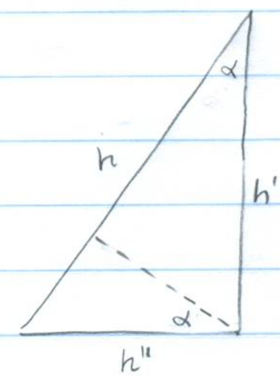
Goldenfeld, Ch. 1

Exercise 1-2

(a) Let the right-angled triangle have area A, hypotenuse h, and smallest angle α. Dimensional analysis requires



$$A = h^2 f(\alpha)$$

for some function f. A perpendicular to the hypotenuse that passes through the opposite vertex divides the triangle into two smaller right triangles, both with smallest angle α. Their areas must satisfy

$$A' = h'^2 f(\alpha)$$

$$A'' = h''^2 f(\alpha)$$

Their areas add up to that of the original triangle:

$$A = A' + A''$$

Cancelling the common factors of $f(\alpha)$, we get

$$h^2 = h'^2 + h''^2$$

Goldenfeld, Chapter 1

Exercise 1-2

The radius R of the shock wave and the elapsed time T have dimension

$$[R] = L \qquad [T] = T$$

The energy E released in the blast and the density ρ of the air have dimensions

$$[E] = ML^2/T^2$$

$$[\rho] = M/L^3$$

If E and ρ are the only variable that determine R(T), dimensional analysis implies

$$R(T) = \text{constant} \times \left(\frac{ET^2}{\rho}\right)^{1/5}$$

A fit to the data in Table 1.1 using the Mathematica command FindFit gives

$$R(T) = (36.4\,m)\left(\frac{T}{10^{-3}s}\right)^{2/5}$$

Assuming that the constant coefficient in the scaling relation for R is of order 1, we get an estimate for the energy E

$$\left(\frac{E}{\rho}\right)^{1/5} = (36.4\,m)\left(\frac{1}{10^{-3}s}\right)^{2/5}$$

$$E = (36.4\,m)^5 \frac{1}{(10^{-3}s)^2}\,\rho^5$$

The density ρ of air at atmosphere pressure and room temperature is approximately $1.28\ kg/m^3$. The resulting energy is

$$E = 8.2\times10^{13}\ kg\cdot m^2/s^2$$

One ton of TNT is equal to 4.2×10^9 J. The energy is therefore about 19.5 kilotons.

Goldenfeld

Chapter 1.1

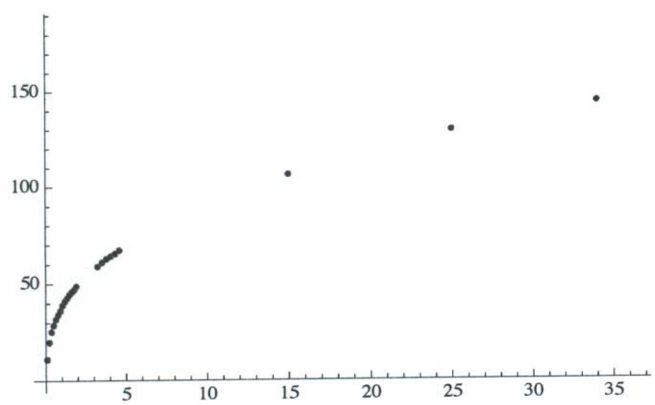
Exercise 1-2

data from Table 1.1 on radius R (in meters) of blast wave after time T (in microseconds)

In[12]:=
```
blastwave = List[
    {0.10, 11.1},
    {0.24, 19.9},
    {0.38, 25.4},
    {0.52, 28.8},
    {0.66, 31.9},
    {0.80, 34.2},
    {0.94, 36.3},
    {1.08, 38.9},
    {1.22, 41.0},
    {1.36, 42.8},
    {1.50, 44.4},
    {1.65, 46.0},
    {1.79, 46.9},
    {1.93, 48.7},
    {3.26, 59.0},
    {3.53, 61.1},
    {3.80, 62.9},
    {4.07, 64.3},
    {4.34, 65.6},
    {4.61, 67.3},
    {15.0, 106.5},
    {25.0, 130.0},
    {34.0, 145.0},
    {53.0, 175.0},
    {62.0, 185.0}];
```

In[2]:=
```
ListPlot[blastwave]
```

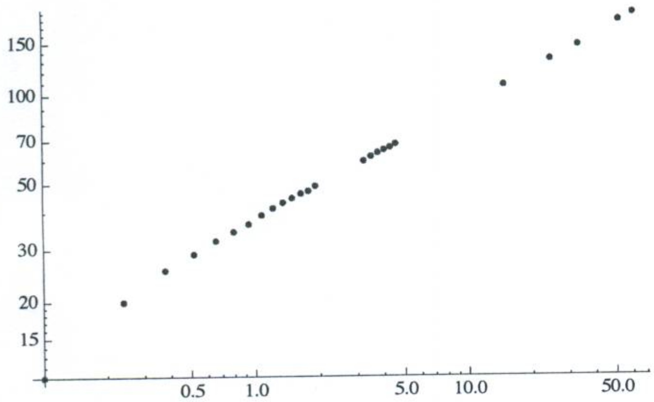
Out[2]=



F2a

In[3]:=
```
data = ListLogLogPlot[blastwave]
```

Out[3]=



make a log-log fit of the blast wave data

In[4]:=
```
length = Length[blastwave]
blastwaveLogLog = Table[{Log[blastwave[[i, 1]]], Log[blastwave[[i, 2]]]}, {i, 1, length}];
```

Out[4]= 25

In[6]:=
```
fit = FindFit[blastwaveLogLog, a + (2 / 5) x, {a}, x]
```

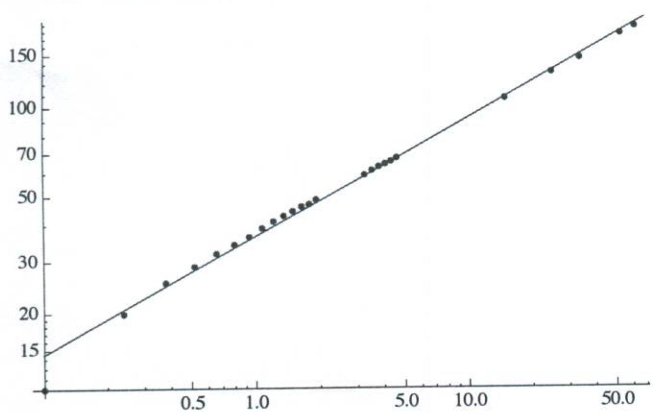
Out[6]= $\{a \to 3.59557\}$

In[7]:=
```
afit = a /. fit
```

Out[7]= 3.59557

In[8]:=
```
curve = LogLogPlot[Exp[afit] t^(2 / 5), {t, 0.1, 70}];
Show[data, curve]
```

Out[9]=



estimate energy released in blast in Joules and in tons of TNT

In[10]:=
```
(36.4 m)^5 / (10^(-3) s)^2 * (1.28 kg / m^3)
```

Out[10]= $\frac{8.17931\times10^{13}\ \mathrm{kg}\ m^2}{s^2}$

In[11]:=
```
% / (4.2 * 10^9 kg * m^2 / s^2)
```

Out[11]= 19474.6