Baierlein, Thermal Physics

Chapter 9

20. BEC with sodium. The second experiment to produce BEC in a dilute gas used sodium atoms. The number density was $N/V = 10^{20}$ atoms/m^3. The mass of a sodium atom is $m = 3.82 \times 10^{-26}$ kg. As with the rubidium experiment, only one state of intrinsic angular momentum was populated.

(a) If the trap that confined the atoms were adequately approximated by a box with rigid walls, at what temperature would you expect BEC to set in (as one lowered the temperature)?
(b) How low a temperature would be required for 90 percent of the atoms to be in the single-particle ground state?
(c) The common, stable isotope of sodium has 12 neutrons and is the isotope referred to above: ^{23}Na. The unstable isotope ^{21}Na has ten neutrons, the same nuclear spin, and a half-life of 23 seconds. In the following, suppress the possibility of radioactive decay.

A box of volume 1 cm^3 contains 10^{14} sodium atoms at a temperature $T = 1.3 \times 10^{-6}$ K. The atoms form a dilute gas, and only one state of intrinsic angular momentum is populated. Determine whether the heat capacity C_V is an increasing or decreasing function of temperature if

(i) all atoms are ^{23}Na atoms;
(ii) half are ^{23}Na and half are ^{21}Na.

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Problem 20

(a) The critical temperature is

$$kT_c = 3\left(\frac{3}{2}\right)^{-2/3} \frac{h^2}{2\pi m}\left(\frac{N}{V}\right)^{2/3}$$

Setting $m = 3.82\times10^{-26}$ kg and $n = 10^{20}/m^3$, we obtain

$$T_c = 1.75\ \mu K$$

(b) The condensate fraction is

$$\frac{N_0}{N} = \left(1 - \frac{T}{T_c}\right)^{3/2}$$

This is equal to 0.90 at the temperature

$$T = 0.068\,T_c = 0.12\ \mu K$$

(c) The heat capacity C_V is an increasing function of T if $T < T_c$ and it is a decreasing function of T if $T > T_c$.

(i) C_V is an increasing function of T since $1.3\ \mu K$ is less than $T_c = 1.75\ \mu K$

(ii) The two isotopes are like 2 spin states.
The critical temperature for 2 spin states is

$$kT_c = \left(2\zeta\left(\tfrac{3}{2}\right)\right)^{-2/3} \frac{h^2}{2\pi m} \left(\frac{N}{V}\right)^{2/3}$$

$$T_c = 2^{-2/3}\,(1.75\,\mu K)$$

$$= 1.10\,\mu K$$

C_V is an increasing function of T since $1.3\,\mu K$ is less than T_c.