Lecture – Thurs., 4/20/06

ANNOUNCEMENTS
• Please turn in HW1 by the end of class.
• Solutions to HW1 will be posted after class.
• HW2 will be posted in the next couple of days.

TODAY
• Regression Models

Models for Environmental Data

MODELS FOR MULTIVARIATE DATA

LINEAR REGRESSION MODELS
Goal: Explain the variation in a response variable $Y$ using explanatory variables $X_1, X_2, \ldots, X_p$
Regression Models

Least Squares Estimates
Goal: Estimate $\alpha$ and $\sigma^2$ by minimizing the sum of squares of the ‘residuals’

- Least Squares Estimate of $\alpha$

Regression Models

- Estimating $\sigma^2$
Regression Models

NORMAL LINEAR REGRESSION MODEL

Maximum Likelihood Estimation

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Example

Chlorophyll-a, measure of the density of algal cells in water, is commonly used to monitor the quality (clarity) of lake water -- the higher the levels of chlorophyll-a, the poorer the water quality. A researcher is interested assessing the relationship between chlorophyll-a and certain chemicals, in particular phosphorus and nitrogen, which are known stimulate algal growth.

Data

<table>
<thead>
<tr>
<th>Case</th>
<th>Chlorophyll-a</th>
<th>Phosphorus</th>
<th>Nitrogen</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>95.0</td>
<td>329.0</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>39.0</td>
<td>211.0</td>
<td>6</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>25</td>
<td>64.0</td>
<td>67.0</td>
<td>19</td>
</tr>
</tbody>
</table>
Regression Models

**TRANSFORMATIONS** (Dependent Variable)

Box-Cox Transformations

- applicable only when the dependent variable is positive (can shift the data so that this condition is satisfied)

R function:

```r
library(MASS)
boxcox(CH ~ PH + NT, data=lakes.data)
```

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Regression Models

Variance Stabilizing Transformations

The δ-method:

Let \( X \) be a random variable with mean \( \mu_X \) and variance \( \sigma_X^2 \), and let

\[
Y = g(X),
\]

where \( g( \cdot ) \) is a twice differentiable function. It can be shown (using a Taylor series expansion) that
Regression Models

Variance Stabilizing Transformations Cont.
Let $X$ be a random variable, and suppose that $\sigma_X^2 = \text{Var}(X) = V(\mu_X)$.

GOAL: Find a function $g(\cdot)$ so that for $Y = g(X)$, $\sigma_Y^2 \sim \text{constant}$.

EXAMPLE
Let $X \sim \text{Poisson}(\lambda)$. We want to find a function $g(\cdot)$, such that for $Y = g(X)$, $\sigma_Y^2 = \text{constant}$.