students have the same absolute score changes.)

Expansion. In this case, we assume that students with above average pre-test scores have larger score changes than students with below average pre-test scores. Here, the post-test score distribution will be flatter than the pre-test score distribution, $\sigma_y > \sigma_x$. To simplify the calculation, we further assume that a student’s score change is linearly related to the difference between the student’s pre-test score and the class average of the pre-test score. Thus, the post-test score distribution is again symmetrical, and we can identify symmetrical student pairs (see Fig. 6 for definitions of the variables for the student pair).

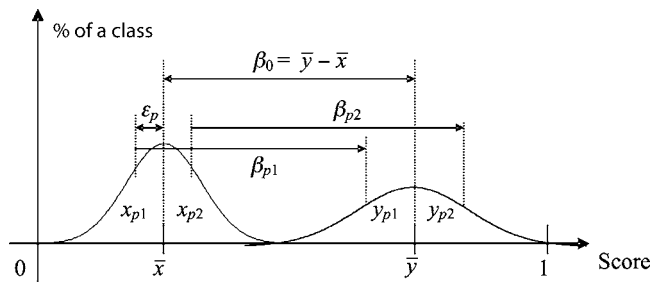


Fig. 6. A class going through a Normal Expansion process—students with low pre-test scores achieve smaller absolute score improvement than students with high pre-test scores.

Denote $\bar{\mathbf{g}}_E$ as the average of the individual student gains under expansion. For the p th pair of students, $\bar{\mathbf{g}}_{E,p}$ can be calculated by

$$\bar{\mathbf{g}}_{E,p} = \frac{1}{2} \left(\frac{\beta_{p1}}{(1 - \bar{x} + \epsilon_p)} + \frac{\beta_{p2}}{(1 - \bar{x} - \epsilon_p)} \right), \quad (9)$$

where

$$\beta_{p2} > \beta_{p1} \text{ and } \beta_0 = \beta_{p0} = \frac{\beta_{p1} + \beta_{p2}}{2}. \quad (10)$$

Denote Δ_p as the absolute value of the difference between β_0 and the p th pair of students' score changes. Then we can write

$$\beta_{p1} = \beta_0 - \Delta_p \text{ and } \beta_{p2} = \beta_0 + \Delta_p. \quad (11)$$

With the additional assumption that no student has a perfect pre- or post-test score, we can rewrite Eq. (9) as Eq. (12). The relation will hold as long as $(1 - \bar{x}) > \epsilon_p$.

$$\bar{\mathbf{g}}_{E,p} = \frac{\beta_0}{(1 - \bar{x}) - \epsilon_p^2/(1 - \bar{x})} + \frac{\epsilon_p \cdot \Delta_p}{(1 - \bar{x})^2 - \epsilon_p^2} > \bar{\mathbf{g}}_{0,p} > \mathbf{g}. \quad (12)$$

Therefore, we can obtain the relation

$$\bar{\mathbf{g}}_E > \bar{\mathbf{g}}_0 > \mathbf{g}. \quad (13)$$

If we calculate the correlation between the individual student's normalized gains and the pre-test scores for a class that undergoes expansion, a larger (compare to constant shift) positive correlation can be expected.

Contraction. This case assumes that the students with above average pre-test scores have smaller score changes than the students with below average pre-test scores, but all students still remain on the same side of the distribution curves of the pre- and post-tests. Denote the average gain by $\bar{\mathbf{g}}_C$. Then the post-test's score distribution will be sharper than that of the pre-test, $\sigma_y < \sigma_x$. To simplify the calculation, we further assume that a student's absolute score change is proportional to the difference between the student's pre-test score and the class average of the pre-test scores. Therefore, the post-test score distribution is symmetrical, and we can identify symmetric student pairs (see Fig. 7 for definitions of the variables for the student pair). Note that $\Delta_p = |\beta_0 - \beta_{p1}| = |\beta_0 - \beta_{p2}|$. In this case we have

$$\beta_{p2} < \beta_{p1}. \quad (14)$$

Similarly, we can write

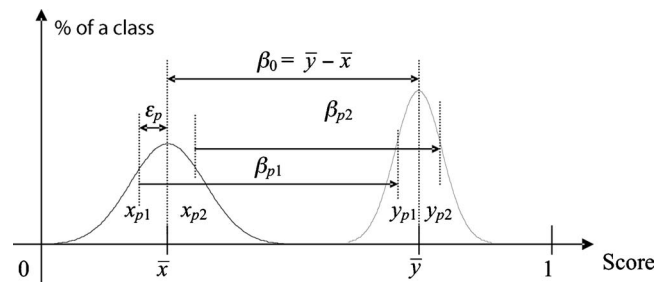


Fig. 7. A class going through a Normal Contraction process—students with low pre-test scores achieve larger absolute score improvement than students with high pre-test scores; however, all students stay at the same sides of the class score distributions for both pre- and post-tests.

$$\beta_{p1} = \beta_0 + \Delta_p \text{ and } \beta_{p2} = \beta_0 - \Delta_p. \quad (15)$$

Because we have assumed that all students stay at the same side of the distribution curves for both pre- and post-tests, we have $\beta_{p1} - \beta_{p2} < 2\epsilon_p$ or $\Delta_p < \epsilon_p$. For the p th pair of students, $\bar{\mathbf{g}}_{C,p}$ can be calculated as

$$\bar{\mathbf{g}}_{C,k} = \frac{\beta_0}{(1 - \bar{x}) - \epsilon_p^2/(1 - \bar{x})} - \frac{\epsilon_p \Delta_p}{(1 - \bar{x})^2 - \epsilon_p^2}. \quad (16)$$

By using similar arguments as for the case of expansion, we have

$$\bar{\mathbf{g}}_E > \bar{\mathbf{g}}_0 > \bar{\mathbf{g}}_C. \quad (17)$$

However, a general relation between $\bar{\mathbf{g}}_C$ and $\mathbf{g}$ cannot be determined. In this case the correlation between the individual student's normalized gains and the pre-test scores will be smaller than that of constant shift and can be negative.

Abnormal shift. An abnormal shift describes the case in which students with below average pre-test scores have above average post-test scores and students with above average pre-test scores have below average post-test scores; that is, students exchange sides on the distribution curve after instruction. In this case students with higher pre-test scores have much smaller score changes than students with lower pre-test scores. Figure 8 shows an example where the pre and post distributions have the same shape and students exist in symmetrical pairs.

Denote $\bar{\mathbf{g}}_A$ as the average of individual student gains assuming an abnormal shift. For the p th pair of students we denote Δ'_p as the absolute value of the difference between β_0 and the pair of students' score changes ($\Delta'_p \equiv \beta_1 - \beta_0 = \beta_0 - \beta_2 \geq 0$). We then have

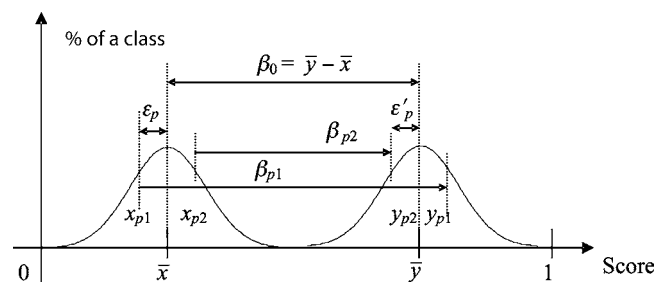


Fig. 8. A class going through an Abnormal Shift process—students with low pre-test scores achieve larger absolute score changes than students with high pre-test scores, and students switch sides on the class score distributions of pre- and post-tests.

Table I. Results of a computer simulation for a class going through four different changes in pre- and post-tests. The data assumes a normal distribution with $\bar{x}=0.39$, $\sigma_x=0.145$, and $\bar{y}=0.59$.

Processes	σ_y	σ_y/σ_x	$\bar{g}$	g	$\bar{g}-g$	$(\bar{g}-g)/g$
Constant Shift	0.145	1.00	0.355	0.330	0.025	7.6%
Expansion	0.264	1.80	0.388	0.329	0.059	17.9%
	0.219	1.50	0.391	0.329	0.062	18.8%
	0.146	1.00	0.354	0.329	0.025	7.6%
Contraction	0.131	0.90	0.347	0.330	0.017	5.2%
	0.098	0.67	0.330	0.330	0.000	0.0%
	0.073	0.50	0.318	0.330	-0.012	-3.6%
	0.044	0.30	0.306	0.329	-0.024	-7.0%
Abnormal Shift	0.218	1.50	0.256	0.330	-0.074	-22.4%
	0.146	1.00	0.269	0.330	-0.061	-18.5%
	0.073	0.50	0.281	0.330	-0.049	-14.9%
	0.044	0.30	0.286	0.329	-0.044	-13.1%

$$\begin{aligned}\bar{g}_{A,p} &= \frac{1}{2} \left(\frac{\beta_0 + \Delta'_p}{(1 - \bar{x} + \varepsilon_p)} + \frac{\beta_0 - \Delta'_p}{(1 - \bar{x} - \varepsilon_p)} \right) \\ &= \frac{\beta_0(1 - \bar{x}) - \varepsilon_p \cdot \Delta'_p}{(1 - \bar{x})^2 - \varepsilon_p^2}.\end{aligned}\quad (18)$$

For the case of contraction with the same pre-test distribution and same post-test average score (see Fig. 7) it is easy to see that $\Delta'_p > \Delta_p$. Note that we can rewrite Eq. (16) as

$$\bar{g}_{C,p} = \frac{\beta_0(1 - \bar{x}) - \varepsilon_p \cdot \Delta_p}{(1 - \bar{x})^2 - \varepsilon_p^2}.\quad (19)$$

By comparing Eq. (19) with Eq. (18), we conclude that

$$\bar{g}_C > \bar{g}_A.\quad (20)$$

In Fig. 8 we define a new variable, $\varepsilon'_p \equiv |y_p - \bar{y}|$. Therefore, we can write

$$\beta_0 = \beta_2 + \varepsilon_p + \varepsilon'_p,\quad (21)$$

which gives

$$\Delta'_p = \beta_0 - \beta_2 = \varepsilon_p + \varepsilon'_p > \varepsilon_p.\quad (22)$$

To compare $\bar{g}_{A,p}$ with g , let's assume $\bar{g}_{A,p}$ is larger than g ,

$$\bar{g}_{A,p} = \frac{\beta_0(1 - \bar{x}) - \varepsilon_p \Delta'_p}{(1 - \bar{x})^2 - \varepsilon_p^2} > \frac{\beta_0}{1 - \bar{x}} = g,\quad (23)$$

which gives $\beta_0(1 - \bar{x})^2 - \varepsilon_p \Delta'_p(1 - \bar{x}) > \beta_0(1 - \bar{x})^2 - \beta_0 \varepsilon_p^2$. Therefore $\beta_0 > \Delta'_p / \varepsilon_p (1 - \bar{x}) > (1 - \bar{x})$. Because $\beta_0 = \bar{y} - \bar{x} < 1 - \bar{x}$ the assumption in Eq. (23) cannot hold. Therefore, for an abnormal shift we have

$$g > \bar{g}_A.\quad (24)$$

The correlation between the individual student's normalized gains and the pre-test scores in this case will most likely be negative.

In summary, we conclude that

$$\bar{g}_E > \bar{g}_0 > [\bar{g}_C, g] > \bar{g}_A.\quad (25)$$

Note that g is the normalized gain calculated with the class' average scores, whereas, $\bar{g}_X$ is the average of individual student's normalized gains under different population assumptions.

If we use the data shown in Fig. 3, we find $\bar{g}=0.364$ and $g=0.435$. This result suggests some abnormal shifts, as can be seen from the scatter plot in Fig. 4; quite a few students with medium to high pre-test scores have zero or below zero score changes, smaller than that of many students with lower pre-test scores. Note that there are many students having negative gains, which is not included in our analysis; however, it can be shown that the relation in Eq. (25) is also valid in this condition.

Simulation results. To see how the derived relations appear numerically, we have done some computer simulations. Table I shows the simulation results for the four cases we have discussed. The class size is chosen to be 100. The pre-test score distribution is randomly generated according to a Gaussian distribution with $\bar{x}=0.39$, $\bar{y}-\bar{x}=0.2$, and $\sigma_x=0.15$. If an individual student's pre-test score is smaller than 0 or greater than 1, the data is rejected and a new score is generated. The post-test scores are generated based on the pre-test scores with the different models listed in Table I. For example, in the case of constant shift a constant value of the score change is added to the pre-test score to obtain the matched post-test score. The post-test scores are also truncated to be between 0 and 1. The conditioning of the data (e.g., the truncations) will bias the simulation results. However, because the standard deviation of the pre-test score is small, the probability for problematic data points is $\approx 2\%$. Each record in Table I is the average result of 1000 simulated classes. The simulation calculates the gains for both positive and negative values. The purpose of the simulation is to provide a computational validation of the theoretical analysis and give a rough estimate of what range of magnitude researchers can expect when dealing with data. The results in Table I show that the computational results are consistent with the analytical predictions, and that the differences between the two methods of calculating the normalized gains are often at 10% level.

V. DISCUSSION

Our main purpose was to understand the difference between the two ways of calculating average gain. We made several assumptions including a normal distribution of scores and no students scoring lower on the post-test than on the pre-test. In addition, we discussed only highly idealized changes that might result from instruction. Nonetheless, this discussion provided some useful information. For example, by comparing $\bar{g}$ and g , we may be able to make inferences about how a group of students has changed: If $\bar{g}$ is greater than g , we can infer that students with low pre-test scores tend to have either smaller or similar score improvement than students with high pre-test scores. On the other hand, if $\bar{g}$ is smaller than g , students with low pre-test scores tend to have larger score improvements than students with high pre-test scores.

Several issues related to $\bar{g}$, g , and error analysis are important to this discussion. These issues are discussed in Ref. 4. In addition, error propagation from the measured scores to the calculated normalized gains and the many types of uncertainties embedded in the pre-post measurements are not discussed here. A detailed discussion of these issues will be presented in a separate paper.

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