

Schwartz Chapter 21: Problems 1*, 3

Problem 1

(A) “Write down all the superficially divergent amplitudes in QED at 2-loops”.

Omit the vacuum-energy Green function which has no external legs,
Omit the 1-photon and 3-photon Green functions, which are 0 by Furry’s
Theorem.

For each of the four remaining superficially divergent 1PI Green functions,
draw the one-loop diagram and all the two-loop diagrams. (For the 1PI
4-photon Green function, draw the diagrams up to permutations of the
external legs.)

(B) “Prove that all of the UV divergences can be removed by the same four
counterterms required to remove the one-loop divergences.”

For each of the four 1PI Green functions,

(1) Draw all the one-loop diagrams with insertions of one-loop counterterms.
(For the 1PI 4-photon Green function, draw the diagrams up to permutations
of the external legs.)

(2) Assume the one-loop counterterm diagrams cancel all UV divergences from
subdiagrams of the two-loop diagrams. The expansion of the Green function in
powers of an external momentum then produces decreasingly divergent
integrals. Show that the remaining UV divergences can be cancelled by tree
diagrams with two-loop counterterms. (This is just a reiteration of the
argument at one loop.)