Epistemics in the Lab:
Two Papers

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Epistemics

1. $\varepsilon\pi\iota\sigma\tau\eta\mu\eta$ (episteme). Knowledge.
2. Epistemic Game Theory:
   Game theory that takes beliefs *very seriously*.
3. Epistemic Experiments:
   Experiments that take beliefs *very seriously*.
   - Measure 1st & 2nd order beliefs about strategies & utilities

My work so far:

1. 5 different 2×2 normal-form games
   - *Why* does Nash equilibrium/rationalizability fail?
2. Extensive-form games: The Centipede
   - *How* do beliefs evolve through the game?

1.) Normal Form Games

Question:
Do people play equilibrium/rationalizability/etc? If not, why not?

- Non-Selfish Utilities? → Nash with Fehr-Schmidt
- Wrong Beliefs? → Level-k
- Not best responding ('irrational')? → QRE

How *do* people play games??

- Traditional Approach: Measure strategies and fit models
- Epistemic Approach: Try to measure beliefs and preferences

One Little Change

In the EGT framework, s_i depends on θ_i.

- s_i part of your type, not necessarily consistent with Nash, etc.
- Pure strategies only
  - Mixing is in our beliefs, not our actions
- Players have beliefs ('conjectures') over S_{-i}
  - $Pr(s_j) = Pr(\{\theta_j \text{ that would play } s_j\})$
- Conjectures may or may not be 'correct'
- Players may or may not be rational
  - **Rational:** $s_i(\theta_i)$ is best response, given θ_i's conjectures

The EGT agenda:

[Assumptions about rationality etc] $\iff$ [Solution concept]

Aumann & Brandenburger (1995)

Aumann & Brandenburger (1995) Theorem A

Theorem

Fix $n = 2$. Suppose the realized type profile is θ with conjectures $\phi(\theta)$. If utilities, conjectures, and rationality are all mutual knowledge at θ, then $(\phi_2(\theta_2), \phi_1(\theta_1))$ is a (mixed-strategy) Nash equilibrium of the game with utilities $U_1(\cdot|\theta_1)$ and $U_2(\cdot|\theta_2)$.

Contrapositive: If conjectures aren't in Nash equilibrium, then at least one of the following is not mutual knowledge:

1. utilities
2. rationality
3. conjectures

Lesson

To ask if people are in equilibrium, we **must** look at these objects!

This paper: elicit all the stuff needed to see if people are in equilibrium.

If not, what are they doing? What's failing?

The Experiment

- 150 subjects were paired anonymously with 1 opponent.
- Play five 2 × 2 one-shot games with no feedback.
- Last 74 subjects: risk & ambiguity questions at end
- For each game, I elicit (on paper, in this order):
    1. Chosen action ($\boldsymbol{s_i}(\theta_i)$)
    2. Preferences over outcomes ($\boldsymbol{u_i}(\cdot|\theta_i)$) (cardinal & ordinal)
    3. i's beliefs about $\boldsymbol{u_j}$ (cardinal & ordinal)
    4. Conjecture about $\boldsymbol{s_j}$ ($\boldsymbol{\phi_i}(\cdot|\theta_i)$)
    5. i's beliefs about $\boldsymbol{\phi_j}$.
    6. i's beliefs about j's rationality
- RPS Payment: For each pair, pick *one* decision for payment.

Example Observation

	L	R
U	$^{\$}10, 10$	$^{\$}1, 15$
D	$^{\$}15, 1$	$^{\$}5, 5$

Game Form (PD)

	35%	65%
>35%	$80, 80$	$5, 95$
65%	$95, 5$	$80, 80^*$

Row's Game

	0%	∨100%
0%	$55^*, 70$	$0^*, 100$
100%	$100, 0$	$15^*, 30$

Column's Game

	35%	65%
>35%	$3, 3$	$1, 4$
65%	$4, 1$	$3, 2$

Row's Ordinal Game

	0%	∨100%
0%	$3, 3$	$1, 4$
100%	$4, 1$	$2, 2$

Column's Ordinal Game

Eliciting Preferences

100 questions, 1 randomly chosen for payment:

1. **Cheeseburger** vs. 0% chance at \$20
2. **Cheeseburger** vs. 1% chance at \$20

   ⋮

36. **Cheeseburger** vs. 36% chance at \$20
37. **Cheeseburger** vs. 37% chance at \$20
38. Cheeseburger vs. **38% chance at \$20**

   ⋮

99. Cheeseburger vs. **99% chance at \$20**
100. Cheeseburger vs. **100% chance at \$20**

Easier question: What's your *probability value* for a cheeseburger? **37%**

$$u(\text{Chsbgr}) = p \cdot \underbrace{u(\$20)}_{=1} + (1-p) \cdot \underbrace{u(\$0)}_{=0} = p.$$

Eliciting Beliefs in a Game

What's your *probability* that COL plays LEFT? **52%**

1. **$20 if COL plays L** vs. 0% chance at $20
2. **$20 if COL plays L** vs. 1% chance at $20

⋮

51. **$20 if COL plays L** vs. 51% chance at $20
52. **$20 if COL plays L** vs. 52% chance at $20
53. $20 if COL plays L vs. **53% chance at $20**

⋮

99. $20 if COL plays L vs. **99% chance at $20**
100. $20 if COL plays L vs. **100% chance at $20**

Eliciting $u(\cdot)$ in a Game

What's your *probability value* for (YOU:\$15 THEY:\$1)? **63%**

1. **YOU:\$15 THEY:\$1** vs. 0% chance at YOU:\$20 THEY:\$20
2. **YOU:\$15 THEY:\$1** vs. 1% chance at YOU:\$20 THEY:\$20

   ⋮

62. **YOU:\$15 THEY:\$1** vs. 62% chance at YOU:\$20 THEY:\$20
63. **YOU:\$15 THEY:\$1** vs. 63% chance at YOU:\$20 THEY:\$20
64. YOU:\$15 THEY:\$1 vs. **64% chance at YOU:\$20 THEY:\$20**

   ⋮

99. YOU:\$15 THEY:\$1 vs. **99% chance at YOU:\$20 THEY:\$20**
100. YOU:\$15 THEY:\$1 vs. **100% chance at YOU:\$20 THEY:\$20**

Thus, $u(\$15, \$1) = 63$

Game 2: Symmetric Coordination

First, a game theory success story.

	97%	3%
97%	${}^{\$}15, 15$	${}^{\$}1, 1$
3%	${}^{\$}2, 2$	${}^{\$}5, 5$

(Percentages are action-choice frequencies.)

Game 2: Symmetric Coordination - Cardinal Utilities

	97%	3%
97%	$^{\$}15, 15$	$^{\$}1, 1$
3%	$^{\$}2, 2$	$^{\$}5, 5$

- Strategies: 97% U/L
- Belief of Strategies: $\geq 90\%$ sure L/U
- Utilities: Almost all say $(\$15, \$15) \succ (\$5, \$5) \succ (\$2, \$2) \succeq (\$1, \$1)$
- Belief of Utilities: very accurate
- Rationality: 96%
- Belief of Rationality: 65-95% sure opponent is rational

Game 1: Dominance Solvable

Now let's look at iterated dominance...

	25%	75%
100%	$10, 5	$15, 15
0%	$5, 10	$1, 1

ROW follows its dominant strategy...

...so why are 25% of COL playing L??

Game 1: Dominance Solvable - Conjectures

	25%	75%
100%	$^{\$}10,5$	$^{\$}15,15$
0%	$^{\$}5,10$	$^{\$}1,1$

- Belief of Strategies:
    - Play R: $p(U) = 94\%$. Play L: $p(U) = 85\%$.
- Utilities: 93% 'selfish'
- Belief of Utilities: 93% quite sure opponent is 'selfish'
- Rationality:
    - Play R: 98%. Play L: 21% (all non-selfish).

Conclusion: 20% of players are non-EU. Loss aversion?

Game 5: Asymmetric Coordination

	49%	51%
93%	$15, 5	$2, 1
7%	$1, 2	$5, 10

Why are 51% of COL playing Right?

- Preferences?
  - Only for 1.3% of subjects.
- EU with wrong beliefs or risk aversion?
  - No. Beliefs are accurate. Most are not rational.

Conclusion: Non-EU regret aversion?

Game 4: Asymmetric Matching Pennies - The Story

	44% **(38% Rat'l)**	56% (90% Rat'l)
(81% Rat'l) 88%	$^{\$}15, 5$	$^{\$}5, 10$
(63% Rat'l) 12%	$^{\$}5, 10$	$^{\$}10, 5$

Why are 44% of COL playing Left?

1. Preferences: **All** non-selfish are rational.

	Rational	Irrational
Selfish	5	20
Non-Selfish	7	0

2. Beliefs:
   - Mean $p(\mathrm{Up})$ is 78%

Conclusion: Altruism in strategies $\neq$ altruism in outcomes?
(Or altruism changed between strategy choice & elicitation.)

Game 3: Prisoners Dilemma

Finally, the prisoners' dilemma.

	35% C	65% D
26% C	$^{\$}10, 10$	$^{\$}1, 15$
74% D	$^{\$}15, 1$	$^{\$}5, 5$

Game 3: Prisoners Dilemma - Preference Types

Type	**Ex: ROW**	**Actual**	**Believed**
Selfish	$\begin{array}{ccc} & L & R \\ U & ^{\$}10,10 & ^{\$}1,15 \\ D & ^{\$}\mathbf{15,\ 1} & ^{\$}\mathbf{5,\ 5} \end{array}$	103	115
CondCoop	$\begin{array}{ccc} & L & R \\ U & ^{\$}\mathbf{10,\ 10} & ^{\$}1,15 \\ D & ^{\$}15,1 & ^{\$}\mathbf{5,\ 5} \end{array}$	29	24
UncondCoop	$\begin{array}{ccc} & L & R \\ U & ^{\$}\mathbf{10,\ 10} & ^{\$}\mathbf{1,\ 15} \\ D & ^{\$}15,1 & ^{\$}5,5 \end{array}$	14	4
Crazy	$\begin{array}{ccc} & L & R \\ U & ^{\$}10,10 & ^{\$}\mathbf{1,\ 15} \\ D & ^{\$}\mathbf{15,\ 1} & ^{\$}5,5 \end{array}$	4	2

This is a Bayesian Game

Game 3: Prisoners Dilemma - Preference Types

Self-similarity result:

Selfish thinks others are more likely to be Selfish

CondCoop thinks others are more likely to be CondCoop

...

(χ^2 p-value: < 0.0001)

This is a Bayesian Game with Correlated Types

Game 3: Prisoners Dilemma - Preference Types

Action Choices & Rationality:

Type	C	D	% Rational
Selfish	18	83	82%
CondCoop	18	11	72%
UncnCoop	8	6	57%
Crazy	1	3	50%
χ^2 p-value:	< 0.001		0.090

This is a Bayesian Game with Correlated Types and Some Non-EU Players

Game 3: Prisoners Dilemma - Preference Types

Action Choices & Rationality:

Type	C	D	% Rational
Selfish	**18**	83	82%
CondCoop	18	11	72%
UncnCoop	8	**6**	57%
Crazy	1	3	50%
χ^2 p-value:	< 0.001		0.090

78% of players have a dominant strategy...
but **21%** of them don't follow it!

Inconsistency between elicited preferences and strategy choice.

1. Uncertainty about preferences?
2. Non-consequentialism?
3. Bad elicitation procedures?

Game 3: Prisoners Dilemma - Preference Types

How much rational cooperation is there
in the one-shot prisoner's dilemma?

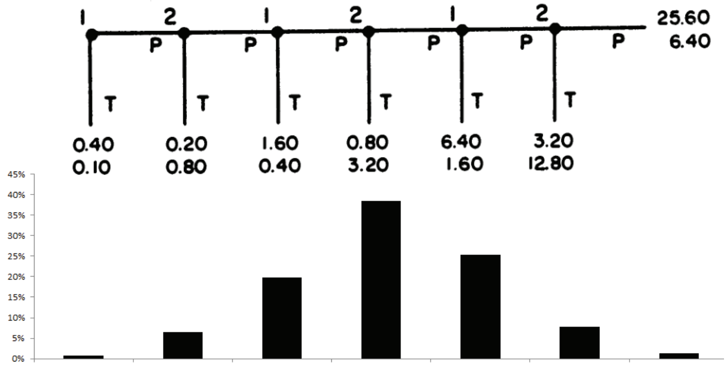
30% cooperate.
53% of those do so rationally.

Summary of Normal-Form Games

- Nash equilibrium *can* work, but only in ‘easy’ games.
- Respect for Bayesian games (P.D.)
- ‘Irrational’ behavior seems to vary by game
- Beliefs clearly inconsistent with Level-k

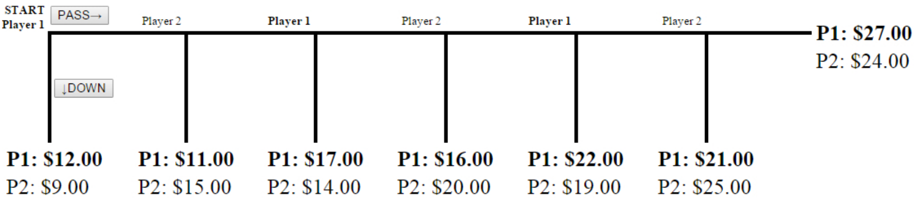
- WARNING: Confound with reliability of elicitation procedure.

2.) Extensive-Form Games: The Centipede



McKelvey & Palfrey (1992): Reputation w/ noisy actions & heterog. blfs
Fey, McKelvey & Palfrey (1996), Kawagoe & Takizawa (2012): AQRE

Epistemic Theory



Claim: Common knowledge of rationality ⇒ backwards induction

Reny (1992): Wrong. If Pass ⇒ irrational, Pass can be a best response!

EGT question: How do players update beliefs about rationality?
Expmntl question: Are beliefs consistent with proposed models (QRE, eg)?

The Experimental Design

Have subjects play 4 centipede game forms. Elicit:

1. Before each game:
    1.1 Own utilities for each outcome
    1.2 Guess of others' utilities
2. At each node:
    2.1 Action choice (for node owner)
    2.2 Future action plan (s_i)
    2.3 1st order belief of s_{-i}
    2.4 2nd order belief of s_i (best guess)
    2.5 Belief of rationality

The Research Plan

Phase 1 Search for the right game form, satisfying:

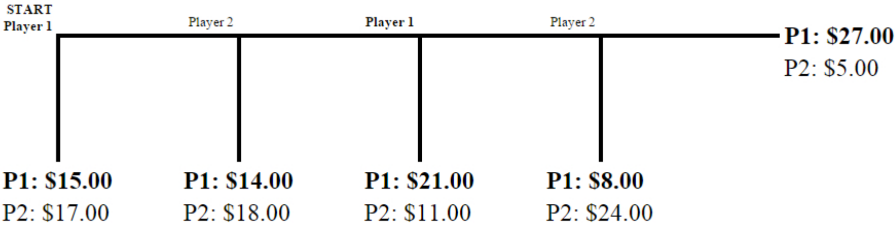
1. elicited utilities are actually centipede game utilities
2. players don't play Down immediately

Phase 2 Collect data on chosen game form

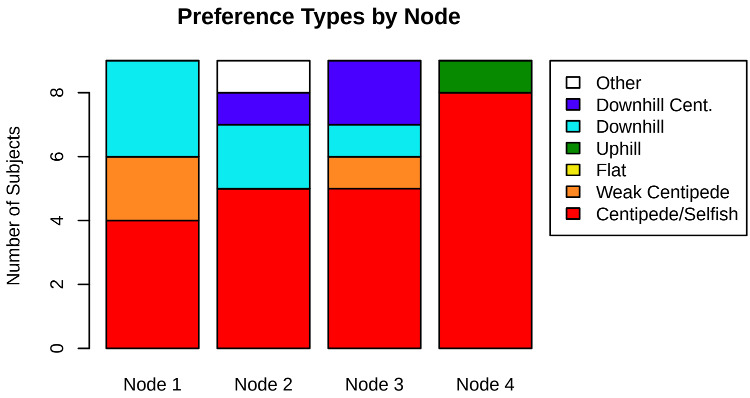
1. not done yet

The First Attempt: Treatment 1

Constant-sum 4-node centipede game form:



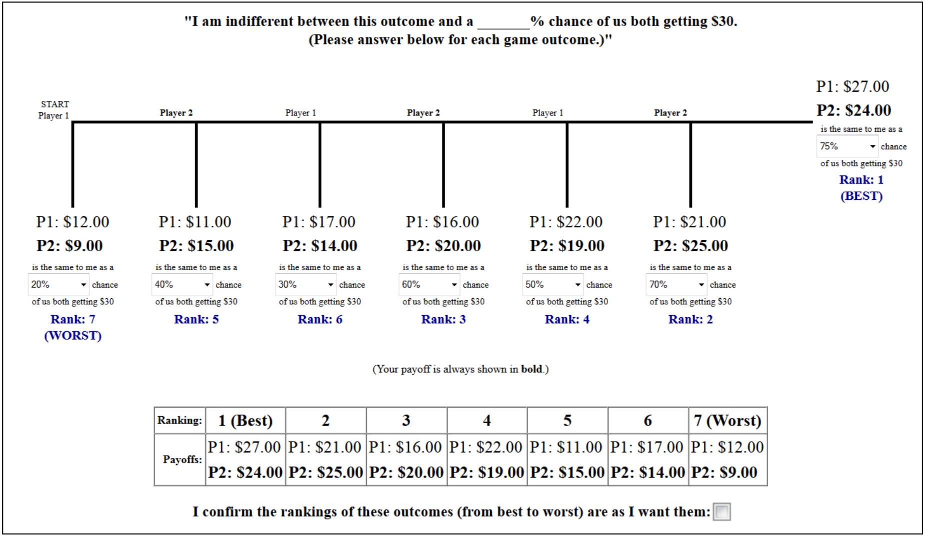
Utility Types: Treatment 1



Type 'at node 1' (e.g.) is based on utility at nodes 1, 2, and 3.

8 of 18 have centipede prefs at all nodes (3 plr1, 5 plr2)

Eliciting Utility



"I am indifferent between this outcome and a _______% chance of us both getting $30.
(Please answer below for each game outcome.)"

P1: $12.00
P2: $9.00
is the same to me as a 20% ▾ chance of us both getting $30
Rank: 7 (WORST)

P1: $11.00
P2: $15.00
is the same to me as a 40% ▾ chance of us both getting $30
Rank: 5

P1: $17.00
P2: $14.00
is the same to me as a 30% ▾ chance of us both getting $30
Rank: 6

P1: $16.00
P2: $20.00
is the same to me as a 60% ▾ chance of us both getting $30
Rank: 3

P1: $22.00
P2: $19.00
is the same to me as a 50% ▾ chance of us both getting $30
Rank: 4

P1: $21.00
P2: $25.00
is the same to me as a 70% ▾ chance of us both getting $30
Rank: 2

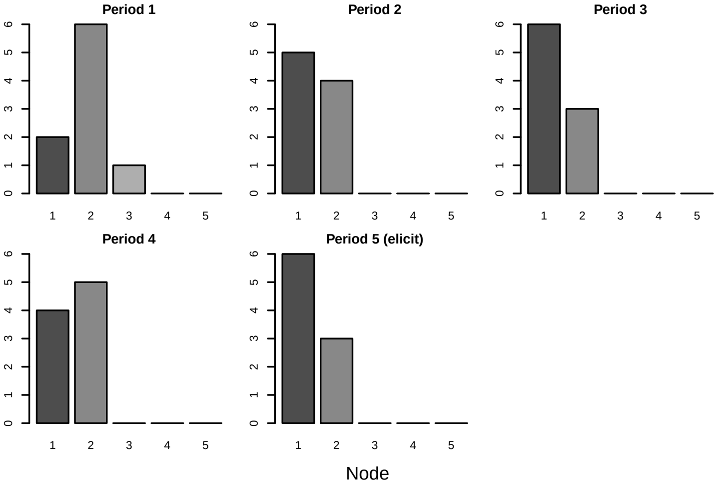
P1: $27.00
P2: $24.00
is the same to me as a 75% ▾ chance of us both getting $30
Rank: 1 (BEST)

(Your payoff is always shown in **bold**.)

Ranking:	**1 (Best)**	**2**	**3**	**4**	**5**	**6**	**7 (Worst)**
Payoffs:	P1: $27.00 **P2: $24.00**	P1: $21.00 **P2: $25.00**	P1: $16.00 **P2: $20.00**	P1: $22.00 **P2: $19.00**	P1: $11.00 **P2: $15.00**	P1: $17.00 **P2: $14.00**	P1: $12.00 **P2: $9.00**

I confirm the rankings of these outcomes (from best to worst) are as I want them: ☐

Outcomes: Treatment 1

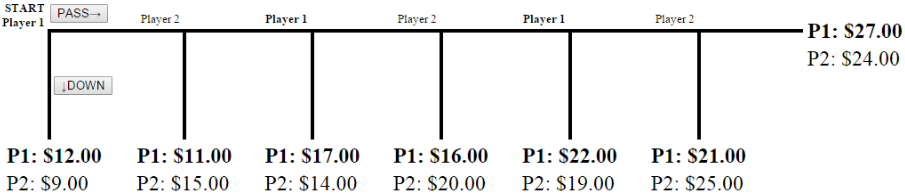


	1	2	3	4	5
Period 1	2	6	1	0	0
Period 2	5	4	0	0	0
Period 3	6	3	0	0	0
Period 4	4	5	0	0	0
Period 5 (elicit)	6	3	0	0	0

A victory for backwards induction! (Similar to past findings)

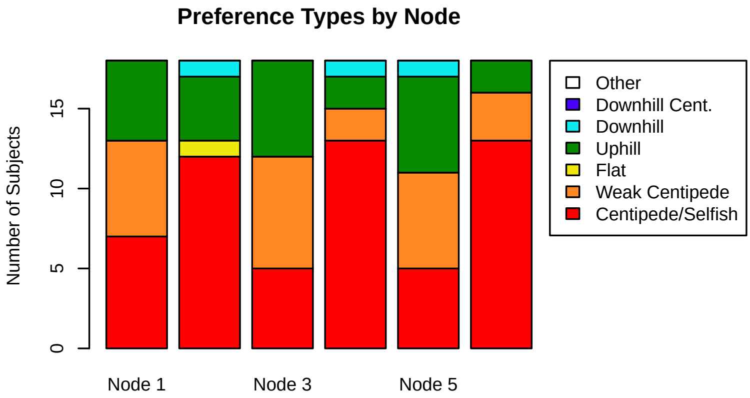
The Second Attempt: Treatment 2

Increasing-sum 6-node centipede game form:



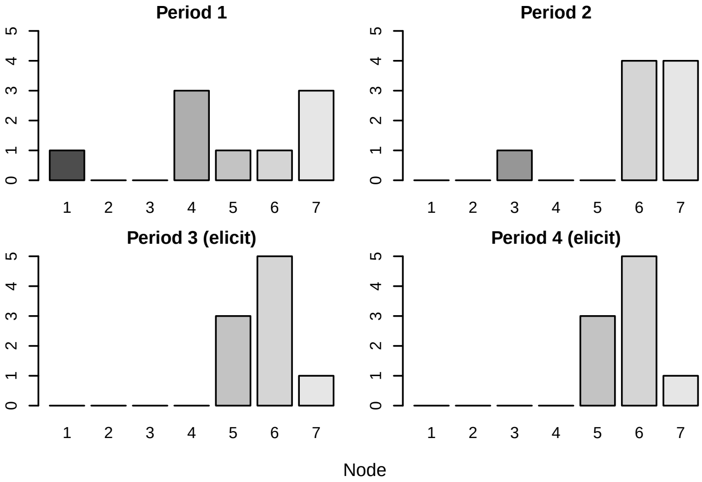
Pass: risks $1 to gain $5.

Utility Types: Treatment 2



13 of 36 have centipede prefs at all nodes (2 plr1, 11 plr2)

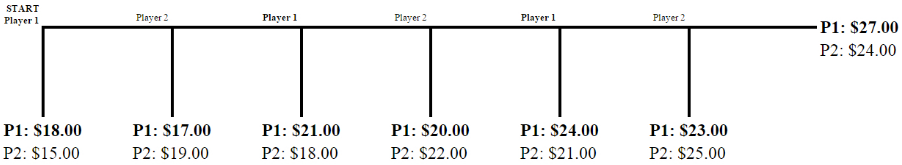
Outcomes: Treatment 2



Is this really a centipede game?

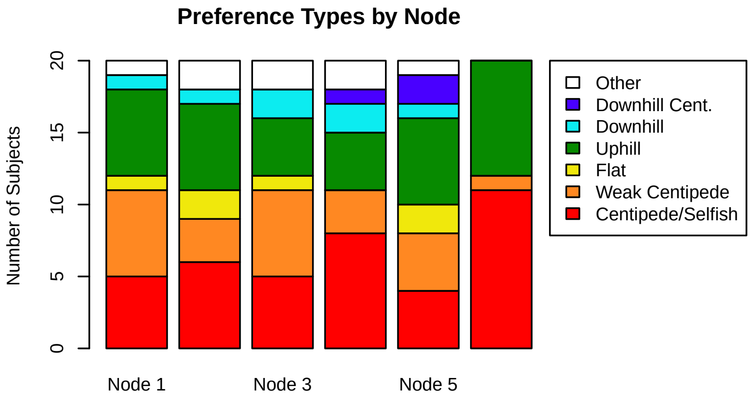
The Third Attempt: Treatment 3

Increasing-sum 6-node centipede game form:



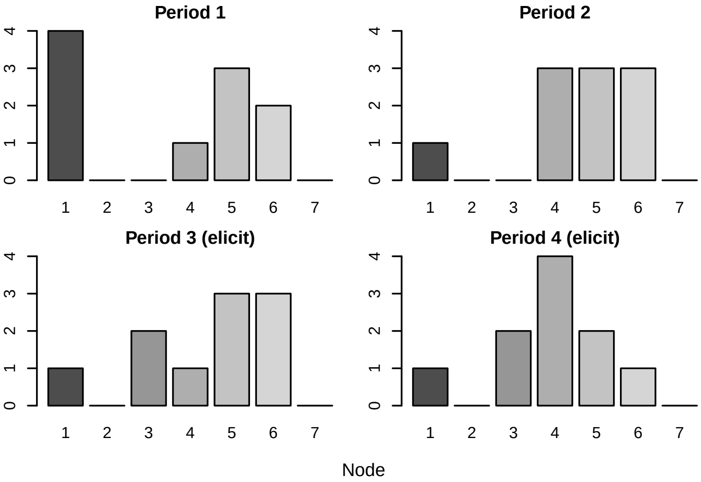
Pass: risks $1 to gain $3.

Utilities: Treatment 3



8 of 40 have centipede prefs at all nodes (2 plr1, 6 plr2) Not a centipede game.

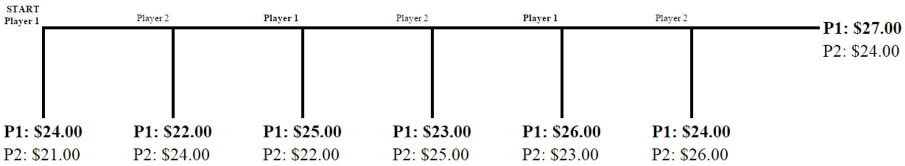
Outcomes: Treatment 3



Looks like McKelvey-Palfrey data...

The Fourth Attempt: Treatment 4

Increasing-sum 6-node centipede game form:



START
Player 1

Player 2

Player 1

Player 2

Player 1

Player 2

P1: $27.00
P2: $24.00

P1: $24.00
P2: $21.00

P1: $22.00
P2: $24.00

P1: $25.00
P2: $22.00

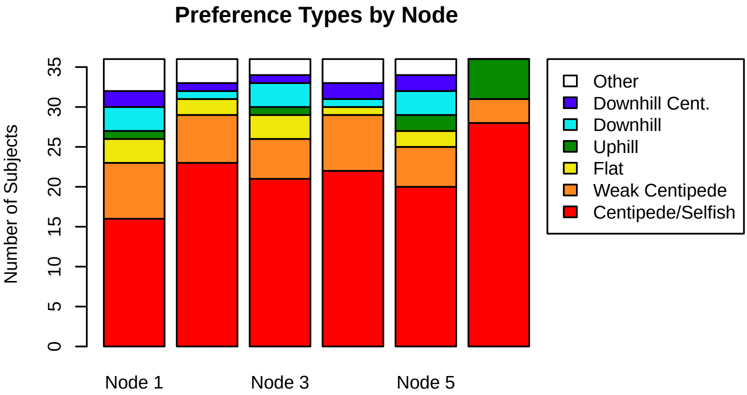
P1: $23.00
P2: $25.00

P1: $26.00
P2: $23.00

P1: $24.00
P2: $26.00

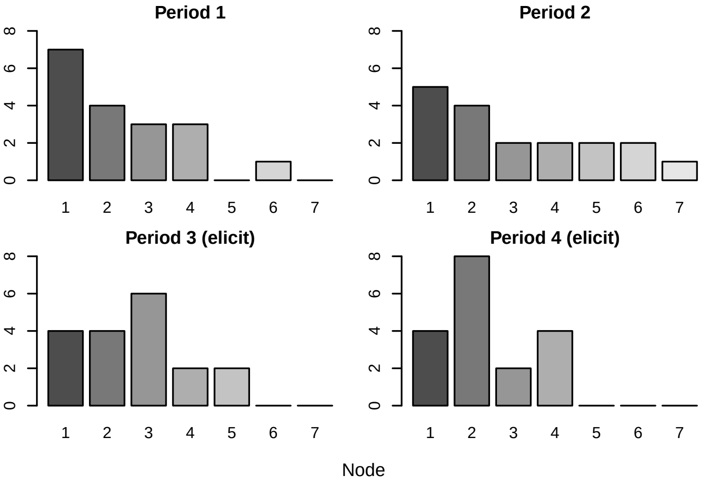
Pass: risks $2 to gain $1.

Utilities: Treatment 4



29 of 72 have centipede prefs at all nodes (11 plr1, 18 plr2)

Outcomes: Treatment 4

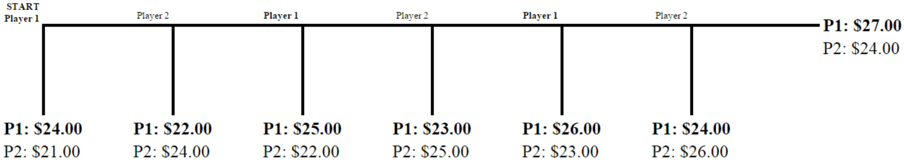


Conclusion of Phase 1

Conclusion 1: I will proceed with Treatment 4
(Haven't collected Phase 2 data yet...
so no statistical tests)

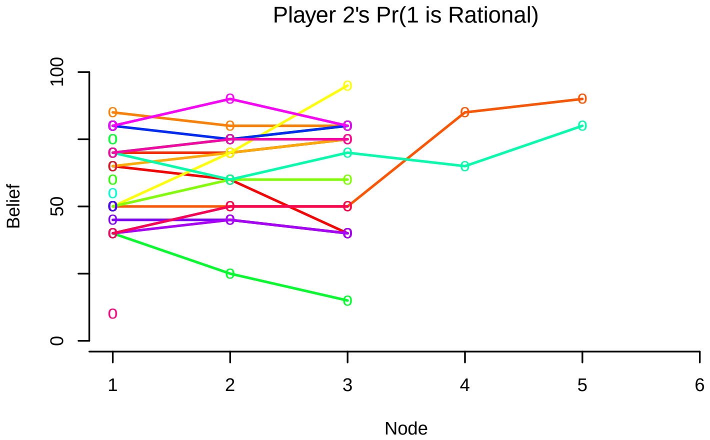
Conclusion 2: It's hard to find a centipede game!

Beliefs About Rationality



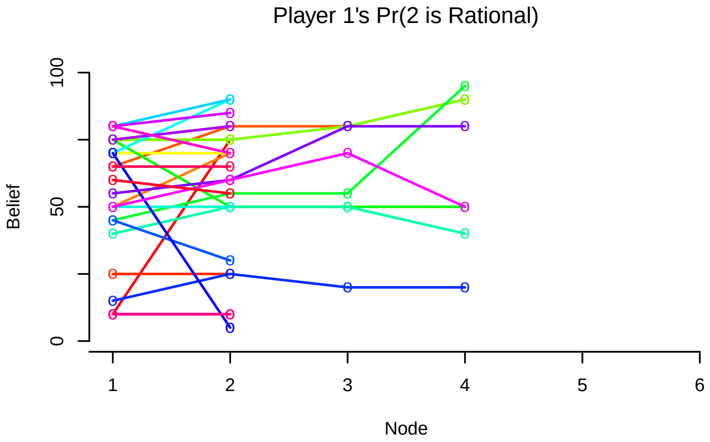
Do beliefs about rationality shift dramatically at node 2??

Beliefs About Rationality: Player 2



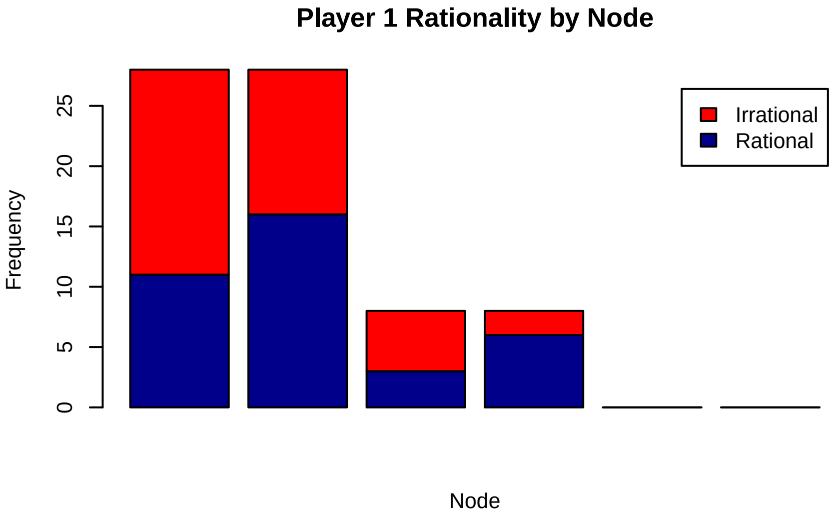
(1) Node 1 vs 2. (2) Upward trend. (3) Initial beliefs.

Beliefs About Rationality: Player 1

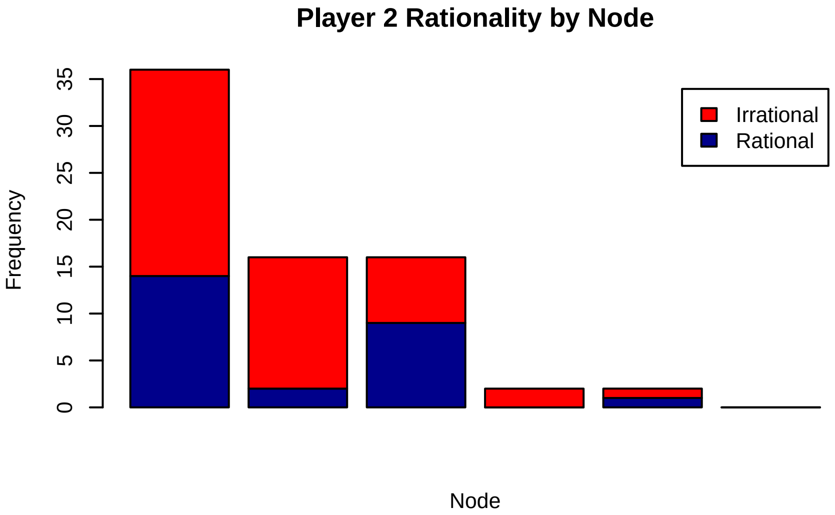


Compare nodes 2 vs. 3

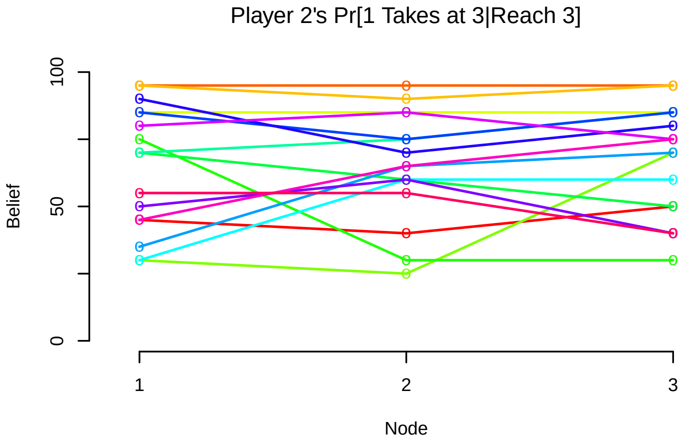
Actual Rationality: Player 1



Actual Rationality: Player 2

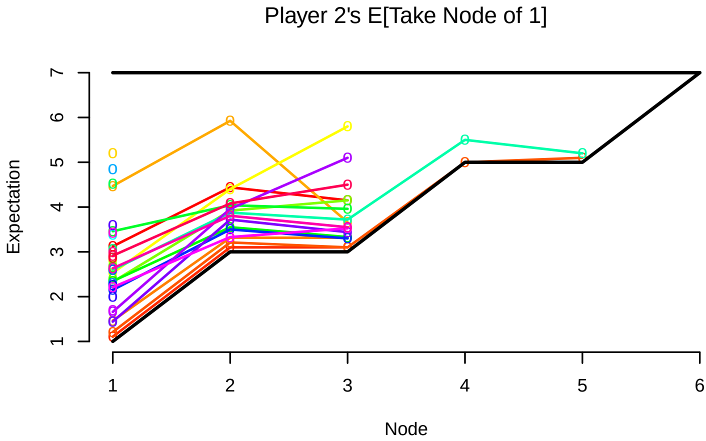


Beliefs about Others' Actions: Player 2



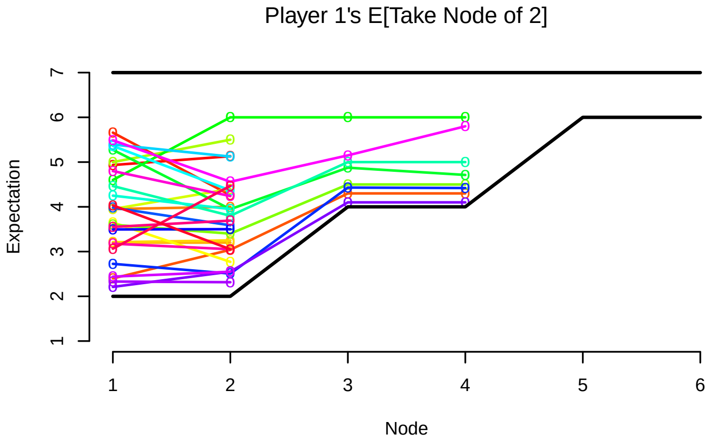
Beliefs about node 3, from nodes 1, 2 & 3 (of matches that reached 3)

Beliefs about Others' Actions: Player 2



Expected take node, assuming 2 always passes

Beliefs about Others' Actions: Player 1



Expected take node, assuming 1 always passes

Contingent Plan Transitions: Player 2

		Plan at Node 2					
		T@1	**2**	**4**	**6**	**7**	**Tot.**
Plan at Node 1	**2**	2	8	2	0	0	12
	4	5	3	6	0	0	14
	6	1	0	2	5	1	9
	7	0	1	0	0	0	1
	Tot.	8	12	10	5	1	36

		Plan at Node 2					
		T@1	**2**	**4**	**6**	**7**	**Tot.**
Plan at Node 3	**T@1**	8	–	–	–	–	8
	T@2	–	12	–	–	–	12
	4	–	–	9	1	0	8
	6	–	–	1	4	0	6
	7	–	–	0	0	1	2
	Tot.	8	12	10	5	1	36

Contingent Plan Transitions: Player 1

		Plan at Node 2				
		T@1	**3**	**5**	**7**	**Tot.**
Plan at Node 1	**1**	8	–	–	–	8
	3	–	8	3	0	11
	5	–	2	10	2	14
	7	–	0	1	2	3
	Tot.	8	10	14	4	36

		Plan at Node 2				
		T@1	**3**	**5**	**7**	**Tot.**
Plan at Node 3	**T@1**	8	–	–	–	8
	T@2	–	2	7	3	12
	3	–	7	1	0	8
	5	–	1	5	0	6
	7	–	0	1	1	2
	Tot.	8	10	14	4	36

Conclusions

1. Centipedes are *elusive*
2. Rationality hovers around 50%
3. Beliefs about rationality heterogeneous, but stable
4. Beliefs about actions are stable
5. Strategies (plans) don't change often

The FRPD Story:

- Kagel & McGee (2015)
- Cox, Jones, Pflum & Healy (2015)

Selected Literature Review

- McKelvey & Palfrey (1992)
- Fey, McKelvey & Palfrey (1996)
- Palacios-Huerta & Voliz (2009) and Levitt, List & Sadoff (2011)
- Mezhvinsky (2015WP)
- Wang (2015WP)

Fin