

Bessel's Correction

The Root Mean Square estimator for σ^2 of an unknown mean μ is naively given as:

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2 \quad \text{where} \quad \bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$$

Bessel showed that this estimator is biased (skewed), not surprising since it depends on x_i^2 . Bessel's corrected formula is given below (for proof see Barlow).

$$\sigma^2 = \frac{1}{(N-1)} \sum_{i=1}^N (x_i - \bar{x})^2 \quad \text{where} \quad \bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$$

Show in the Monte Carlo generated plot below are averaged σ^2 of 1 million trials each of σ^2 s estimated from N samples of a random gaussian distribution with $\mu=0$ and $\sigma^2=1$. Red points have Bessel's correction while the blue points use the naïve formula. The curve superimposed on the blue points is $(N-1)/N$.

