Sample Questions

1. An ecologist is interested in monitoring the health of a 5km² forest over time.

   (a) She decides to randomly select tree seedlings in the forest and measure their height every 3 months. Explain how using a sampling design based on randomly sampling seedlings within 100m² quadrants could be beneficial?

   Convenience - takes less time to collect random samples within a small area.

   (b) At one point in her study, the ecologist decides that she would like to determine the average number of seedlings per 100m² in the forest. She decides to count the total number of seedlings in each of her quadrants and assume that the total counts follow a Poisson distribution with mean $\lambda$. The maximum likelihood estimate of $\lambda$ is $\hat{\lambda} = \bar{n} = (n_1 + \cdots + n_k)/k = 49$, where $n_i$ is the number of seedlings observed in quadrant $i$. Find the MLE of the standard deviation of the number of seedlings in each quadrant.

   \[
   \text{Use invariance property of the MLE:} \\
   N_i \sim \text{Poisson} (\lambda) \\
   \hat{\lambda} = \bar{n} \\
   \text{sd}(N_i) = \sqrt{\text{Var}(N_i)} = \sqrt{\frac{\bar{N} \cdot \overline{\text{Var}(N_i)}}{\text{Var}(\bar{N})}} = \sqrt{\frac{\lambda}{\lambda}} = \sqrt{121} \\
   \]

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2. Consider a linear logistic model used in an analysis of a toxicological experiment. Let \( y_i, \quad i = 1, \ldots, n, \) be the number of deaths out of \( n_i \) individuals exposed to a dose of \( x_i \). Assume that \( y_i \sim Bin(n_i, \pi_i) \), where

\[
\text{logit}(\pi_i) = \log \left( \frac{\pi_i}{1 - \pi_i} \right) = \alpha + \beta x_i.
\]

(a) If \( \beta \neq 0 \), show that \( LD50 = -\alpha/\beta \).

\[
LD50 \Rightarrow \pi = .5 \\
\text{logit (.5)} = \log_2 (1) = 0 = \alpha + \beta x \\
\Rightarrow x = -\frac{\alpha}{\beta}
\]

(b) Why can the \( LD50 \) not be determined if \( \beta = 0 \)?

If \( \beta = 0 \), there is no relationship b/t dose and the probability of death.

(c) Suppose that two chemicals are compared using their respective \( LD50 \)s. If chemical A is more potent than chemical B (i.e., for a given dose, chemical A is expected to kill more individuals than chemical B), what does this imply about the relationships the two chemicals \( LD50 \)s?

Chemical A is more potent than chemical B \( \Rightarrow \) \( \alpha(A) > \alpha(B) \)

Therefore, since \( \text{logit}(\cdot) \) is an increasing fcn.

\[
\alpha(A) + \beta(A) x > \alpha(B) + \beta(B) x
\]

Let \( x = LD50(A) \Rightarrow \) that

\[
0 = \alpha(A) + \beta(A) LD50(A) > \alpha(B) + \beta(B) LD50(A)
\]

Since \( \alpha(B) + \beta(B) LD50(B) = 0 \),

\[
\alpha(B) + \beta(B) LD50(B) > \alpha(B) + \beta(B) LD50(A)
\]

\[
\Rightarrow LD50(A) - LD50(B) > 0
\]

\[
\Rightarrow \beta(B) > 0 \quad (\text{always the case for toxic substances})
\]

\[
LD50(B) - LD50(A) > 0, \text{ or } LD50(B) > LD50(A)
\]
3. Researchers designed an experiment to explore the impact of ingestion of two potentially carcinogenic chemicals. A total of 50 rats were randomly assigned to two treatment groups. Out of the 20 rats fed 0.2mg of chemical A, 15 died. For the group of 30 rats who were fed 0.2mg chemical B, 20 died.

(a) Test the independence of exposure to the potentially carcinogenic chemical and death, using Pearson’s $\chi^2$ statistic.

\[
E_{11} = \frac{15 \times 20}{50} = 6 \\
E_{12} = \frac{35 \times 20}{50} = 14 \\
E_{21} = \frac{30 \times 15}{50} = 9 \\
E_{22} = \frac{30 \times 35}{50} = 21
\]

\[
x^2 = \sum \frac{(O_i - E_i)^2}{E_i} = \frac{(6-5)^2}{6} + \frac{(14-15)^2}{14} + \frac{(10-9)^2}{9} + \frac{(21-20)^2}{21} = 0.43
\]

\[P(X^2 > 1.431) = .51 \Rightarrow \text{no statistically significant evidence of dependence.}\]

(b) A reporter for the Columbus Dispatch obtained the data from the experiment and wrote an article that stated that since the relative risk of chemical A to chemical B is greater than 1, people should be more concerned about eating food that contains chemical A than food that contains chemical B. Explain why this conclusion is potentially not valid?

This conclusion may not be valid since the toxic effects of the chemicals on rats may differ from their effects on humans.