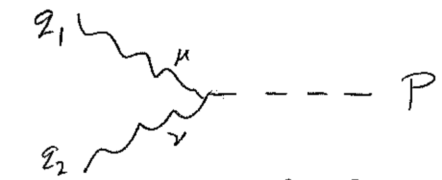
Higgs Production in Electron Collisions

The Higgs boson couples to photons through an effective interaction term $\mathcal{L}_{\text{int}} = g_{H\gamma\gamma} H F_{\mu\nu} F^{\mu\nu}$. The Feynman rule for the $H\gamma\gamma$ vertex for photon lines with incoming momenta q_1 and q_2 and with Lorentz indices μ and ν is $2ig_{H\gamma\gamma}(q_1 . q_2 g^{\mu\nu} - q_2^\mu q_1^\nu)$.

A. The Higgs can be produced by the reaction $\gamma(q_1)\gamma(q_2) \to H(P)$. Draw the Feynman diagram for this reaction, labelling the momenta of the lines and the Lorentz indices for the vertex.



B. Write down the matrix element for the reaction $\gamma(q_1)\gamma(q_2) \to H$ for photons with polarization vectors ε_1 and ε_2.

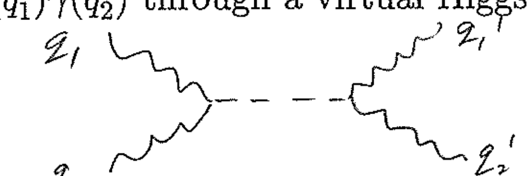
$$i\mathcal{M} = 2ig_{H\gamma\gamma}\left(q_1 q_2 g^{\mu\nu} - q_2^\mu q_1^\nu\right) \varepsilon_{1\mu} \varepsilon_{2\nu}$$

The cross section for Higgs production from the reaction $\gamma(q_1)\gamma(q_2) \to H$ is

$$\sigma[\gamma(q_1)\gamma(q_2) \to H] = \frac{3 g_{H\gamma\gamma}^2 M_H^3 \Gamma_H}{(s - M_H^2)^2 + M_H^2 \Gamma_H^2},$$

where $s = (q_1 + q_2)^2$ is the square of the center-of-mass energy.

C. Draw the Feynman diagram for the elastic scattering reaction $\gamma(q_1)\gamma(q_2) \to \gamma(q_1')\gamma(q_2')$ through a virtual Higgs in the s channel.



D. The imaginary part of the matrix element for $\gamma(q_1)\gamma(q_2) \to \gamma(q_1')\gamma(q_2')$ comes from the imaginary part of the Higgs propagator factor $1/(s - M_H^2 + iM_H\Gamma_H)$. Show that the imaginary part gives the dependence on s of the Breit-Wigner resonance factor in $\sigma[\gamma(q_1)\gamma(q_2) \to H]$.

$$\operatorname{Im} \frac{1}{s - M_H^2 + i M_H \Gamma_H} = \operatorname{Im} \frac{-i M_H \Gamma_H}{(s - M_H^2)^2 + M_H^2 \Gamma_H^2} = -\frac{M_H \Gamma_H}{(s - M_H^2)^2 + M_H^2 \Gamma_H^2}$$

The parton distribution for photons in an electron that undergoes a collision involving a large momentum scale Q is

$$f_{\gamma/e}(x, Q) = \frac{\alpha}{2\pi} \log \frac{Q^2}{m_e^2} \frac{1 + (1-x)^2}{x}.$$

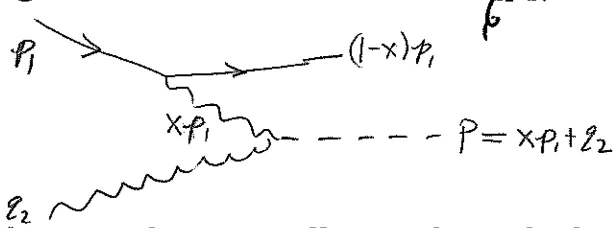
The Higgs can be produced in an electron-photon collision through the emission of a collinear virtual photon from the electron. The cross section is

$$\sigma[e^-(p_1)\gamma(q_2) \to e^- H] = \int_0^1 dx\, f_{\gamma/e}(x, M_H)\sigma[\gamma(xp_1)\gamma(q_2) \to H].$$

E. Express the parton center-of-mass energy $\hat{s}$ for the $\gamma\gamma$ collision in terms of the center-of-mass energy s for the $e^-\gamma$ collision.

$$\hat{s} = (xp_1 + q_2)^2 = 2x p_1 \cdot q_2 = x(p_1+q_2)^2 = x s$$

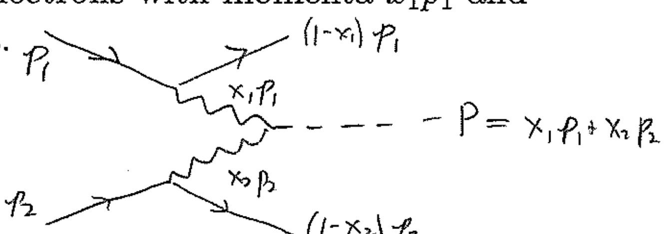
F. Draw the Feynman diagram for the reaction $e^-(p_1)\gamma(q_2) \to e^- H$ with a virtual photon collinear to the incoming electron with momentum $x_1 p_1$, labelling the momenta of all the lines.



G. The Higgs can be produced in an electron-electron collision through the emission of collinear virtual photons from both electrons. Express the cross section for $e^-(p_1)e^-(p_2) \to e^-e^-H$ in terms of a double integral over momentum fractions x_1 and x_2.

$$\sigma[e^-(p_1)e^-(p_2) \to e^-e^-H]$$
$$= \int_0^1 dx_1\, f_{\gamma/e}(x_1, M_H) \int_0^1 dx_2\, f_{\gamma/e}(x_2, M_H)\, \sigma[\gamma(x_1p_1)\gamma(x_2p_2) \to H]$$

H. Draw the Feynman diagram for the reaction $e^-(p_1)e^-(p_2) \to e^-e^-H$ with virtual photons collinear to the incoming electrons with momenta x_1p_1 and x_2p_2, labelling the momenta of all the lines.



I. Express the parton center-of-mass energy $\hat{s}$ for the $\gamma\gamma$ collision in terms of the center-of-mass energy s for the e^-e^- collision.

$$\hat{s} = (x_1p_1 + x_2p_2)^2 = 2x_1x_2\, p_1 \cdot p_2 = x_1x_2(p_1+p_2)^2 = x_1x_2\, s$$