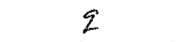
Renormalized Perturbation Theory for Yukawa Model

The Lagrangian for bare perturbation theory in the renormalizable Yukawa model with a Dirac spinor field ψ and a real scalar field ϕ is $\mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$, where

$$\begin{aligned}
\mathcal{L}_{\text{free}} &= i\bar{\psi}\gamma^\mu\partial_\mu\psi - M_0\,\bar{\psi}\psi + \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{2}m_0^2\,\phi^2, \\
\mathcal{L}_{\text{int}} &= -g_0\,\phi\bar{\psi}\gamma_5\psi - \tfrac{1}{24}\lambda_0\,\phi^4.
\end{aligned}$$

A. Write down the Feynman rules for the propagators and vertices.

 $= \dfrac{i}{\not{p} - M_0 + i\epsilon}$  $= -ig_0\gamma_5$  $= -i\lambda_0$

 $= \dfrac{i}{q^2 - M_0^2 + i\epsilon}$

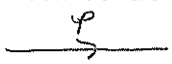
The parameters can be separated into finite parts and counterterms:

$$g_0 = Z_g g = g + \delta g,\ \lambda_0 = Z_\lambda\lambda = \lambda + \delta\lambda,\ M_0 = Z_M M = M + \delta M,\ m_0^2 = Z_{m^2}m^2 = m^2 + \delta m^2.$$

The Lagrangian for renormalized perturbation theory for T-matrix elements is

$$\begin{aligned}
\mathcal{L}_{\text{free}} &= i\bar{\psi}\gamma^\mu\partial_\mu\psi - M\,\bar{\psi}\psi + \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{2}m^2\,\phi^2, \\
\mathcal{L}_{\text{int}} &= -g\,\phi\bar{\psi}\gamma_5\psi - \tfrac{1}{24}\lambda\,\phi^4 - \delta M\,\bar{\psi}\psi - \tfrac{1}{2}\delta m^2\,\phi^2 - \delta g\,\phi\bar{\psi}\gamma_5\psi - \tfrac{1}{24}\delta\lambda\,\phi^4.
\end{aligned}$$

B. Write down the Feynman rules for the propagators and vertices.

 $= \dfrac{i}{\not{p} - M + i\epsilon}$  $= -i\delta M$  $= -ig\gamma_5$  $= -i\delta g\gamma_5$

 $= \dfrac{i}{q^2 - m^2 + i\epsilon}$  $= -i\delta m^2$  $= -i\lambda$  $= -i\delta\lambda$

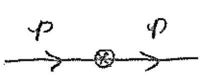
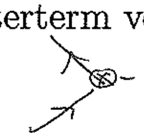
The fields can be rescaled by wavefunction renormalization constants:

$$\psi(x) \longrightarrow \sqrt{Z_\psi}\,\psi(x), \quad \psi(x) \longrightarrow \sqrt{Z_\phi}\,\phi(x).$$

The Lagrangian for renormalized perturbation theory for Green functions is

$$\begin{aligned}
\mathcal{L}_{\text{free}} &= i\bar{\psi}\gamma^\mu\partial_\mu\psi - M\,\bar{\psi}\psi + \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{2}m^2\,\phi^2, \\
\mathcal{L}_{\text{int}} &= -g\,\phi\bar{\psi}\gamma_5\psi - \tfrac{1}{24}\lambda\,\phi^4 - (Z_\psi\sqrt{Z_\phi}Z_g - 1)g\,\phi\bar{\psi}\gamma_5\psi - \tfrac{1}{24}(Z_\phi^2 Z_\lambda - 1)\lambda\,\phi^4 \\
&\quad + (Z_\psi - 1)(i\bar{\psi}\gamma^\mu\partial_\mu\psi - M\bar{\psi}\psi) - Z_\psi(Z_M - 1)M\,\bar{\psi}\psi \\
&\quad + (Z_\phi - 1)(\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{2}m^2\phi^2) - \tfrac{1}{2}Z_\phi(Z_{m^2} - 1)m^2\,\phi^2.
\end{aligned}$$

C. Write down the Feynman rules for the four counterterm vertices.

 $= i\left[(Z_\psi - 1)(\not{p} - M) - Z_\psi(Z_M - 1)M\right]$  $= -i\left(Z_\psi\sqrt{Z_\phi}Z_g - 1\right)g\gamma_5$

 $= i\left[(Z_\phi - 1)(q^2 - m^2) - Z_\phi(Z_{m^2} - 1)m^2\right]$  $= -i\left(Z_\phi^2 Z_\psi - 1\right)\lambda$

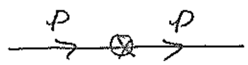
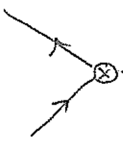
The renormalization constants can be expressed as

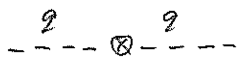
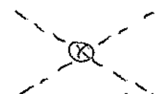
$$Z_g = 1+\delta_g,\ Z_\lambda = 1+\delta_\lambda,\ Z_M = 1+\delta_M,\ Z_{m^2} = 1+\delta_{m^2},\ Z_\psi = 1+\delta_\psi,\ Z_\phi = 1+\delta_\phi.$$

Renormalized perturbation theory at next-to-leading order (NLO) in g^2 and λ requires only the expansion of $\mathcal{L}_{\rm int}$ to 1st order in the counterterms:

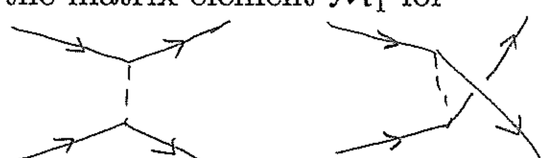
$$\begin{aligned}\mathcal{L}_{\rm int} \;=\; & -g\,\phi\bar\psi\gamma_5\psi - \tfrac{1}{24}\lambda\,\phi^4 - (\delta_g+\delta_\psi+\tfrac12\delta_\phi)g\,\phi\bar\psi\gamma_5\psi - \tfrac{1}{24}(\delta_\lambda+2\delta_\phi)\lambda\,\phi^4 \\ & +\delta_\psi(i\bar\psi\gamma^\mu\partial_\mu\psi - M\bar\psi\psi) - \delta_M\, M\,\bar\psi\psi \\ & +\delta_\phi(\tfrac12\partial_\mu\phi\partial^\mu\phi - \tfrac12 m^2\phi^2) - \tfrac12\delta_{m^2}\, m^2\,\phi^2 + \ldots.\end{aligned}$$

E. Write down the Feynman rules for the four counterterm vertices required at NLO.

 $= i\left[\delta_\psi(\not p - M) - \delta_M M\right]$  $= -i(\delta_g+\delta_\psi+\frac12\delta_\phi)g\gamma_5$

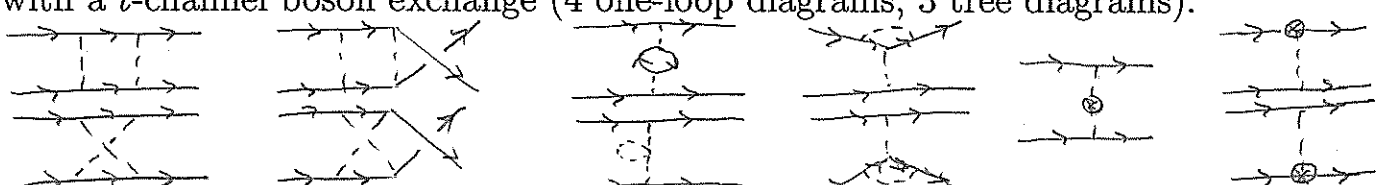
 $= i\left[\delta_\phi(q^2-m^2) - \delta_{m^2}m^2\right]$  $= -i(\delta_\lambda+2\delta_\phi)\lambda\phi^4$

F. Draw the two Feynman diagrams for the matrix element $\mathcal{M}_1$ for fermion-fermion scattering at order g^2.



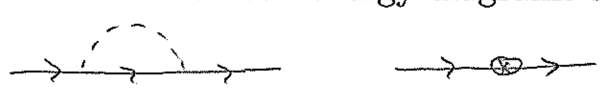
There are 18 Feynman diagrams for the matrix element $\mathcal{M}_2$ for fermion-fermion scattering at NLO in g^2 and λ.

G. Draw the 4 NLO two-boson exchange diagrams and the 7 NLO diagrams with a t-channel boson exchange (4 one-loop diagrams, 3 tree diagrams).



The matrix element $\mathcal{M}$ for fermion-fermion scattering at NLO is obtained by multiplying the sum $\mathcal{M}_1+\mathcal{M}_2$ of the 18 Feynman diagrams by $\left(\sqrt{Z_f}\right)^4$, and then expanding to 2nd order in g^2 and λ. The fermion residue factor Z_f can be determined from fermion self-energy diagrams.

H. Draw the two fermion self-energy diagrams at order g^2.



I. Draw the 9 fermion self-energy diagrams at NLO (4 two-loop diagrams, 4 one-loop diagrams, 1 tree diagram).