Z^0 Production in Photon Collisions

The cross section for Z^0 production from the reaction $e^+(p_1)e^-(p_2) \to Z^0$ is

$$\sigma[e^+(p_1)e^-(p_2) \to Z^0] = \frac{12\pi B_{ee}\Gamma_Z^2\, s}{M_Z^2\left[(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2\right]},$$

where $s = (p_1+p_2)^2$ is the square of the center-of-mass energy and B_{ee} is the branching fraction of Z^0 into e^+e^-.

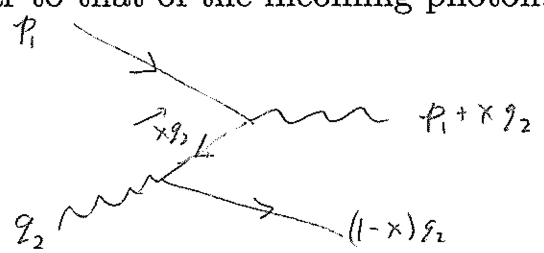
A. The parton distribution for electrons in a photon that undergoes a collision involving a large momentum scale $Q \gg m_e$ is

$$f_{e^-/\gamma}(x,Q) = \frac{\alpha}{2\pi}\log\frac{Q}{m_e}\left[x^2+(1-x)^2\right].$$

Use the charge conjugation symmetry of QED to deduce the parton distribution $f_{e^+/\gamma}(x,Q)$ for positrons in a photon.

$$f_{e^+/\gamma}(x,Q) = f_{e^-/\gamma}(1-x,Q) = \frac{\alpha}{2\pi}\log\frac{Q}{m_e}\left[x^2+(1-x)^2\right]$$

B. The Z^0 can be produced in a positron-photon collision through the splitting of the photon into a collinear e^+e^- pair. Draw the Feynman diagram for the reaction $e^+(p_1)\gamma(q_2) \to e^+Z^0(P)$ with a virtual electron with momentum xq_2 collinear to that of the incoming photon. Label the momenta of all the lines.



C. Express the cross section for $e^+(p_1)\gamma(q_2) \to e^+Z^0$ as an integral over the longitudinal momentum fraction x of electrons in the photon.

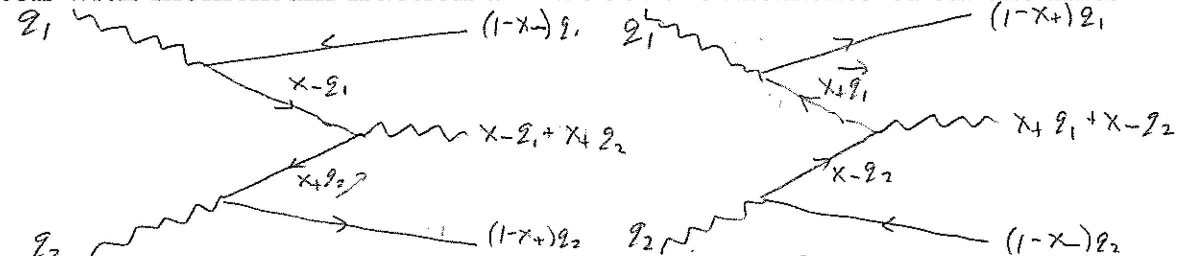
$$\sigma[e^+(p_1)\gamma(q_2) \longrightarrow e^+ Z^0]$$

$$= \int_0^1 dx\, f_{e^-/\gamma}(x, M_Z)\, \sigma[e^+(p_1)\, e^-(xq_2) \longrightarrow Z^0]$$

D. Express the parton center-of-mass energy $\hat{s}$ for the e^+e^- collision in terms of the center-of-mass energy s for the $e^+\gamma$ collision. Express $\hat{s}$ in terms of M_Z.

$$\hat{s} = (p_1 + xq_2)^2 = 2x\, p_1\cdot q_2 = x(p_1+q_2)^2 = x s$$

E. The Z^0 can be produced in a photon-photon collision through the splitting of both photons into collinear e^+e^- pairs. Draw the two Feynman diagrams for the reaction $\gamma(q_1)\gamma(q_2) \to e^+e^-Z^0(P)$ in which there is a virtual e^+ collinear to one photon with momentum fraction x_+ and a virtual e^- collinear to the other photon with momentum fraction x_-. Label the momenta of all the lines.



F. Express the cross section for $\gamma(q_1)\gamma(q_2) \to e^+e^-Z^0$ as a double integral over momentum fractions x_+ and x_-. (There are two terms in the cross section.)

$$\sigma[\gamma(q_1)\gamma(q_2) \to e^+e^- Z_0]$$
$$= \int_0^1 dx_+ f_{e^+/\gamma}(x_+,M_Z)\int_0^1 dx_- f_{e^-/\gamma}(x_-,M_Z)\Big(\sigma[e^+(x_+q_1)e^-(x_-q_2)\to Z^0]$$
$$+\sigma[e^+(x_+q_2)e^-(x_-q_1)\to Z^0]\Big)$$

G. Express the parton center-of-mass energy $\hat{s}$ for the e^+e^- collision in terms of the center-of-mass energy s for the $\gamma\gamma$ collision. Express $\hat{s}$ in terms of M_Z.

$$\hat{s} = (x_\pm q_1 + x_\mp q_2)^2 = 2x_\pm x_\mp q_1\cdot q_2 = x_+x_-(q_1+q_2)^2 = x_+x_-s$$

H. Calculate the integral $\int_0^1 dx\, f_{e^-/\gamma}(x,Q)$, which gives the mean number of collinear electrons in a photon that undergoes a collision with a large momentum scale Q.

$$\int_0^1 dx\, f_{e^-/\gamma}(x,Q) = \frac{\alpha}{2\pi}\log\frac{Q}{m_e}\int_0^1 dx\,[x^2+(1-x)^2] = \frac{2}{3}\cdot\frac{\alpha}{2\pi}\log\frac{Q}{m_e}$$

I. The parton distributions for a photon satisfy the momentum sum rule

$$\int_0^1 dx\, x\left[f_{\gamma/\gamma}(x,Q) + f_{e^-/\gamma}(x,Q) + f_{e^+/\gamma}(x,Q)\right] = 1.$$

The parton distribution for photons in the photon at the momentum scale Q has the simple form $f_{\gamma/\gamma}(x,Q) = F(Q)\,\delta(1-x)$ up to corrections of order α^2. Use the momentum sum rule to deduce $f_{\gamma/\gamma}(x,Q)$ through order α.

$$F(Q) + \int_0^1 dx\, x\,[f_{e^-/\gamma}(x,Q) + f_{e^+/\gamma}(x,Q)] = 1$$
$$F(Q) = 1 - 2\,\frac{\alpha}{2\pi}\log\frac{Q}{m_e}\int_0^1 dx\, x\,[x^2+(1-x)^2]$$
$$= 1 - \frac{1}{3}\cdot\frac{\alpha}{\pi}\log\frac{Q}{m_e} \implies f_{\gamma/\gamma}(x,Q) = \left[1 - \frac{\alpha}{3\pi}\log\frac{Q}{m_e}\right]\delta(1-x)$$