
ANNOUNCEMENTS
• HW3 is available on the course website

TODAY
• Periodicities
• Stationarity
• Autoregressive Model
• R Example – Canadian lynx data

Time-Series – Descriptive Methods

Periodic (Seasonal) Process: $X_t = s_t + \varepsilon_t$

Estimating Periodicities (assuming the period, $p$, is known)
• Ad hoc approach: average observations with a time separation $p$
• Regression on sine/cosine terms

Removing Periodicities
• differencing at separations of $p$
• smoothing

QUESTIONS
How can we estimate/remove periodicities if we don’t know the period, $p$? What if there is more than one cyclical component of the time-series?

Sunspots Example

Autocorrelation at lag $k$:

SAMPLE AUTOCORRELATION FUNCTION
(sometimes called the correlogram)

MODELING TIME SERIES

GOAL
• Fit a statistical model to time-series data that enables us to understand the mechanism that generated the data and/or allows us to predict future values.

STATIONARITY
• Time series develop over time in different ways:
  - Consider the process $X_t = \mu_t + s_t + \varepsilon_t$, where $\varepsilon_t$ is a “white noise” process.
DEFINITIONS

1. The discrete time series \( \{X_t; t = 1, 2, \ldots\} \) is said to be **strictly stationary** if \((X_{t_1}, \ldots, X_{t_i})\) and \((X_{t_1+k}, \ldots, X_{t_i+k})\) have identical distributions for all \(t_1, t_2, \ldots, t_i\) and \(k\).

2. The discrete time series \( \{X_t; t = 1, 2, \ldots\} \) is said to be **second-order stationary** if \(\mu, \sigma^2, \) and \(\text{Cov}(X_t, X_{t+s})\), for \(s=1, 2, \ldots\), do not depend on \(t\).

**TIME-SERIES ANALYSES**

1. Remove any trend and seasonality components.
2. Fit a *second-order stationary* time-series model to the residuals.

**AUTOREGRESSIVE PROCESS OF ORDER p**

\[
X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \cdots + \alpha_p X_{t-p} + \epsilon_t
\]

where the \(\epsilon_t\)s are independent and have mean 0 and constant variance \(\sigma^2\).