Inflation Targeting as Constrained Discretion[*]

Junhan Kim[†]

March 24, 2011

Abstract

This paper suggests a simple framework for modeling inflation targeting as constrained discretion. Although it is widely claimed that inflation targeting has been successful in maintaining low and stable inflation, an announcement of an inflation target does not by itself mean that central bankers are precommiting to how they conduct monetary policy. In comparison to the assumption of many theoretical studies, central banks conduct monetary policy in a discretionary fashion and rarely precommit to a rule in reality. Therefore the central bank in this paper is modeled as discretionary, yet faced with a constraint, that an average of future inflation over a certain horizon should be kept on or near the pre-announced target level. It is natural to add this constraint to the central

banker's optimization problem, since inflation targeting involves one way or another an evaluation of the performance over a certain horizon. So it is argued that the better outcome of inflation targeting does not come from a commitment, but from 'constrained discretion'. This paper also sheds some light on optimal targeting horizon.

JEL Classification: C61, E52, E61

1 Introduction

Since 1990 many countries have adopted inflation targeting as a monetary policy framework. One of the core elements, and sometimes argued as the only element, in inflation targeting is an announcement of a numerical target level for inflation. Inflation targeting proponents argue that by announcing the numerical target, the public presume that central banks are committed to achieving the target, and therefore they can anchor inflation expectations at the target level. Therefore it is claimed that inflation targeting has been successful in maintaining low and stable inflation because central banks have committed to it.

Simply announcing a numerical target, however, does not necessarily imply that central banks are committed to stabilizing inflation. In contrast to the underlying assumption of many theoretical studies, central banks conduct monetary policy in a discretionary fashion and rarely precommit to a rule in the real world. Uncertainties and incomplete knowledge of the economy, among many other factors, prevent them from setting the course of policy in advance. Whenever things do not turn out the way they expected, they change the way they conduct policy. In addition, there do not appear to be any central banks that claim that price stability is their one and only goal. So there is always room for central bankers to be discretionary.

As (Kydland and Prescott 1977) and (Barro and Gordon 1983) show, discretionary monetary policy produces a sub-optimal outcome. That is, actual inflation is greater than the optimal level. A discretionary central bank is one that reoptimizes every period assuming that inflation expectations are given at the particular point in time. Since inflation expectations are assumed to be given, increasing output at the expense

of higher inflation seems to improve welfare. Rational agents, however, would not keep the inflation expectation unchanged if they knew that central banks re-optimize every period to take advantage of the 'given' expectations. So this spiral of actual and expected inflation makes actual inflation end up higher without having any effect on output. Of course, if central banks can commit, optimality may be attained. This is because if central banks make a promise not to re-optimize every period, they can control inflation expectations. As mentioned above, however, it is very unlikely in reality that central banks would commit to a rule.

This is why there have been numerous studies on how to improve the performance of central banks when there is no precommitment mechanism. One of the most prominent examples is the conservative central banker argument set out by (Rogoff 1985). He argues that appointing a 'conservative' central banker, one who believes that stabilizing inflation is more important than it really is, would mitigate problems from discretion. Following a similar line of argument, (Svensson 1997) argues for a conservative target, which means a lower target level for inflation than the optimal level. The similarity between these two beyond the word 'conservative', is that they are both arguing for an objective function for central bankers which differs from the social welfare function. From a slightly different perspective, (Svensson 1999) and (Cecchetti and Kim 2005) argue that targeting price-level or a hybrid of price level and inflation is better at stabilizing inflation in the long-run than inflation targeting. So the core message of these authors is that central banks should be charged with an objective (or a target) which makes the outcome close to optimality.

The case of targeting the price level instead of inflation, even though it is inflation that matters, is not without theoretical support. The optimal policy with an uncon-

strained commitment in a New-Keynesian model as analyzed in (Clarida, Gali and Gertler 1999) supports price-level targeting. The advantage of 'keeping the promise' comes from including a history dependent term in the optimality condition. Delegating an objective function to central banks becomes more explicit in (Walsh 1995), who suggests an optimal contract in which central bankers are punished (rewarded) for higher (lower) inflation.

There is little doubt that these suggestions would indeed improve the performance of discretionary central banks. However, as (Mankiw 2005) points out, delegating central banks an objective function that is different from society's welfare function will invite some criticism. He even calls this 'two wrongs make one right.' It is giving central banks a wrong objective, albeit useful one.

This is where the present paper starts. I suggest a simple framework of modeling inflation targeting as discretion but constrained discretion, and argue that optimality can be attained without an artificial objective or a contract. The constraint represents a performance evaluation for central bankers, so this is not at all unnatural, since inflation targeting involves the evaluation of central bankers one way or another. In this sense I contend that the framework in this paper is indeed an implementation of 'constrained discretion' as in (Bernanke and Mishkin 1997).

This paper is organized as follows. First, a simplified central banker's optimization problem is presented as a benchmark. Then, I add a medium term constraint that represents a performance evaluation for central bankers. The constraint is that an average inflation over a certain horizon be on or near the target. Second, I compare the optimality condition with those from previous studies. This is quite in line with previous studies. That is, the more persistent a shock is, the more active monetary

5

policy should be. Third, I derive the optimal horizon for different set of parameters and do some welfare analysis. I round off the paper with some conclusions.

2 Model

2.1 Optimization Problem for Central Bankers

Let's first take a look at the following optimization problem for the central bank.

$$\min \quad \frac{1}{2}(1-\beta)E_t \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \lambda y_{t+i}^2], \tag{1}$$

$$\text{s.t.} \quad \pi_{t+i} = \beta E_{t+i}\pi_{t+i+1} + \alpha y_{t+i} + \epsilon_{t+i}, \tag{2}$$

$$\epsilon_{t+i} = \rho\epsilon_{t+i-1} + u_{t+i}, \tag{3}$$

where π denotes inflation, y denotes the output gap, E denotes the expectation operator, ϵ denotes a shock process, u denotes an i.i.d random process, and β, κ, ρ are parameters. Equation (1) is the loss function, which is the discounted sum of squared inflation deviation from the target, which is normalized to 0 without loss of generality, and output gap squared, weighted by a parameter λ. Despite some criticism that this type of loss function is ad-hoc or a mere device for the sake of simplicity, we think of this as a representation of the social welfare, since (Rotemberg and Woodford 1997) show that the loss function of this form can be derived from the social welfare. Minimizing the inflation deviation is to lower the relative price distortion due to price stickiness, and stabilizing the output gap represents lowering the welfare cost from

output fluctuation. Equation (2) is the so called New-Keynesian Phillips curve, which relates inflation, inflation expectation and output gap. This can be derived with an assumption of sticky prices such as (Calvo 1983). In this setup, only Equation (2) represents the dynamics of the economy since the demand side of the economy can be abstracted. The demand side is redundant since it is implicitly assumed that changing a policy instrument, usually an interest rate, does not involve any cost. The demand side would become important if changing interest rate is not costless, for example, when the interest rate is bounded by a non-zero constraint, or for some reason central banks prefer stable interest rates.

In order to grasp the intuition analyzed in the next section, the solution for the optimization with a medium-term constraint, it is crucial to make a distinction between discretion and commitment solutions for the optimization problem described above. There are three different solution concepts: discretion, commitment to a simple rule, and commitment without constraint. In fact, this is the same optimization problem for the central banker studied by (Clarida, Gali and Gertler 1999), and they elaborated three different concept solutions. The first is when the central bank is discretionary, meaning that the central bank re-optimizes the objective every period. It is therefore optimal for the central bank to make a promise for the future but not keep the promise made in the past. Agents know that the central bank would do so, so they form the expectation accordingly. The first order conditions are as follows.

$$\pi_t = \mu_t,$$
$$\lambda y_t = -\alpha \mu_t,$$

where μ is the Lagrange multiplier for the Phillips curve.[1] These can be arranged to

$$\pi_t = -\frac{\lambda}{\alpha}y_t. \tag{4}$$

This condition means that the central bank should keep the cost of changing 1%p of inflation, μ, equal to the benefit of changing 1%p of output gap, $(\lambda/\alpha)y_t$.

The second type of the solution is when the central bank can commit to a simple rule, which is defined as a policy reaction to (an) observable state variable(s), such as the one suggested by (Taylor 1993). However, the policy instrument is not the interest rate in this setup, and the only observable state variable at time t is the shock ϵ_t. So, the rule can be represented by endogenous variables, π_t and y_t being a function of the state variable ϵ_t such as

$$\pi_t = a\epsilon_t,$$

$$y_t = b\epsilon_t,$$

where a and b are undetermined coefficients. When solved together with equations (2) and (3), we get

$$\pi_t = -\frac{\lambda(1-\beta\rho)}{\alpha}y_t. \tag{5}$$

---

[1]For comparison with other solutions, I use the Lagrange method for the discretionary solution instead of the usual value function approach. Since there is no endogenous state variable, the Lagrange method is simpler.

This resembles conservative central banker argument by (Rogoff 1985). That is, when the central bank is optimizing the loss function with a higher-than-society weight on inflation, $1/\lambda(1-\beta) > 1/\lambda$, better outcome than the discretionary solution such as (4) can be attained.

The last case is when central banks can pre-commit to a policy without limiting to the case of a simple rule as described above. The first order conditions for this case[2] are

$$\pi_t = \mu_t - \mu_{t-1},$$

$$\lambda y_t = \alpha\mu_t,$$

which can be arranged to

$$\pi_t = -\frac{\lambda}{\alpha}(y_t - y_{t-1}). \tag{6}$$

This is similar to Equation (4) with only one difference. The right hand side is $y_t - y_{t-1}$ instead of y_t. Here μ_{t-1} is the Lagrange multiplier at time $t-1$ and also represents the cost at time t of keeping the promise made at time $t-1$. So if we modify the original optimization problem for the central bank to incorporate the cost of keeping the promise explicitly, it becomes the following optimization problem, and when solved as a discretionary solution, it produces exactly the same result as the original problem.

---

[2] The first order conditoin for the the first period is Equation(4).

$$\text{min} \quad \frac{1}{2}(1-\beta)E_t \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \lambda y_{t+i}^2] - \mu_{t-1}\pi_t, \tag{7}$$

$$\text{s.t.} \quad \pi_{t+i} = \beta E_{t+i}\pi_{t+i+1} + \alpha y_{t+i} + \epsilon_{t+i}, \tag{8}$$

$$\epsilon_{t+i} = \rho\epsilon_{t+i-1} + u_{t+i}. \tag{9}$$

Another thing to note is that Equation (6) is very closely related to price-level targeting. Equation (6) is modified as follows.

$$
\begin{aligned}
p_t - p_{t-1} &= -\frac{\lambda}{\alpha}(y_t - y_{t-1}), \\
p_t &= -\frac{\lambda}{\alpha}y_t.
\end{aligned}
$$

The better outcome turns out when central banks tie their hands in a well designed way.

2.2 Constrained Optimization

In this section, we introduce what we call 'a medium-term constraint.' The optimization problem for the central bank can be modified as follows.

$$\min \quad \frac{1}{2}(1-\beta)E_t \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \lambda y_{t+i}^2], \tag{10}$$

$$\text{s.t.} \quad \pi_{t+i} = \beta E_t \pi_{t+i+1} + \alpha y_{t+i} + \epsilon_{t+i}, \tag{11}$$

$$\epsilon_{t+i} = \rho \epsilon_{t+i-1} + u_{t+i}, \tag{12}$$

$$E_t \sum_{h=0}^{H-1} \pi_{t+h} = 0, \tag{13}$$

where Equation (13) is the medium-term constraint, which requires the central bank to keep the inflation over a certain horizon $H \geq 2$ to be on the target, which is normalized to zero for simplicity. Without this constraint, this optimization is exactly the same as the one studied in the previous section. The inclusion of this constraint can be justified by an accountability clause, often cited as another important element in the inflation targeting framework together with the announcement of the target level.[3] That is, if the central bank misses the pre-announced target, there will be some consequences. For example, the governor is to be fired, or he needs to write a letter to the finance minister explaining the reason of the miss. One must, however, determine what the target miss means. Usually the evaluation of the performance happens every year or so. However, for some central bankers, they are to be evaluated over more than a year. Often it ranges up to 2 to 5 years. One of the rationales for multi-period evaluation of the performance is that the monetary policy entails time lag. There is a time gap between the time of policy change and the time when the

---

[3]Svensson writes, in The New Palgrave Dictionary of Economics, that inflation targeting "is characterized by an announced numerical inflation target, an implementation of monetary policy that gives a major role to an inflation forecast and has been called 'inflation-forecast targeting', and a high degree of transparency and accountability."

effects finally take place.

This requirement that we impose on central banks when they minimize the loss function is slightly different from what central banks have to meet in practice. In practice, central bankers are to meet the target every year, or if it is a 'medium-term' target, the target sets over years. Usually they take form of an average of year on year inflation on calendar year.

It is worth noting that Equation (13) has two characteristics. The first is that the average of inflation is forward looking, unlike in (Nessen and Vestin 2005) who suggest a targeting framework that involves an average of inflation over a certain horizon in the past. It is well known that it is optimal for monetary policy to be inertial. However, we would not worry about problems from discretionary policy, if there was a technology to enforce central bankers to pursue a backward looking objective. The second is that the average of inflation is not based on a fixed calendar year. Instead, it is a moving average. That is, the constraint at time t is the average from t to $t + H - 1$, and the constraint at time $t + 1$ is the average from $t + 1$ to $t + H$. Although average inflation for a calendar year may have some significance in practice, it is not clear what central banks would (should) do on the last day if the average up until that day is off the target.

The first order conditions for this optimization problem are as follows.

$$\pi_t + \mu_t + \nu = 0, \tag{14}$$

$$\lambda y_t - \alpha \mu_t = 0, \tag{15}$$

$$E_t \sum_{h=0}^{H-1} \pi_{t+h} = 0,$$

where ν is the Lagrange multiplier for Equation (13). Equation (14) means that the loss from changing inflation is the sum of the cost of deviating from the Phillips curve, μ_t, and the cost of deviating from the medium-term constraint, ν. To solve for ν, Equation (14) is to be averaged over t to $t + H - 1$, and we get

$$\nu = \frac{1}{H} \sum_{h=0}^{H-1} E_t \mu_{t+h}.$$

Here, we utilize $E_t \sum_{h=0}^{H-1} \pi_{t+h} = 0$. When this equation is put into Equation (15), we finally get the following condition.

$$\pi_t = -\frac{\lambda}{\alpha}\left(y_t - \frac{1}{H} \sum_{h=0}^{H-1} E_t y_{t+h}\right). \tag{16}$$

This equation can be compared to Equation (4). In Equation (4), the optimality is to equalize the changes in inflation and changes in output gap. In Equation (16), the output gap change is replaced with the output gap change normalized by the average of expected output gap over the horizon.

2.3   Comparison with Other Studies

In this section, I show how the optimization problem with the medium-term constraint can be compared to other optimization problems in the literature. First, I compare it with (Svensson 1999), and (Cecchetti and Kim 2005).

In order to make the comparison easier, let's first write down the determination of price level under medium-term targeting.

$$
\begin{aligned}
p_t \equiv \sum_{i=0}^{\infty} \pi_{t-i} &= -\frac{\lambda}{\alpha} \sum_{i=0}^{\infty} \left(y_{t-i} - \frac{1}{H} \sum_{h=0}^{H-1} E_{t-i} y_{t-i+h} \right), \\
&= -\frac{\lambda}{H\alpha} \sum_{i=0}^{\infty} \sum_{h=0}^{H-1} \left(y_{t-i} - E_{t-i} y_{t-i+h} \right), \\
&= \frac{\lambda}{H\alpha} \sum_{j=1}^{H-1} \sum_{h=1}^{H-j} E_{t-j+1} y_{t+h} - \frac{\lambda}{H\alpha} \sum_{i=0}^{\infty} \sum_{h=1}^{H-1} \left(y_{t-i} - E_{t-i-h} y_{t-i} \right). \quad (17)
\end{aligned}
$$

The first term of Equation (17) is the sum of the expected future output gaps. The second term is the sum of the expectation errors. If we ignore the second term for now since expectation errors disappear if we assume certainty of future output gaps, this equation becomes the relationship between price level and the sum of expected future output gap. Equation (17) becomes trivial when H is 1, that is, $p_t = 0$ for all time, so we focus on the cases where H is greater than 2. When H is 2, the first term of the equation becomes the following.

$$
p_t = \frac{\lambda}{2\alpha} y_{t+1}. \quad (18)
$$

14

This resembles that of (Svensson 1999), who suggests the price level targeting as follows.

$$p_t - p_t^* = -by_t, \tag{19}$$

where b are functions of parameters. One of the most important differences between Equation (19) and Equation (18) is that while Equation (19) relates the price level and the current output gap, Equation (18) relates the price level and the *future* output gap. However, (Svensson 1999) assumes a New Classical Phillips curve, in which inflation expectation is $E_{t-1}\pi_t$, not $E_t\pi_{t+1}$ as in the New Keynesian Phillips curve. To make the comparison more meaningful, let's derive the optimization problem for the price level targeting with the New Keynesian Phillips curve, then compare the results.

$$
\begin{aligned}
\min \quad & \frac{1}{2}(1-\beta)E_t\sum_{i=0}^{\infty}\beta^i[p_{t+i}^2 + \lambda y_{t+i}^2], \\
\text{s.t.} \quad & (p_{t+i} - p_{t+i-1}) = \beta E_t(p_{t+i+1} - p_{t+i}) + \alpha y_{t+i} + \epsilon_{t+i}, \\
& \epsilon_{t+i} = \rho\epsilon_{t+i-1} + u_{t+i}.
\end{aligned}
$$

The first order condition[4] can be expressed as

---

[4]Unlike inflation targeting, now p_{t-1} is another state variable, so we cannot use simple Lagrangian to solve this problem. Instead, we have to use quadratic value function and undetermined coefficient. See (Kiley 1998) for similar solution.

$$p_t = -kE_t y_{t+1}, \tag{20}$$

where k is a function of parameters. Equation (20) seems very close in appearance to Equation (17) or (18), in that the price level is related with the future output gap.[5]

A hybrid of the price level targeting and inflation targeting suggested in (Cecchetti and Kim 2005) also bears resemblance to our results. Their first order condition is as follows.

$$p_t = \eta p_{t-1} - dy_t, \tag{21}$$

where $0 < \eta < 1$ is the degree of hybrid targeting, where $\eta = 1$ means inflation targeting, while $\eta = 0$ means price level targeting, and d is a function of parameters. When arranged, Equation (21) can be written as follows.

$$p_t = -d \sum_{i=0}^{\infty} \eta^i y_{t-i}. \tag{22}$$

Now we can compare this with our result. In our result, if H is anywhere between 2 to infinity, say 10, (17) becomes

[5]New Keynesian Philips curve implies that under price level targeting, inflation should be followed by deflation, so price change at time t would have an impact on future output gap.

$$p_t = \frac{\lambda}{10\alpha} \sum_{j=1}^{9} \sum_{h=1}^{10-j} E_{t-j+1} y_{t+h}. \tag{23}$$

Again, the second term of Equation (17) is ignored. Equations (22) and (23) are very similar except that in Equation (22) the output gaps are averaged over the past, where as in Equation (23) the output gaps are averaged over the future. This difference is due to the difference in the Phillips curve assumed in each paper. It is easy to see that when H goes infinite, Equation (16) becomes pure inflation targeting since $(1/H) \sum_{h=0}^{H-1} y_{t+h} \to 0$ as $H \to \infty$.

$$\pi_t = -\frac{\lambda}{\alpha} y_t.$$

Although this paper suggests a framework that encompasses previous studies, there are some others that contrast with this paper on the issues such as the definitions of horizon, solution methods or time consistency. (Smets 2003) analyzes the optimal horizon in a slightly different framework and with a different solution method as well. In comparison to the average inflation over a horizon as in this paper, the horizon in his paper is defined as the one over which inflation is brought back to target. That is, instead of $\sum_{h=0}^{H-1} E_t \pi_{t+h} = \pi^*$, the constraint regarding the horizon H is $E_t \pi_{t+H} = \pi^*$. This contraint, together with an assumption that the central bank can commit to a rule, requires different solution method than in this paper. Since the H is a fixed point in time in his case, this becomes a non-recursive optimization problem. That

is, the public, believing that the central bank will stick to bringing inflation back to target by the last period of the horizon, form iterated expectations backward from the last period of the horizon. Using (Marcet and Marimon 1998) this problem can be converted into a recursive one, and can be solved with a conventional solution method. Also, even though this definition of horizon is in line with (Batini and Nelson 2001), as in many accounts of inflation targeting practice, the horizons in many countries range 1 to 3 years, whereas policy decisions are made at least quarterly or even monthly, especially when it is branded with 'medium term'. This leaves central banks with multi-period intra-horizon decision intervals. Since it would be hard to justify that inflation at the end of the year is more important than any other time of the year, central banks take into account inflation not just at a certain point in time but rather average inflation over a certain 'horizon' when they make decisions. In this regard, (Nessen and Vestin 2005) is closer to this paper. As mentioned above, however, history dependence from averaging over past inflation implies commitment by central banks.

Another difference of this paper with (Smets 2003) is on the time consistency. If the central banker's problem is solved under the assumption of commitment, time inconsistency can be avoided by assumption. Whenever there is no credible commitment mechnism present, time inconsistency becomes an issue. This issue is elaborated in (Leitemo 2003), in which time consistency problem can occur if inflation forecast horizon and policy horizon, the horizon over which interest rate is held constant, do not coincide. That is, policy stance should change as the policy horizon rolls forward within the inflation forecast horizon. Although the horizon in our framework also rolls forward and therefore the time inconsistency problem does occur, credibility of the central bank may not be in danger. As the horizon rolls forward an averge infla-

tion forecasts over the horizon varies more smoothly than a single inflation forecast. Therefore, under our framework, the policy changes would be due more to new shocks than to rolling horizon. If the horizon becomes infinity, time inconsistecy will be no long an issue as this case is pure inflation targeting under descretion.

3 Optimal Horizon

3.1 Derivation of Optimal Horizon

Before we proceed to the derivation of optimal horizon, we need to find the final solution. That is, we need to write endogenous variables, π_t and y_t as functions of a state variable, ϵ_t. If we write $y_t = b\epsilon_t$, where b is undetermined coefficient, then

$$\pi_t = -\frac{\lambda}{\alpha} b (\epsilon_t - \frac{1}{H} \sum_{h=0}^{H-1} \epsilon_{t+h}).$$

If we use Equations (2) and (3), this can be written as follows.

$$y_t = -\frac{1}{\frac{\lambda}{\alpha}(1 - \frac{1-\rho^H}{H(1-\rho)})(1 - \beta\rho) + \alpha} \epsilon_t, \tag{24}$$

$$\pi_t = \frac{\frac{\lambda}{\alpha}(1 - \frac{1-\rho^H}{H(1-\rho)})}{\frac{\lambda}{\alpha}(1 - \frac{1-\rho^H}{H(1-\rho)})(1 - \beta\rho) + \alpha} \epsilon_t. \tag{25}$$

To find an optimal horizon, in which the loss is minimized, the loss needs to be expressed in terms of H.

19

$$(1-\beta)E_t \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \lambda y_{t+i}^2] \quad \simeq \quad \mathrm{Var}\pi_t + \lambda \mathrm{Var}y_t,$$

$$= \quad \frac{[\frac{\lambda}{\alpha}(1 - \frac{1-\rho^H}{H(1-\rho)})]^2 + \lambda}{[\frac{\lambda}{\alpha}(1 - \frac{1-\rho^H}{H(1-\rho)})(1-\beta\rho) + \alpha]^2} \mathrm{Var}\epsilon_t.$$

So the loss minimizing H can be found by taking a derivative of the loss with respect to H.

$$\frac{dLoss}{dH} = 0,$$

and we get the following first order condition.

$$\frac{1-\rho^H}{H} = \beta\rho(1-\rho). \tag{26}$$

Since this expression does not have a closed form solution for H with respect to ρ, the persistence of an exogenous shock ϵ, we derive the optimal horizon H by numerical approximation. Figure 1 is the optimal horizon for different values of ρ.

That is, the more persistent a shock is, the shorter the horizon should be. This is because high persistence of shocks means longer deviation of inflation and output from the long-run equilibrium. So as to lower the cost of deviation from the long-run equilibrium, central banks should be more active in reverting inflation back to 'normal' by having a short horizon.

20

This interpretation is quite in line with (King 1999), who relates horizon and a choice of targeting regime between inflation targeting and price level targeting. He lays out the following relation.

$$\pi_t^{**} = \pi^* + \frac{1}{H}(\frac{p_t - p_t^*}{p_t^*}),$$

where π_t^{**} is the inflation central banks target at time t, π^* is the slope of the desired predetermined price path, and p_t^* is the level of that path at time t. So $1/H$ acts as the speed of adjustment of the price level towards the desired path. If the horizon is 1, the regime is the price level targeting, under which central banks target the price level right on the desired path every period. And if the horizon goes infinite, it is the inflation targeting regime, under which the price level is never reverted to the path. In other words, 'bygones is bygones.'

The results of this paper, that the optimal horizon is inversely related with the persistence of the shock to inflation, can be understood by putting together the definition of horizon by (King 1999) and the analysis from (Cecchetti and Kim 2005), who argue that the more persistence the shock to inflation is, the closer the optimal targeting regime becomes to price level targeting.

$$\eta^* = \frac{1 - \rho}{2\rho}.$$

Also, if we relate Equations (16) and (5), we have the relationship between the

horizon and coservativeness of central bankers in the spirit of (Rogoff 1985). These results are summarized in Table 1.

3.2   Impulse Responses and Horizon

Figure 2 (a) and (b) show theoretical impulse responses of output and inflation to a cost push shock with different horizons, respectively. Parameters are well within ranges from other studies, and changes in them would not make much difference in the shapes of impulse responses, even if chosen differently.

As the horizon, H, becomes longer, the impulse reponse of output gets smaller in absolute value and that of inflation gets larger. This implies that the longer horizon allows inflation to vary more. This implication bears on welfare analysis in next section.

3.3   Welfare Analysis

So far we have analyzed how we can set up an optimization problem for central bankers without any modification of the objective function by adding a constraint, which represents accountability of central banks. And then we derive the optimal horizon, which is the horizon that minimizes the loss. In this section, we show that under the optimal horizon, the solution is the second best as in (Rogoff 1985). As in 2.1 endogenous variables, π_t and y_t can be written as functions of the only state variable, ϵ_t.

$$\pi_t = a\epsilon_t,$$

$$y_t = b\epsilon_t.$$

Using this as a constraint, we can solve the optimization problem for central banks and find the following first order condition.

$$\pi_t = -\frac{\alpha}{\lambda(1 - \beta\rho)} y_t. \tag{27}$$

As (Clarida, Gali and Gertler 1999) show, this is in line with (Rogoff 1985). That is, in order to avoid inflation bias from discretion, appointing a Governor whose weight on inflation stabilization, $1/[1-\lambda(1-\beta\rho)]$, is greater than that of the society, $1/(1-\lambda)$. This result is the second best result since it is inferior to the result from commitment without qualification, yet better than that of discretion. In order to find a condition under which the medium-term targeting suggested in this paper becomes the second best, we need to solve for a and b, and equate these solutions to Equations (14) and (15).

Using the solved a and b, we can attain the following expressions. [6]

---

[6]See (Clarida et el. 1999) for derivation.

$$\pi_t = \frac{\lambda(1-\beta\rho)}{\lambda(1-\beta\rho)+\alpha^2}\epsilon_t,$$

$$y_t = -\frac{\alpha}{\lambda(1-\beta\rho)+\alpha^2}\epsilon_t.$$

When compared with Equations (14) and (15), it is easy to find the following condition.

$$1-\beta\rho = 1 - \frac{1-\rho^H}{(1-\rho)H}. \tag{28}$$

Here, the right hand side is the degree to which the current output gap is normalized by the average of the expected future output gap. Therefore, this condition implies that the degree of conservativeness, $1-\beta\rho$, and the degree of normalization by the future output gap are equivalent in achieving the better outcome. In addition, it is noteworthy that this condition is exactly the same as Equation (26), which means that if the horizon is optimally chosen, medium-term targeting yields the second best result.

This can be seen in Figure 3. The loss when the horizon is infinite, which can be thought of as inflation targeting, is denoted by A. The loss when the horizon is 2, which is very close to price level targeting, is denoted by B. And the loss when the horizon is optimal, which means that H satisfies Equation (28), is denoted by C. Therefore the gap, $A - C$, or $B - C$, is the gains from constrained discretion when the horizon is chosen optimally.

Table 2 compares the losses of unconstrained commitment case, short horizon close to price level targeting ($H = 2$), inflation targeting ($H = \infty$), and constrained disrection case with optimal horizon ($H = H^*$) under different sets of parameters. First, we can see the loss from unconstrained commitment case is much lower than any other cases. This implies that there will be significant welfare gains if the central bank can commit to a rule credibly. As discussed above, however, central banks in reality are rarely able to commit to a rule, so we rule out this case as inpractical. Second, it is clear that as ρ, persistence of shock, increases, gains from having shorter horizon becomes greater. For example, the gain from having optimal horizon when $\rho = 0.8$, is about 40 to 70% with respect to inflation targeting, while it is less than 1% when $\rho = 0.2$. The gain from having optimal horizon, when it is compared to price level targeting, persistence of shock does not affect too much on the level of gains from having an optimal horizon.

It is also interesting to see that this finding shed some light on the claim by (Cecchetti and Kim 2005) that when there is an uncertainty about the persistence of shock, price level targeting is safer than inflation targeting since the gains foregone by not choosing the horizon optimally is much less for price level targeting than inflation targeting.

4   Conclusions

This paper suggests a framework of monetary policy, inflation targeting in particular, as constrained discretion. The contribution of this paper is three folds. First, unlike previous research on cures for the problem from discretionary monetary policy, this

paper suggests a way of formulating the monetary policy framework without distorting central banker's objective. This implies, first that a cure for discretionary monetary policy does not have to be a distortion of central banker's incentive, which in itself invites unnecessary criticism. Second, there is a way, which may have been already practiced, to overcome discretionary monetary policy other than committing to a rule, which is very difficult to implement in practice. That is, central banks need to set up a horizon over which the price level should be brought back to the original path in a way that reflects accountability clause, if any, of delegation of monetary policy.

The third contribution of this paper is that this paper explicitly models the horizon in a simple way, and shows the relationship between the horizon and the conservativeness of a central bank, and the relationship between the horizon and the choice between inflation targeting and price-level targeting. This will help central bankers understand the implication of the horizon they set. For this end, this paper provides the simple way to test the implication of the horizon changes.

For future research, more close attention must be paid to the central banker's optimization problem. Setting up a horizon over which the price level should be brought back to the original path inevitably involves the interplay of discretion and commitment. This interplay will be reflected in the behavior of a central bank from inter-period optimization as well as intra-period optimization. For example, if a central bank decided to bring back the price level to the original path, say over a year, it needs to be clear as to what needs to be done after one year, and every month or even every day within the year.

References

Barro, Robert J., and David B. Gordon. (1983) "Rules, Discretion and Reputation in a Model of Monetary Policy." *Journal of Monetary Economics*, 12:1, 101-21.

Batini, Nicoletta, and Edward Nelson. (2001) "Optimal Horizons for Inflation Targeting." *Journal of Economic Dynamics and Control*, 25:6-7, 891-910.

Bernanke, Ben S., and Frederic S. Mishkin. (1997) "Inflation Targeting: A New Framework for Monetary Policy?." *Journal of Economic Perspectives*, 11:2, 97-116.

Calvo, Guillermo A. (1983) "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics*, 12:3, 383-98.

Cecchetti, G. Stephen, and Junhan Kim. (2005) "Inflation Targeting, Price-Path Targeting and Output Variability." In *Inflation Targeting Debate*, edited by Ben Bernanke and Michael Woodford, pp. 173-200. Chicago: University of Chicago Press.

Clarida, Richard, Jordi Gali, and Mark Gertler. (1999) "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature*, 37:4, 1661-1707.

Kiley, Michael T. (1998) "Monetary policy under Neoclassical and New-Keynesian Phillips Curves, with an Application to Price Level and Inflation Targeting." *Finance and Economics Discussion Series* 1998-27, Board of

Governors of the Federal Reserve System (U.S.).

King, Mervyn. (1999) "Challenges for Monetary Policy : New and Old." *Proceedings*, Federal Reserve Bank of Kansas City, pp. 11-57.

Kydland, Finn E., and Edward C. Prescott. (1977) "Rules Rather Than Discretion: The Inconsistency of Optimal Plans." *Journal of Political Economy*, 85:3, 473-91.

Leitemo, Kai. (2003) "Targeting Inflation by Constant-Interest-Rate Forecasts." *Journal of Money, Credit and Banking*, 35:4, 609-26.

Mankiw, Grogory. (2005) Comments on "Inflation Targeting, Price-Path Targeting and Output Variability." In *Inflation Targeting Debate*, edited by Ben Bernanke and Michael Woodford, 195-99. Chicago: University of Chicago Press.

Marcet, A., and R. Marimon. (1998) "Recursive Contracts." *Economics Working Papers* 337, Department of Economics and Business, Universitat Pompeu Fabra, revised Oct 1998.

Nessen, Marianne, and David Vestin. (2005) "Average Inflation Targeting." *Journal of Money, Credit and Banking*, 37:5, 837-63.

Rogoff, Kenneth. (1985) "The Optimal Degree of Commitment to an Intermediate Monetary Target." *The Quarterly Journal of Economics*, 100:4, 1169-89.

Rotemberg, Julio J., and Michael Woodford. (1997) "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy."

NBER Macroeconomics Annual, 12, 297-346.

Smets, Frank. (2003) "Maintaining Price Stability: How Long is the Medium Term?." *Journal of Monetary Economics*, 50:6, 1293-1309.

Svensson, Lars E.O. (1997) "Optimal Inflation Targets, 'Conservative' Central Banks, and Linear Inflation Contracts." *American Economic Review*, 87:1, 98-114.

Svensson, Lars E.O. (1999) "Price-Level Targeting versus Inflation Targeting: A Free Lunch?" *Journal of Money, Credit and Banking*, 31:3, 277-95.

Taylor, John B. (1993) "Discretion versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy*, 39, 195-214.

Walsh, Carl E. (1995) "Optimal Contracts for Central Bankers." *American Economic Review*, 85:1, 150-67.
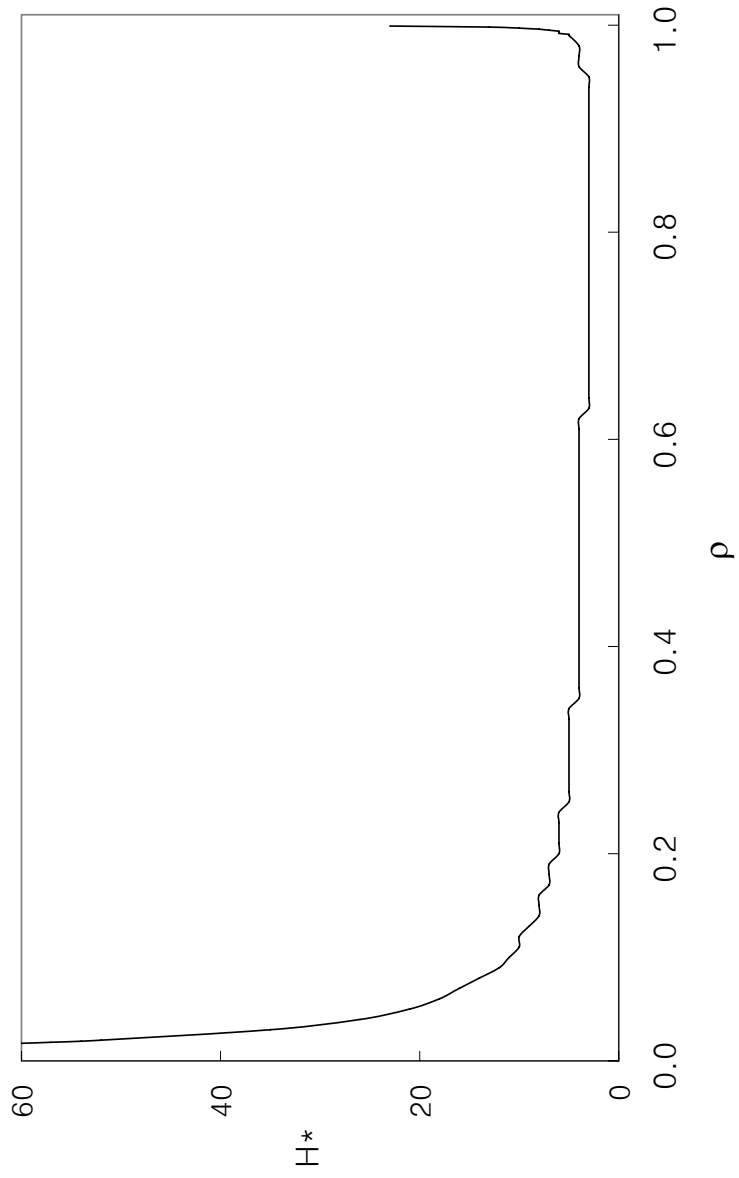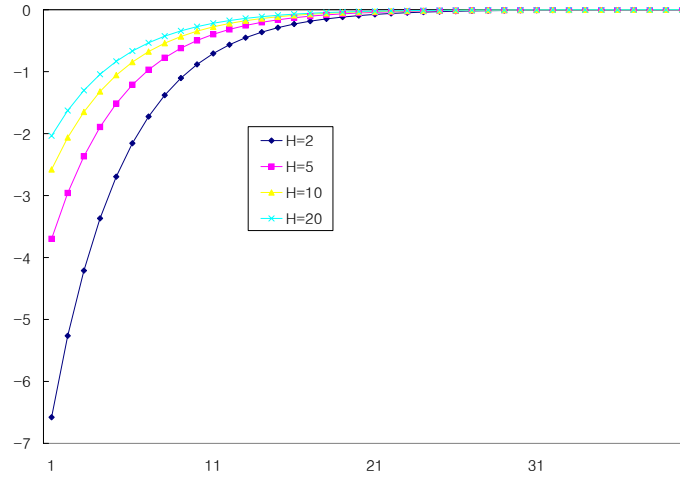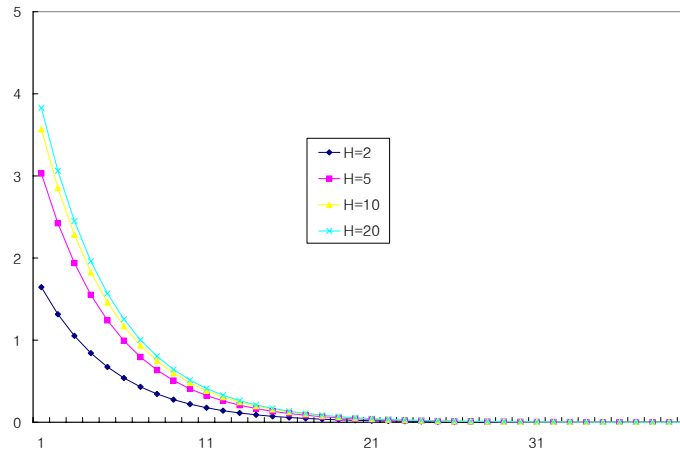
Figure 1: Optimal horizon and the persistence of a shock

(a) Impulse response of output

(b) Impulse response of inflation

Figure 2: Theoretical Impulse Responses to a Cost Push Shock (u_t) : We assume $\rho = 0.8$, $\alpha = 0.1$, $\beta = 0.99$, and $\lambda = 0.25$.
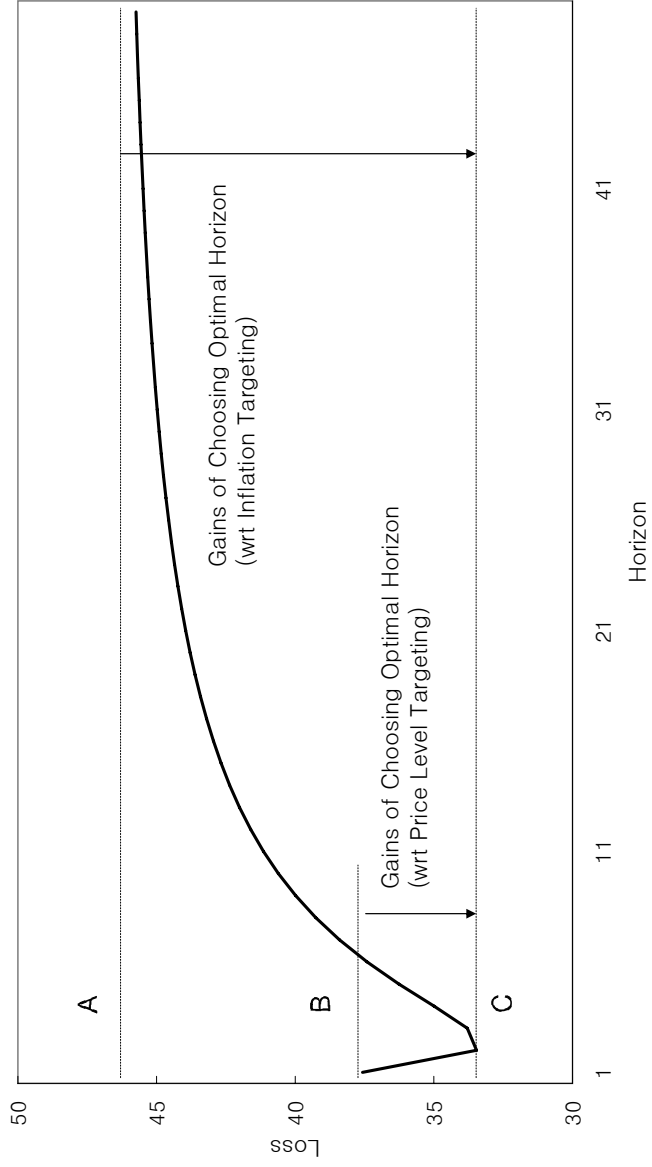
Figure 3: Gains from Constrained Discretion with Optimal Horizon: We assume $\rho = 0.8$, $\alpha = 0.1$, $\beta = 0.99$, and $\lambda = 0.25$

Persistence (ρ)	High	Low
Targeting Regime (η)	PLT	IT
Horizon (H)	Short	Long
Conservativeness (λ)	High	Low

Table 1: Summary of the related literature

Parameters		Optimal	Unconstrained	Discretion			Loss Gain Over (%)		
ρ	α	Horizon (H^*)	Commitment (a)	$H = 2$ (b)	$H = \infty$ (c)	$H = H^*$	(a)	(b)	(c)
				$\lambda = 0.25$					
0.2	0.1	6	1.230	1.600	1.528	1.525	-19.3	5.0	0.2
0.2	0.2	6	0.941	1.442	1.306	1.297	-27.4	11.2	0.7
0.5	0.1	4	3.154	4.945	4.668	4.521	-30.2	9.4	3.3
0.5	0.2	4	2.045	3.621	3.496	3.215	-36.4	12.6	8.8
0.8	0.1	3	19.214	37.572	46.915	33.459	-42.6	12.3	40.2
0.8	0.2	3	8.761	14.446	23.725	13.691	-36.0	5.5	73.3
				$\lambda = 0.75$					
0.2	0.1	6	1.386	1.617	1.588	1.587	-12.6	1.9	0.1
0.2	0.2	6	1.179	1.588	1.500	1.496	-21.2	6.2	0.3
0.5	0.1	4	3.893	5.190	5.029	4.969	-21.7	4.4	1.2
0.5	0.2	4	2.933	4.789	4.504	4.326	-32.2	10.7	4.1
0.8	0.1	3	30.127	55.631	57.432	49.189	-38.8	13.1	16.8
0.8	0.2	3	16.656	32.011	42.779	28.838	-42.2	11.0	48.3

Table 2: Loss Comparison