Schwartz Chapter 14: Problems 5, 6

Problem 14.5 (hints)

(a) A more explicit form for the Schwinger-Dyson equation for the photon field in scalar QED is

$$\Box_x^{\mu\nu} \left\langle A_\nu(x) A_\alpha(y) \phi^\dagger(z) \phi(w) \right\rangle = \ ?,$$

where $\Box^{\mu\nu} = -g^{\mu\nu}\partial^2 + \partial^\mu\partial^\nu - (1/\xi)\partial^\mu\partial^\nu$. It can be derived from the invariance of the functional integral under shifts of the field $A_\mu(x)$. Start with the numerator of the path integral formula for $\left\langle A_\alpha(y)\phi^\dagger(z)\phi(w)\right\rangle$, and use its invariance under the change of variables $A_\mu(x) \to A_\mu(x) + \varepsilon_\mu(x)$, where $\varepsilon_\mu(x)$ is a four-vector function.
(1) Write down the equation that expresses that invariance.
(2) Expand both sides of the equation to first order in ε_μ.
(3) Express the equation in the form

$$0 = \int d^4x \, \varepsilon_\mu(x)(\text{function of } x).$$

(4) Deduce from this that the function of x must vanish.
(5) Divide each term by the denominator of the path integral formula.

(b) A Schwinger-Dyson equation for current conservation in scalar QED is

$$(\partial/\partial x^\mu) \left\langle j^\mu(x) A_\alpha(y) \phi^\dagger(z) \phi(w) \right\rangle = \ ?,$$

where $j^\mu = i(\phi^* \partial^\mu \phi - \partial^\mu \phi)$. It can be derived from the invariance of the functional integral under space-time dependent phase transformation of the field $\phi(x)$. Start with the numerator of the path integral formula for $\left\langle A_\alpha(y)\phi^\dagger(z)\phi(w)\right\rangle$, and use its invariance under the change of variables $\phi(x) \to \exp(i\varepsilon(x))\phi(x)$.