Maximum LogLikelihood Fit- Radioactive Decay

PDF for radioactive decay: $F(t) = Ne^{-\frac{t}{\tau}}$

Normalize in range 0-10 μsec $\int_0^{10} Ne^{-\frac{t}{\tau}} dt = \tau(1-e^{-\frac{10}{\tau}}) \Rightarrow N = \frac{1}{\tau(1-e^{-\frac{10}{\tau}})}$

The Likelihood function is: $L = \prod_i F(t_i) = \prod_i \frac{e^{-\frac{t_i}{\tau}}}{\tau(1-e^{-\frac{10}{\tau}})}$

Thus we want to maximize the logLikelihood:

$$\ln(L) = \sum_i \ln\left(\frac{e^{-\frac{t_i}{\tau}}}{\tau(1-e^{-\frac{10}{\tau}})}\right) = \frac{-\sum_i t_i}{\tau} - N\ln(\tau) - N\ln\left(1-e^{-\frac{10}{\tau}}\right) \qquad \textbf{Eqn.1}$$

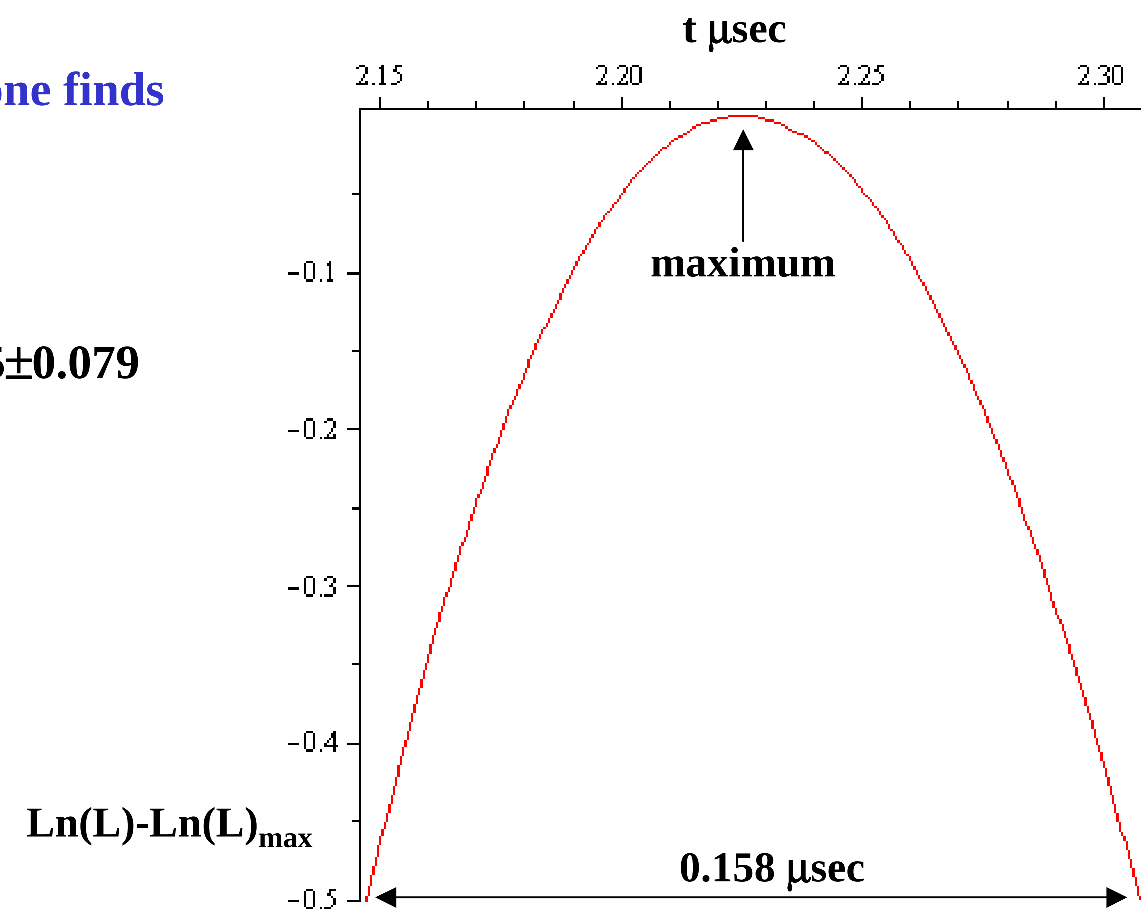
Generate 1000 MC events in Range 0<t<10 μsec with Lifetime 2.2 μsec

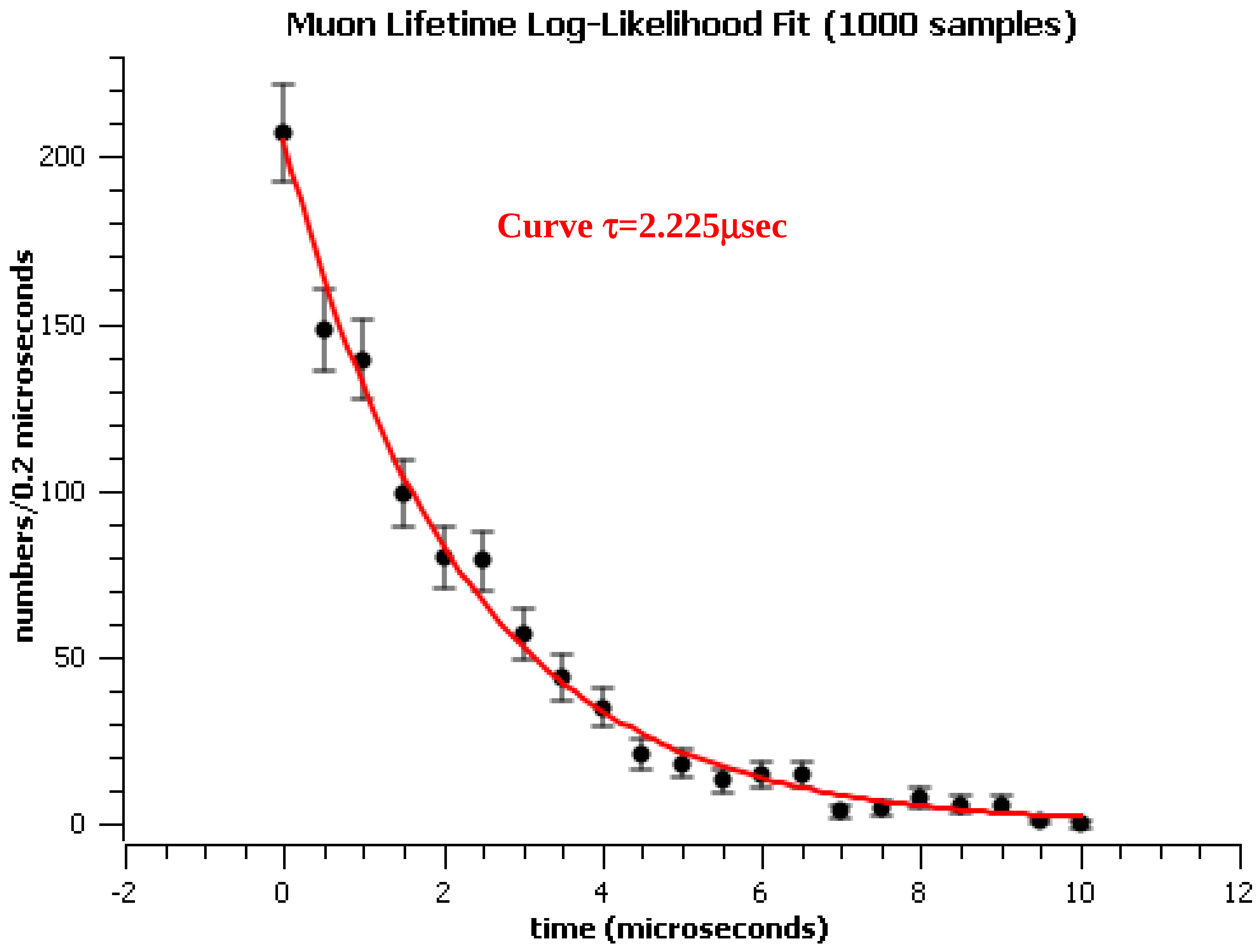
$$N = 1000 \qquad S \equiv \sum_i t_i = 2112.085$$

Maximize Eqn 1: **τ=2.225 μsec** **$\ln(L)_{max}$= -1737.8**

When $\ln(L)-\ln(L)_{max}$= -0.5 one finds the 1σ errors

Thus our fit yields: τ= 2.225±0.079




Muon Lifetime Log-Likelihood Fit (1000 samples)
Curve τ=2.225μsec
numbers/0.2 microseconds
time (microseconds)
0
50
100
150
200
-2
0
2
4
6
8
10
12

Alternate Calculation of σ_τ

Mean Sigma Parent Distribution

$$\int_0^{10} t \frac{e^{-\frac{t}{2.2}}}{2.2(1-e^{-\frac{10}{2.2}})} dt = 2.0927 \qquad \int_0^{10} (t-2.0927)^2 \frac{e^{-\frac{t}{2.2}}}{2.2(1-e^{-\frac{10}{2.2}})} dt = 3.756$$

Differential of Eqn 1, set to zero to get maximum:

$$\frac{S}{N} = \tau - \frac{10\mathrm{e}^{-\frac{10}{\tau}}}{(1-\mathrm{e}^{-\frac{10}{\tau}})}$$

Propagate errors:

$$\sigma_{\frac{S}{N}}^2 = \left(\frac{d(\frac{S}{N})}{d\tau}\right)^2 \sigma_\tau^2 = \left(1 - \frac{100e^{-\frac{10}{\tau}}}{\tau^2(1-e^{-\frac{10}{\tau}})} - \frac{100e^{-\frac{20}{\tau}}}{\tau^2(1-e^{-\frac{10}{\tau}})^2}\right)^2 \sigma_\tau^2$$

Plugging in Numbers $\sigma_{\frac{S}{N}}^2 = 3.765 \quad \tau = 2.2$ **yields** $\sigma_\tau^2 = 6.238$

Using Central Limit Theorem

$$\sigma_{fit} = \frac{\sqrt{6.238}}{\sqrt{1000}} = 0.079$$

To test Baye's Theorem we run the Monte Carlo simulation <u>1 million times</u> and histogram the best fit τ. The resulting plot yields a mean of 2.2 μsec with an RMS of 0.079 μsec. This agrees with our previous single run error bars. Thus we have made a correct determination of the uncertainty of our data. (e.g. 67% of the time our answer will be within ± 0.079 μsec of 2.225 μsec)